SPACE-TIME HYBRIDIZABLE DISCONTINUOUS GALERKIN IN MFEM

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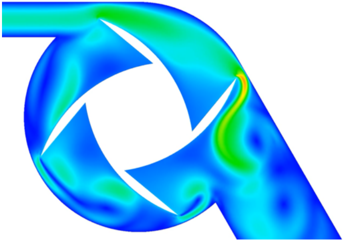
CONTENTS

Time-dependent Navier-Stokes equations

Time-dependent incompressible Navier-Stokes equation

$$\begin{aligned} \partial_t u + u \cdot \nabla u + \nabla p - \nu \nabla^2 u &= f \qquad \text{in } \Omega(t) \times [0, T], \\ \nabla \cdot u &= 0 \qquad \text{in } \Omega(t) \times [0, T], \\ &+ BC + IC \end{aligned}$$



Time-dependent Navier-Stokes equations

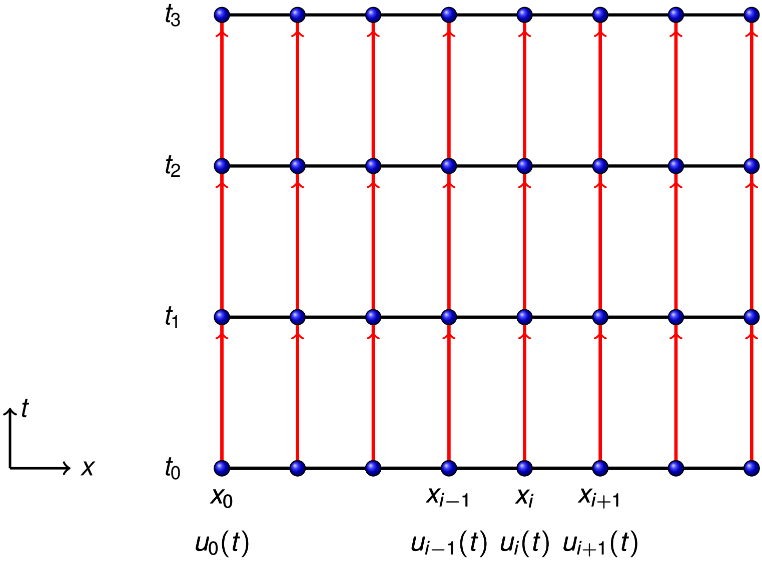
Easy if the domain Ω is fixed

Semidiscretization ($\partial_t u = \mathbf{S}u$) via your favorite spatial method

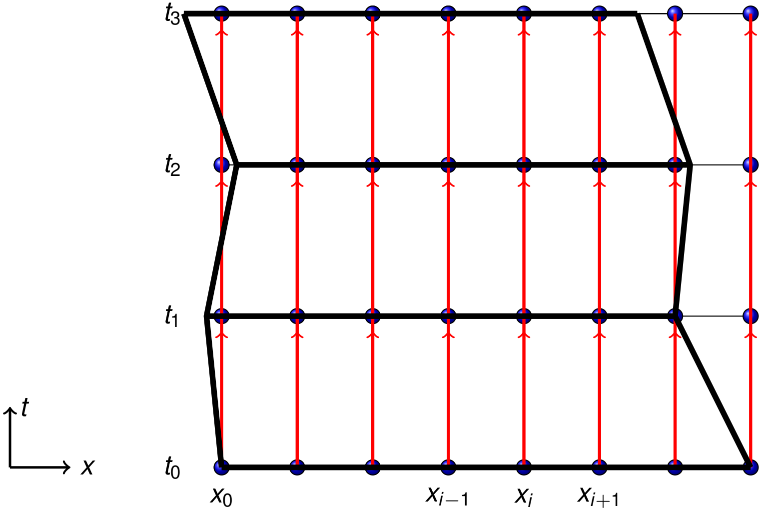
Use your favorite time stepping: Explicit/Implicit Euler, Runge-Kutta, BDF...

Parallelization is restricted by the time-stepping

Method of Lines in 1D



WHAT IF THE DOMAIN IS NOT FIXED?



ARBITRARY LAGRANGIAN EULERIAN VS SPACE-TIME

ARBITRARY LAGRANGIAN EULERIAN

Transform back to a fixed domain

Legacy codes can handle it!

SPACE-TIME (ST)

Transform to a higher dimensional problem

Arbitrary high order both in space and in time

Geometric Conservation Law: the uniform solution stays uniform even if the mesh is moving

