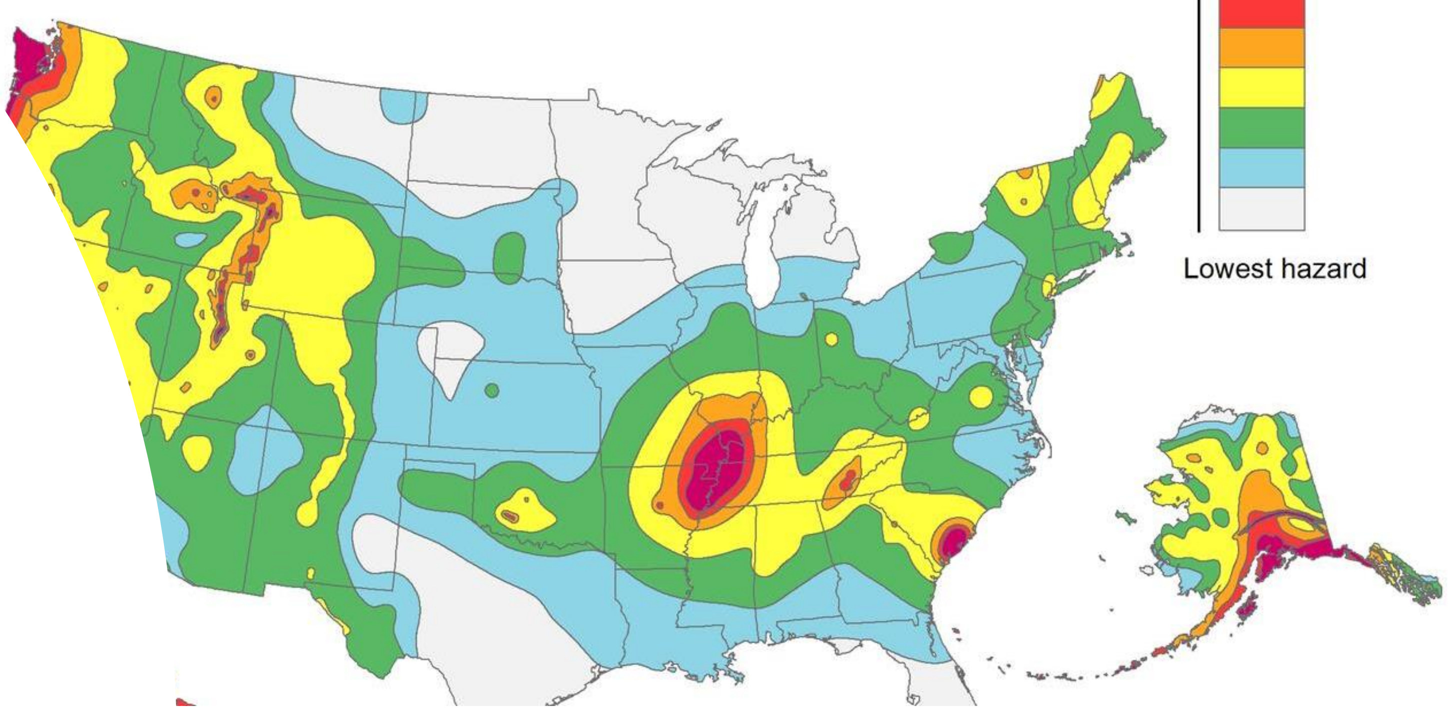
THE UNIVERSITY OF MEMPHIS
Center for Earthquake Research and Information

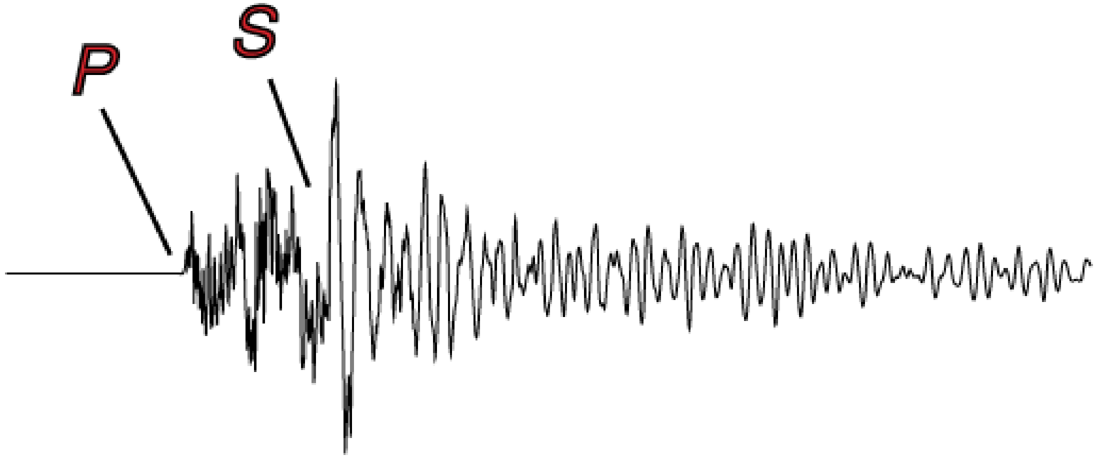
LAGHOST: Development of Lagrangian High-Order Solver for Tectonics