Legacy codes can handle it ... if they can handle matrix diffusion

... and 4D meshes

APPLICATIONS

MOVING DOMAINS ARE EVERYWHERE

Blood flow in arteries

Airplanes, wind turbines

Helicopter blades

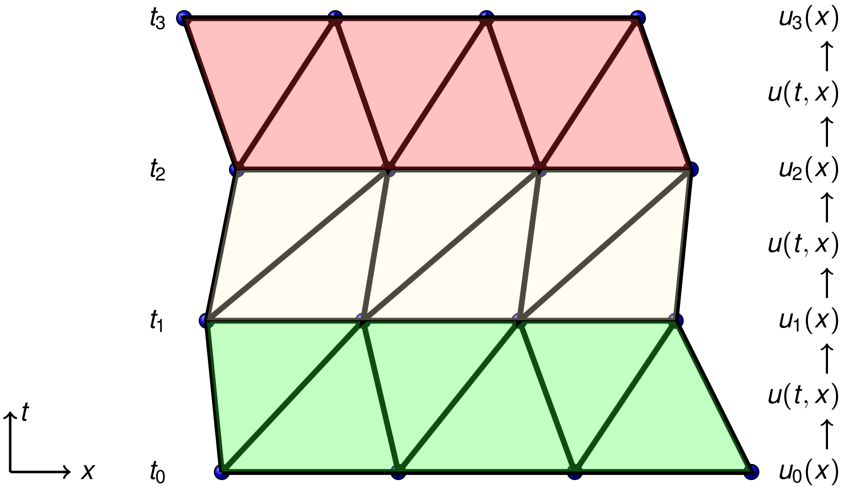
Submarine turbines

Car wheels

Free surface problems

MOTIVATION

UPWIND IN TIME



Space-time algorithm

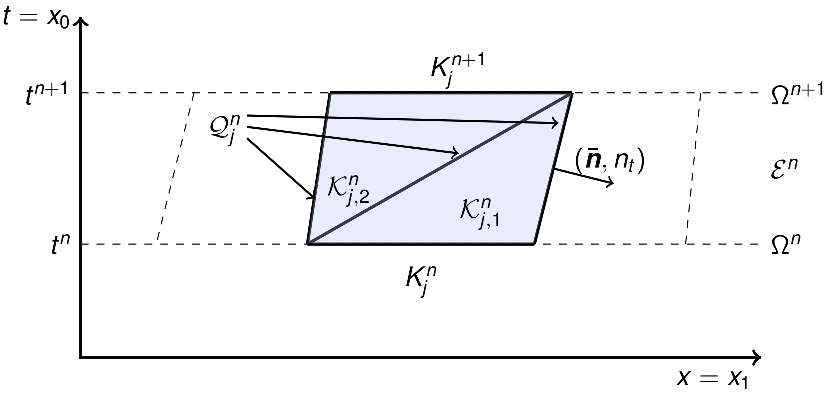
Space-time

Start with $i = 0$ and the initial condition $u(0, x) = u_0(x)$

- Generate a mesh of the domain $\Omega(t_i)$.
- Generate a mesh of the domain $\Omega(t_{i+1})$
- Generate a space-time mesh using these two meshes
- Solve inside the time slab and evaluate the solution at the upper time level
- Use this as an initial condition for the next time slab

All we need is u from the previous time slab

Space-time notations - simplices



How to discretize within the time slab?

Continuous Galerkin methods

Based on the weak form of the PDE

Do not work well for advection dominated problems

Stabilized Continuous Galerkin methods

Adding some stabilization terms for the CG formalism

Work well for advection dominated problems

Do not provide locally conservative solutions for Navier-Stokes

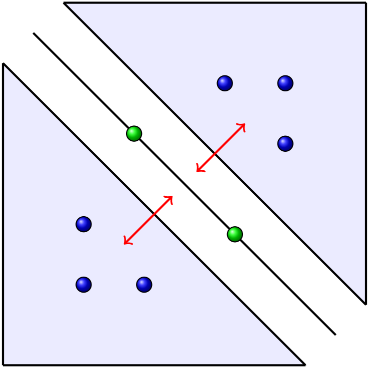
Discontinuous Galerkin methods

Work well for advection dominated problems

Provide locally conservative solutions

High number of unknowns (no shared unknowns between elements)

HDG



$$u_h \in V_h = \left\{ v_h \in L^2(\mathcal{E}^n),\ v_h \in P_k(\mathcal{K}), \forall \mathcal{K} \in \mathcal{T}^n \right\}$$

$$\overline{u}_h \in \overline{V}_h = \left\{ \overline{v}_h \in L^2(\Gamma^n), \overline{v}_h \in P_k(F), \forall F \in \mathcal{F}^n \right\}$$

SPACE-TIME HDG FOR 1D ADVECTION-DIFFUSION

Consider the 1D advection-diffusion equation:

$$\partial_t u + \nabla_x \cdot (\bar{\boldsymbol{a}} u) - \nabla_x \cdot (\nu \nabla_x u) = f, \quad \Omega(t) \subset \mathbb{R}, t \in (0, T)$$

On the space-time domain $\mathcal{E} := \{(\boldsymbol{x}, t) : \boldsymbol{x} \in \Omega(t),\ t \in (0, T)\}$:

$$\nabla_{x,t} \cdot (\boldsymbol{a} u) - \nabla_{x,t} \cdot (\tilde{\nu} \nabla_{x,t} u) = f, \quad \mathcal{E}, \quad \boldsymbol{a} = (\bar{\boldsymbol{a}}, 1), \tilde{\nu} = \begin{bmatrix} \nu & 0 \\ 0 & 0 \end{bmatrix}$$

Finite element spaces on one space-time slab $\mathcal{E}^n$ ($t \in [t_n, t_{n+1}]$):

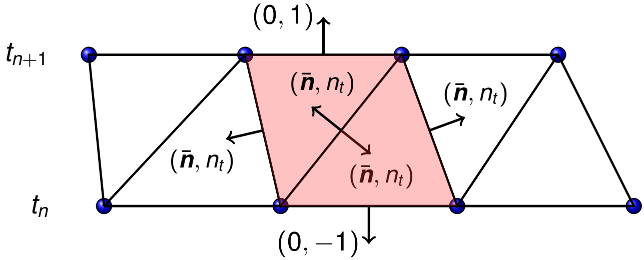
$$V_h := \left\{ v_h \in L^2(\mathcal{E}^n) : v_h \in P_k(\mathcal{K}), \forall \mathcal{K} \in \mathcal{T}^n \right\}$$
$$\overline{V}_h := \left\{ \bar{v}_h \in L^2(\Gamma^n) : \bar{v}_h \in P_k(F), \forall F \in \mathcal{F}^n \right\}$$

ADVECTION

Test with $v_h \in V_h$, apply integration by parts and sum over all elements:

$$-\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{K}}(\boldsymbol{a}u_h)\cdot\nabla_{x,t}v_h+\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\partial\mathcal{K}}\widehat{\boldsymbol{a}\cdot\boldsymbol{n}u_h}v_h=\int_{\mathcal{E}^n}fv_h.$$

How does the space-time normal $\boldsymbol{n}$ look like?



$$\boldsymbol{n}=\begin{cases}(0,1) & K^{n+1}\\ (0,-1) & K^n\\ (\bar{\boldsymbol{n}},n_t) & \mathcal{Q}_{\mathcal{K}}\end{cases}$$

Upwind numerical flux: If $\boldsymbol{a} \cdot \boldsymbol{n} > 0 \Rightarrow$ inside value; otherwise outside value:

- On K^{n+1} : $\boldsymbol{a} \cdot \boldsymbol{n} = (\bar{\boldsymbol{a}}, 1) \cdot (0, 1) = 1 > 0 \Rightarrow u_h$
- On K^n : $\boldsymbol{a} \cdot \boldsymbol{n} = (\bar{\boldsymbol{a}}, 1) \cdot (0, -1) = -1 < 0 \Rightarrow u_n^-$
- On $\mathcal{Q}_\mathcal{K}$: $\boldsymbol{a} \cdot \boldsymbol{n} = (\bar{\boldsymbol{a}}, 1) \cdot (\bar{\boldsymbol{n}}, n_t) = \bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t$, and

$$\widehat{\boldsymbol{a} \cdot \boldsymbol{n} u_h} = (\bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t)(u_h + \eta(\bar{u}_h - u_h)), \text{ with } \eta = \begin{cases} 0, & \bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t > 0 \\ 1, & \bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t < 0 \end{cases}$$

u_n^- is the initial condition or the solution from the previous space-time slab.

Thus,

$$\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\partial\mathcal{K}}\widehat{\boldsymbol{a}\cdot\boldsymbol{n}u_h}v_h = \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{K^{n+1}}u_h v_h - \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{K^n}u_n^- v_h + \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}_{\mathcal{K}}}(\bar{\boldsymbol{a}}\cdot\bar{\boldsymbol{n}}+n_t)(u_h+\eta(\bar{u}_h-u_h))v_h$$

Additional equation for $\bar{u}_h$: continuity of numerical flux in the normal direction across faces:

$$\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}_{\mathcal{K}}}(\bar{\boldsymbol{a}}\cdot\bar{\boldsymbol{n}}+n_t)(u_h+\eta(\bar{u}_h-u_h))\bar{v}_h = 0, \quad \forall\bar{v}_h\in M_h.$$

DIFFUSION

Test with $v_h \in V_h$, apply integration by parts and sum over all elements:

$$\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{K}} \tilde{\nu}\nabla_{x,t}u_h \cdot \nabla_{x,t}v_h - \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\partial\mathcal{K}} \tilde{\nu}\widehat{\nabla_{x,t}u_h} \cdot \boldsymbol{n} v_h.$$

Hybridized interior penalty

$$\tilde{\nu}\widehat{\nabla_{x,t}u_h} \cdot \boldsymbol{n} = \tilde{\nu}\nabla_{x,t}u_h \cdot \boldsymbol{n} + \frac{\alpha\tilde{\nu}}{h}(u_h - \bar{u}_h)\boldsymbol{n}\cdot\boldsymbol{n}$$

The flux is 0 on K^n and K^{n+1}. On $\mathcal{Q}_{\mathcal{K}}$:

$$\tilde{\nu}\boldsymbol{n}\cdot\boldsymbol{n} = \nu\bar{\boldsymbol{n}}\cdot\bar{\boldsymbol{n}}$$

SPACE-TIME HDG DISCRETIZATION FOR ADVECTION-DIFFUSION EQUATION

For $n = 0, 1, \ldots, N-1$: find $(u_h, \bar{u}_h) \in V_h \times M_h$ s.t., $\forall (v_h, \bar{v}_h) \in V_h \times M_h$:

$$
\begin{aligned}
&- \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{K}} (\boldsymbol{a} u_h) \cdot \nabla_{x,t} v_h + \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{K}} \tilde{\nu} \nabla_{x,t} u_h \cdot \nabla_{x,t} v_h + \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{K^{n+1}} u_h v_h \\
&\quad + \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{Q}_{\mathcal{K}}} (\bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t)(u_h + \eta(\bar{u}_h - u_h)) v_h \\
&+ \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{Q}_{\mathcal{K}}} \left(\tilde{\nu} \nabla_{x,t} u_h \cdot \boldsymbol{n} + \frac{\alpha \tilde{\nu}}{h} (u_h - \bar{u}_h) \boldsymbol{n} \cdot \boldsymbol{n} \right) v_h = \int_{\mathcal{E}^n} f v_h + \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{K^n} u_h^- v_h, \\
&\quad \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{Q}_{\mathcal{K}}} (\bar{\boldsymbol{a}} \cdot \bar{\boldsymbol{n}} + n_t)(u_h + \eta(\bar{u}_h - u_h)) \bar{v}_h \\
&\quad + \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{Q}_{\mathcal{K}}} \left(\tilde{\nu} \nabla_{x,t} u_h \cdot \boldsymbol{n} + \frac{\alpha \tilde{\nu}}{h} (u_h - \bar{u}_h) \boldsymbol{n} \cdot \boldsymbol{n} \right) \bar{v}_h = 0.
\end{aligned}
$$

STATIC CONDENSATION

Using U and $\overline{U}$ for the coefficient vectors

BLOCK SYSTEM

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U \\ \overline{U} \end{bmatrix} = \begin{bmatrix} F \\ G \end{bmatrix}$$

A IS BLOCK DIAGONAL

$$\begin{aligned} U &= A^{-1}(F - B\overline{U}) \\ CA^{-1}(F - B\bar{U}) + D\overline{U} &= G \end{aligned}$$

FINAL PROBLEM

$$\begin{aligned} (D - CA^{-1}B)\overline{U} &= G - CA^{-1}F \\ U &= A^{-1}(F - B\overline{U}) \end{aligned}$$

Typically, the number of DoFs is smaller for HDG than for DG

HDG branch on MFEM

Steady HDG

HDG advection: solves the steady advection-reaction equation

HDG Poisson: solves the Poisson problem using the hybridized LDG flux

Integrators add the volume terms and the faces terms as a one-shot integrator

Another approach: add an index to identify the sub-matrix (gets messy)

The code can be extended for problems involving more FESpaces

DG Assembly

Algorithm 1 DG Assembly loop

1: **loop** Over all elements
2: Calculate the volume integrals
3: **end loop**
4: **loop** Over all faces
5: Calculate the face integrals, add contributions to two neighboring elements
6: **end loop**
7: Solve the linear system

Looping over faces only once

HDG ASSEMBLY

Algorithm 2 HDG Assembly/Reconstruction

loop Over all elements
  Calculate the volume integrals
  loop Over all faces of the element
    Calculate the face integrals, add contributions to local matrices
  end loop
end loop
Invert A locally
Calculate Schur complement locally / Reconstruct u locally
Solve the linear system

Looping over interior faces twice - once for both neighboring elements

Schur complement can be calculated without storing A, B and C (storage vs time)

ISSUES & CHALLENGES

FACET SPACES IN MFEM

Norms: loop over elements, not faces

Gridfunction evaluations: are defined over element FESpaces

SPACE-TIME

We create only one mesh, then move the nodes

Diffusion integrators needs to be (slightly) modified

Hexahedra and wedges are easy, tetrahedra are a bit more technical

Hybridization for the Navier-Stokes

Unknowns: $\mathbf{u} = (u, \bar{u})$, $\mathbf{p} = (p, \bar{p})$ where $\bar{u}, \bar{p}$ are the facet unknowns

Finite dimensional spaces

$$u \in V_h := \left\{ v_h \in \left[L^2(\mathcal{E}^n) \right]^d, \ v_h \in \left[P_k(\mathcal{K}) \right]^d, \forall \mathcal{K} \in \mathcal{T}^n \right\}$$

$$\bar{u} \in \bar{V}_h := \left\{ \bar{v}_h \in \left[L^2(\Gamma^n) \right]^d, \ \bar{v}_h \in \left[P_k(F) \right]^d, \forall F \in \mathcal{F}^n, \ \bar{v}_h = 0 \text{ on } \Gamma_D \right\}$$

$$p \in Q_h := \left\{ q_h \in L^2(\mathcal{E}^n), \ q_h \in P_{k-1}(\mathcal{K}), \forall \mathcal{K} \in \mathcal{T}^n \right\}$$

$$\bar{p} \in \bar{Q}_h := \left\{ \bar{q}_h \in L^2(\Gamma^n), \ \bar{q}_h \in P_k(F), \forall F \in \mathcal{F} \right\}$$

HDG vs EDG vs EHDG

Hybridized DG - Discontinuous facet variables

$$\bar{u} \in \bar{V}_h, \qquad \bar{p} \in \bar{Q}_h$$

Embedded DG - Continuous facet variables

$$\bar{u} \in \bar{V}_h^* = \bar{V}_h \cap C(S), \qquad \bar{p} \in \bar{Q}_h^* = \bar{Q}_h \cap C(S)$$

Embedded-Hybridized DG - Continuous facet variables only for the velocity

$$\bar{u} \in \bar{V}_h^* = \bar{V}_h \cap C(S), \qquad \bar{p} \in \bar{Q}_h$$

DISCRETIZATION I

Viscous term: hybridized IP-DG discretization

Pressure terms: standard hybridization

BILINEAR FORMS

$$a_h^n(\mathbf{u},\mathbf{v}) := \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{K}} \nu\nabla u : \nabla v \,\mathrm{d}x + \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}_\mathcal{K}} \frac{\nu\alpha}{h_\mathcal{K}}(u-\bar{u})\cdot(v-\bar{v})\,\mathrm{d}s$$
$$- \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}_\mathcal{K}} \nu\left[(u-\bar{u})\cdot\nabla v n + \nabla u n\cdot(v-\bar{v})\right]\mathrm{d}s,$$
$$b_h^n(\mathbf{p},\mathbf{v}) := -\sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{K}} p\nabla\cdot v\,\mathrm{d}x + \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}_\mathcal{K}} (v-\bar{v})\cdot n\bar{p}\,\mathrm{d}s,$$

Discretization II

Convection: nonlinear upwinding

Trilinear form

$$t_h^n(\mathbf{w},\mathbf{u},\mathbf{v}) := \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{K^{n+1}} u\cdot v\,\mathrm{d}s + \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{Q}^n_{\mathcal{K}}} H(\mathbf{u},w;n_t,n)\cdot(v-\bar{v})\,\mathrm{d}s$$
$$+\int_{\partial\mathcal{E}^N\cap I_n}\max\left(n_t+\bar{w}\cdot n,0\right)\bar{u}\cdot\bar{v}\,\mathrm{d}s - \sum_{\mathcal{K}\in\mathcal{T}^n}\int_{\mathcal{K}}\left(u\partial_t v + u\otimes w:\nabla v\right)\mathrm{d}x,$$

Flux function - Space-time Upwind (NOT on the time levels)

$$H(\mathbf{u},w;n_t,n) = \tfrac{1}{2}\left(u+\bar{u}\right)\left(n_t+w\cdot n\right) + \tfrac{1}{2}\left(u-\bar{u}\right)\left|n_t+w\cdot n\right|.$$

SLAB-BY-SLAB APPROACH

IN SPACE-TIME SLAB n

Find $(u_h, \bar{u}_h, p_h, \bar{p}_h) \in V_h \times \bar{V}_h \times Q_h \times \bar{Q}_h$ such that

$$t_h^n(\mathbf{u}_h, \mathbf{u}_h, \mathbf{v}_h) + a_h^n(\mathbf{u}_h, \mathbf{v}_h) + b_h^n(\mathbf{p}_h, \mathbf{v}_h) - b_h^n(\mathbf{q}_h, \mathbf{u}_h) = \sum_{\mathcal{K} \in \mathcal{T}^n} \int_{\mathcal{K}} f \cdot v_h \, \mathrm{d}x - \int_{\partial \mathcal{E}^N \cap I_n} g \cdot \bar{v}_h \, \mathrm{d}s + \int_{\Omega_n} u_h^- \cdot v_h \, \mathrm{d}s$$

Nonlinearity - Picard Iteration

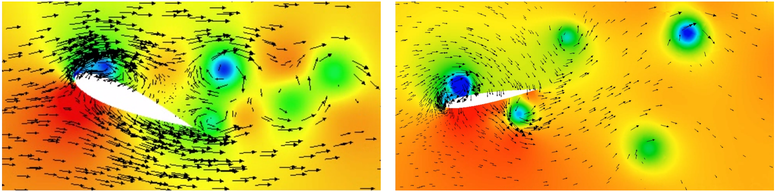
COMPARISON (ON TETRAHEDRA)

	ST-HDG*	ST-EDG†	ST-EHDG†
div-free velocity	✓	✓	✓
div-conforming velocity	✓	×	✓
energy-stable	✓	✓	✓
loc. mom. conserving	✓	×	✓
number of degrees-of-freedom	largest	smallest	significantly < ST-HDG slightly > ST-EDG

*T.L. Horvath and S. Rhebergen, A locally conservative and energy-stable finite element method for the Navier–Stokes problem on time-dependent domains, Int. J. Numer. Meth. Fluids, 89/12 (2019), pp 519-532.

†T.L. Horvath and S. Rhebergen, An exactly mass conserving space-time embedded-hybridized discontinuous Galerkin method for the Navier-Stokes equations on moving domains, J. Comp. Phys. 417 (2020)

Small domain deformation



LARGE DOMAIN DEFORMATION

Large domain deformation → mesh tangling

TYPICAL SOLUTION

Remeshing the domain

Meshing is expensive

Projection of the solution from the old to the new mesh

Expensive and maybe suboptimal rates

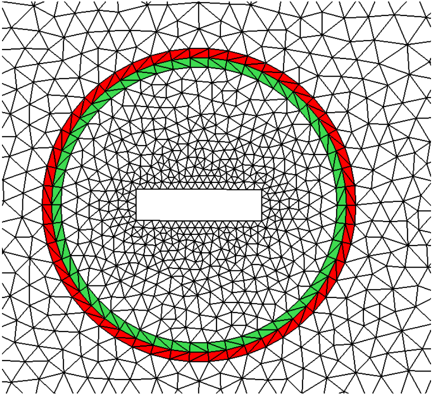
NEW IDEA

Tetrahedra: allow to connect different meshes

Edge flipping (not new)

With precomputable mesh elements (new)

SLIDING MESH GENERATION



Inner region rotates with the object

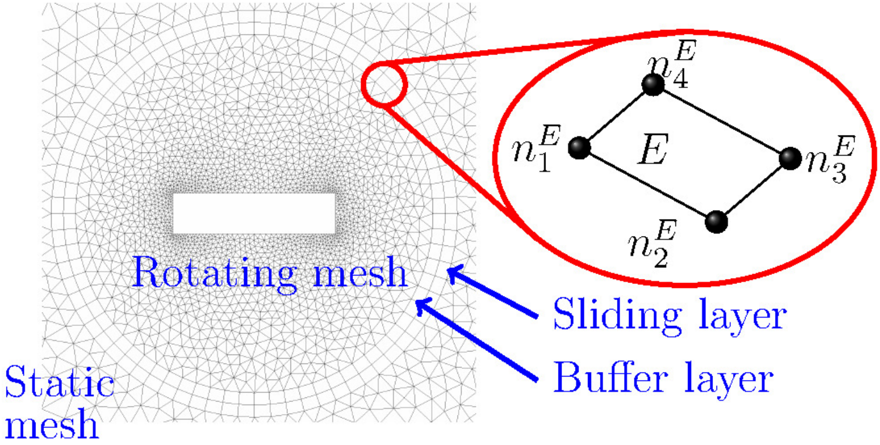
Outer region does not change

In between: sliding layer

Buffer layer allows precomputable meshes

"Build your own mesh"

INITIAL MESH[‡]

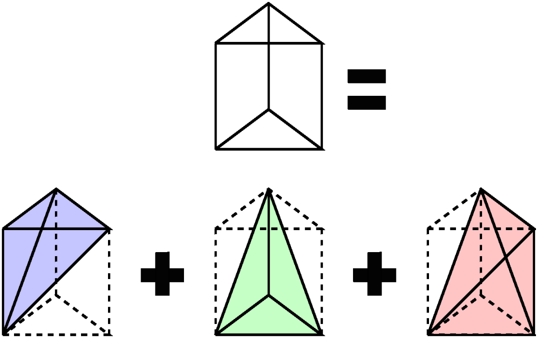


[‡] **T.L. Horvath** and S. Rhebergen: A conforming sliding mesh technique for an embedded-hybridized discontinuous Galerkin discretization for fluid-rigid body interaction (2021), arxiv.org

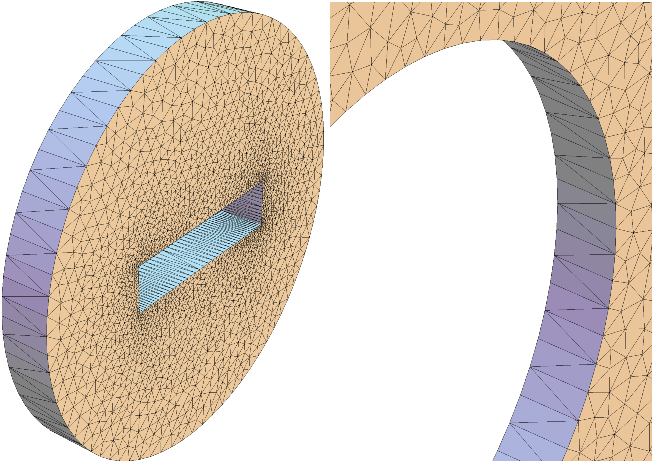
TETRAHEDRAL MESH GENERATION

If no edge flipping: extend to prisms and cut each prism into 3 tetrahedra

Use the diagonal with the smallest node identifier



Rotating and static mesh

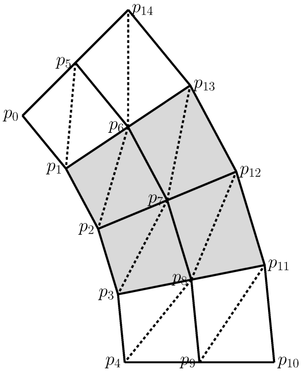


No edge flipping

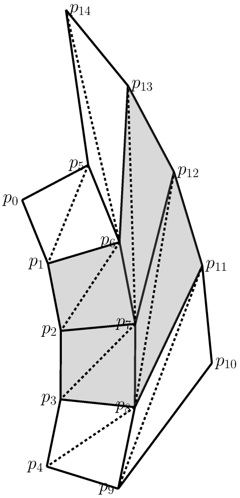
Triangles to tetrahedra

Using a special numbering on the circles: Chainsaw pattern

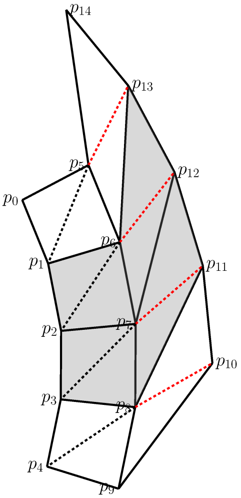
When do we need to flip an edge?



bottom

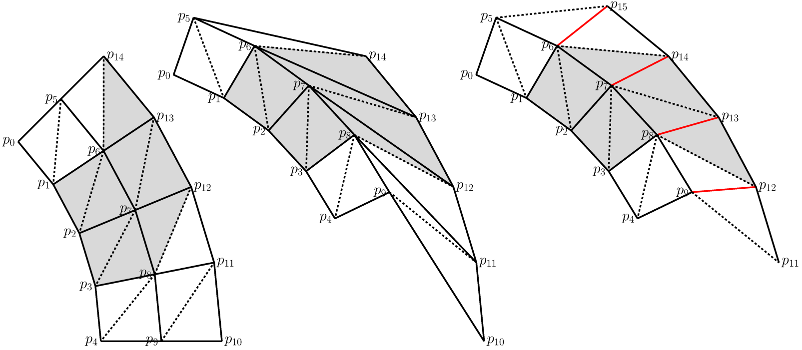


rotated (never used)



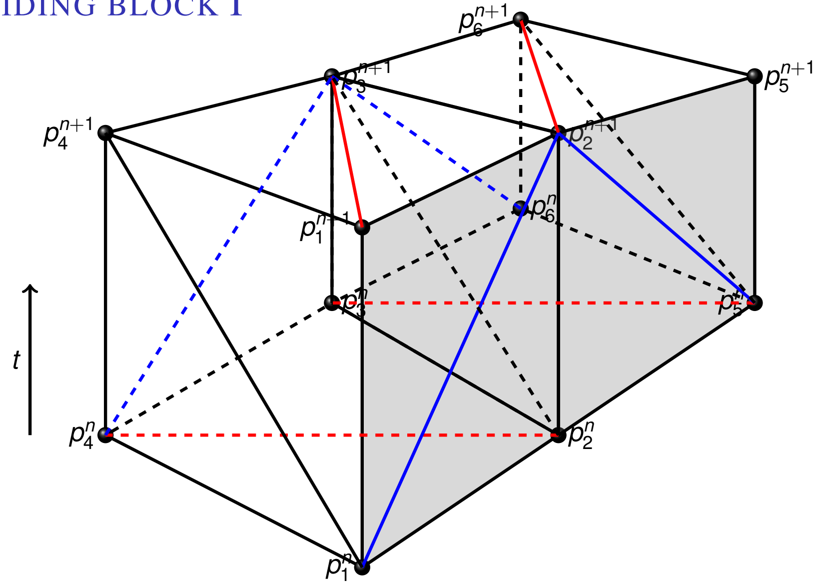
flipped edges

WHEN DO WE NEED TO FLIP AN EDGE?

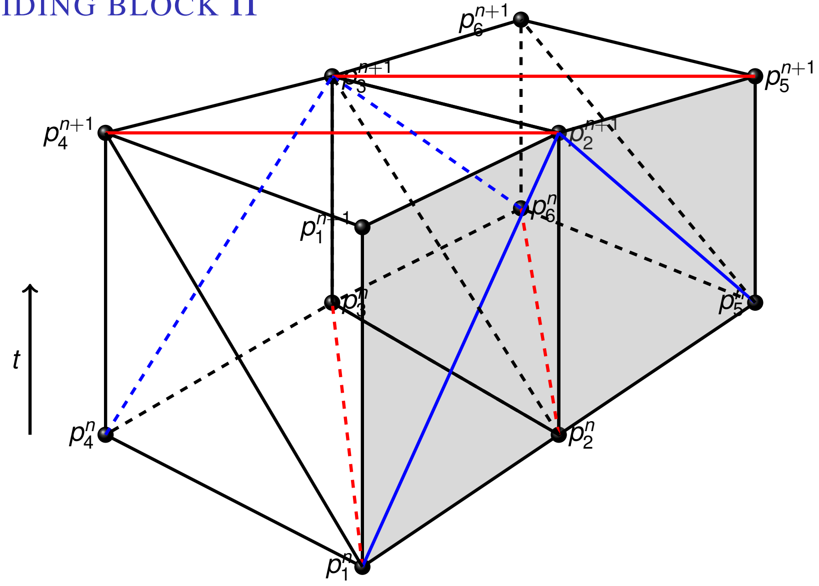


bottom — rotated (never used) — flipped edges

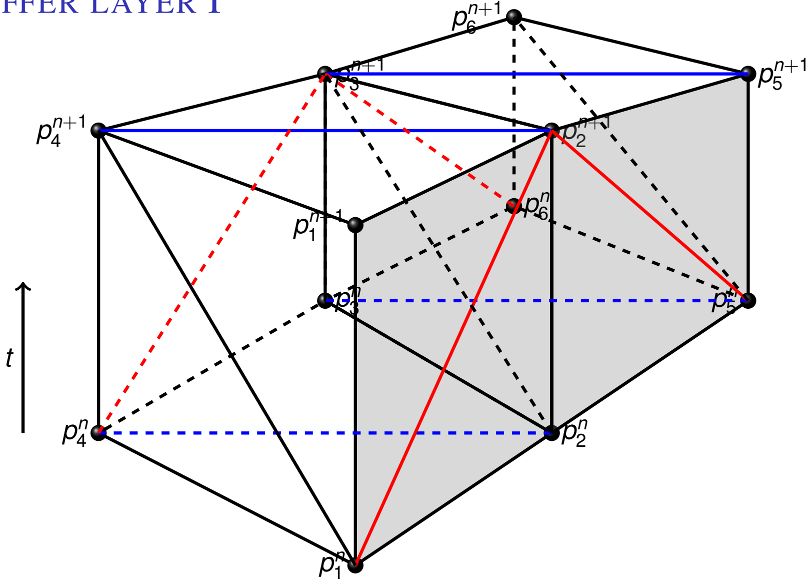
SLIDING BLOCK I



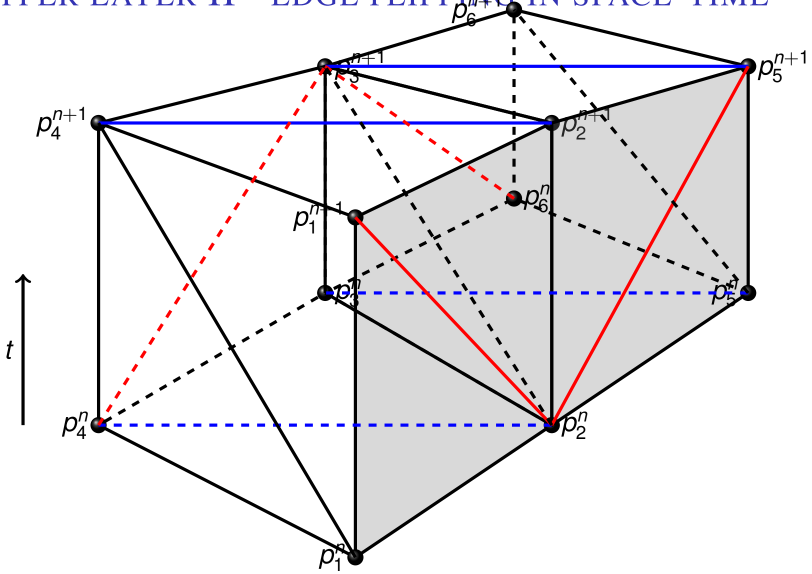
Sliding block II



BUFFER LAYER I



Buffer layer II - edge flipping in space-time



FLUID-RIGID BODY INTERACTIONS

TIME DEPENDENT NAVIER-STOKES

$$\begin{aligned} \partial_t u + u \cdot \nabla u + \nabla p - 2\nu \nabla \cdot \varepsilon(u) &= f \qquad \text{in } \Omega(t) \\ \nabla \cdot u &= 0 \qquad \text{in } \Omega(t) \end{aligned}$$

ST-EHDG

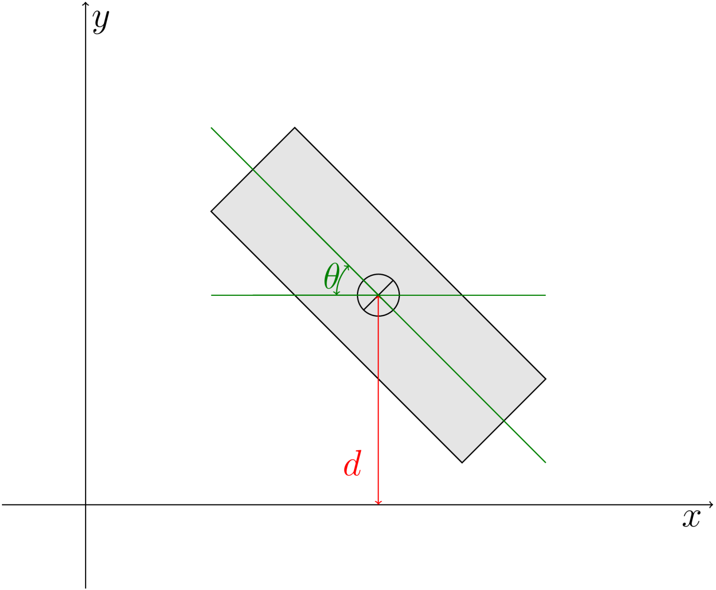
VERTICAL DISPLACEMENT AND ROTATION

$$\begin{aligned} m\ddot{d} + c_y \dot{d} + k_y d &= F_y, \\ I_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta &= M, \end{aligned}$$

where F_y is the lifting force, M is the pitching moment force

Predictor-corrector (BDF2)

VERTICAL DISPLACEMENT AND ROTATION



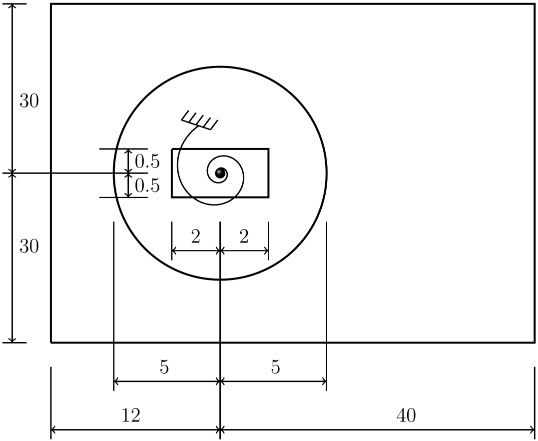
STAGGERED FRBI ALGORITHM

Algorithm 3 Staggered coupling for fluid-rigid body solver in space-time slab $\mathcal{E}_h^n$.

1: Predictor step to obtain an initial guess for the rigid body position
2: **while** Rigid body stopping criterion is not satisfied **do**
3: Update the flow domain and mesh $\mathcal{E}_h^n$
4: **while** Picard stopping criterion is not satisfied **do**
5: Solve Picard iteration to obtain the flow solution $(u_h^{k+1}, \bar{u}_h^{k+1}, p_h^{k+1}, \bar{p}_h^{k+1})$
6: **end while**
7: Set $(u_h, \bar{u}_h, p_h, \bar{p}_h) = (u_h^{k+1}, \bar{u}_h^{k+1}, p_h^{k+1}, \bar{p}_h^{k+1})$
8: Corrector step to update the rigid body position
9: **end while**

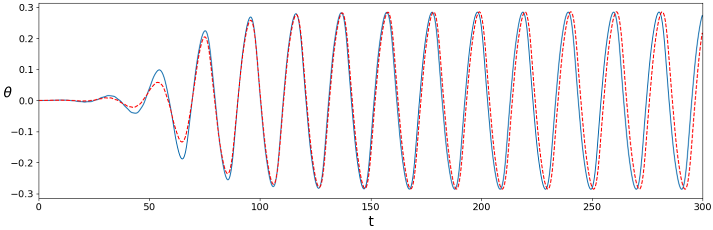
Galloping rectangle

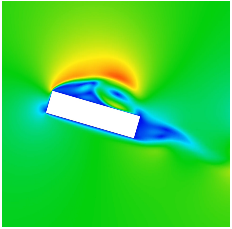
Rectangle with aspect ratio $A = 4$, $u_{in} = [2.5, 0]$



Galloping rectangle

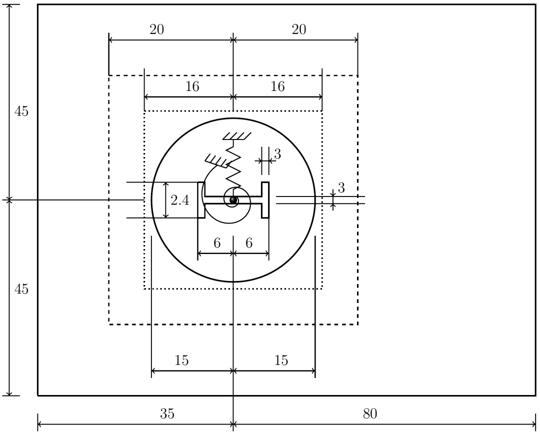
$$\max |\theta| = 0.28$$





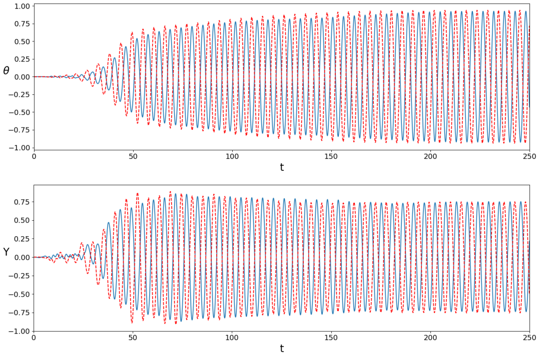
Fluttering bridge

Rotation and vertical displacement, $u_{in} = [10, 0]$



Fluttering bridge

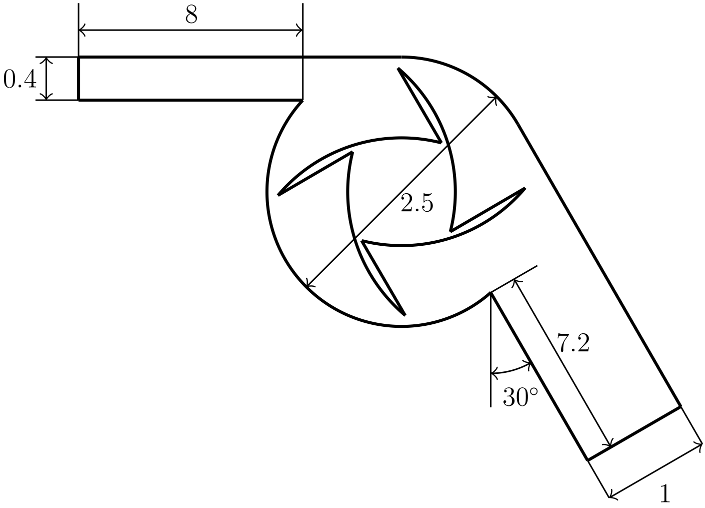
$$\max|\theta| = 0.92, \quad \max|Y| = 0.75$$



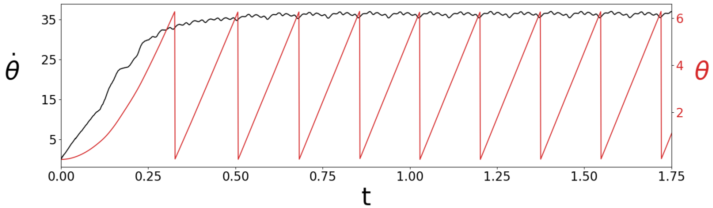
Video

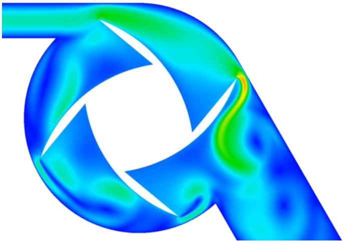
Rotating turbine

- Parabolic inflow, $\max |u_{in}| = 50$



ROTATING TURBINE





CONCLUSION & FUTURE WORK

CONCLUSION

- Space-time HDG in MFEM
- Application to fluid-rigid body interactions
- Sliding grid technique with pre-built blocks on tetrahedra

FUTURE WORK

- 3D problems (4D meshes)
- Fluid-structure interaction

https://thorvath12.github.io/

Thank you!