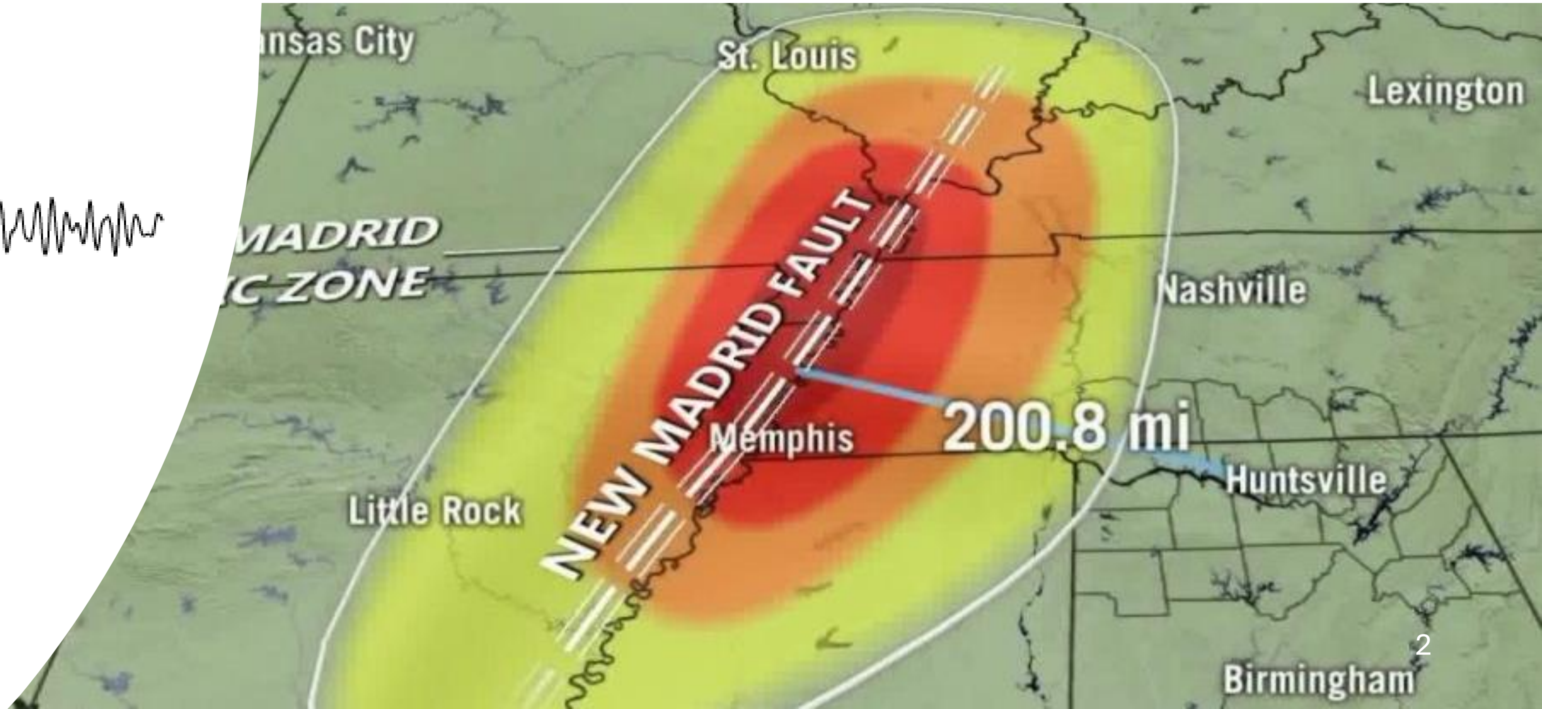
March 5th, 2024

Sungho Lee*, Euneo Choi

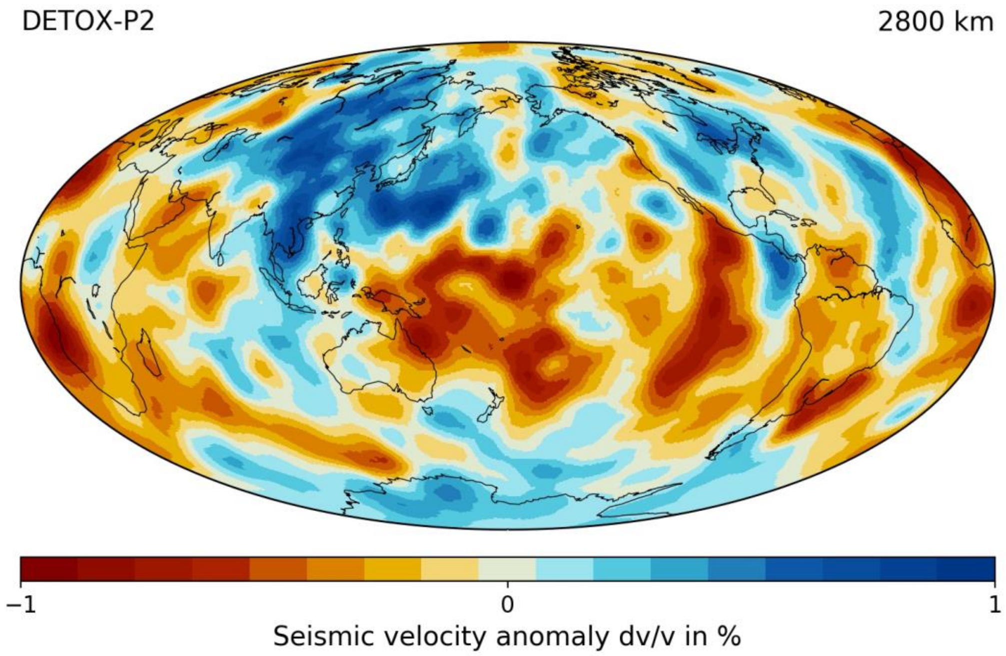
CERI: Center for Earthquake and Research Information

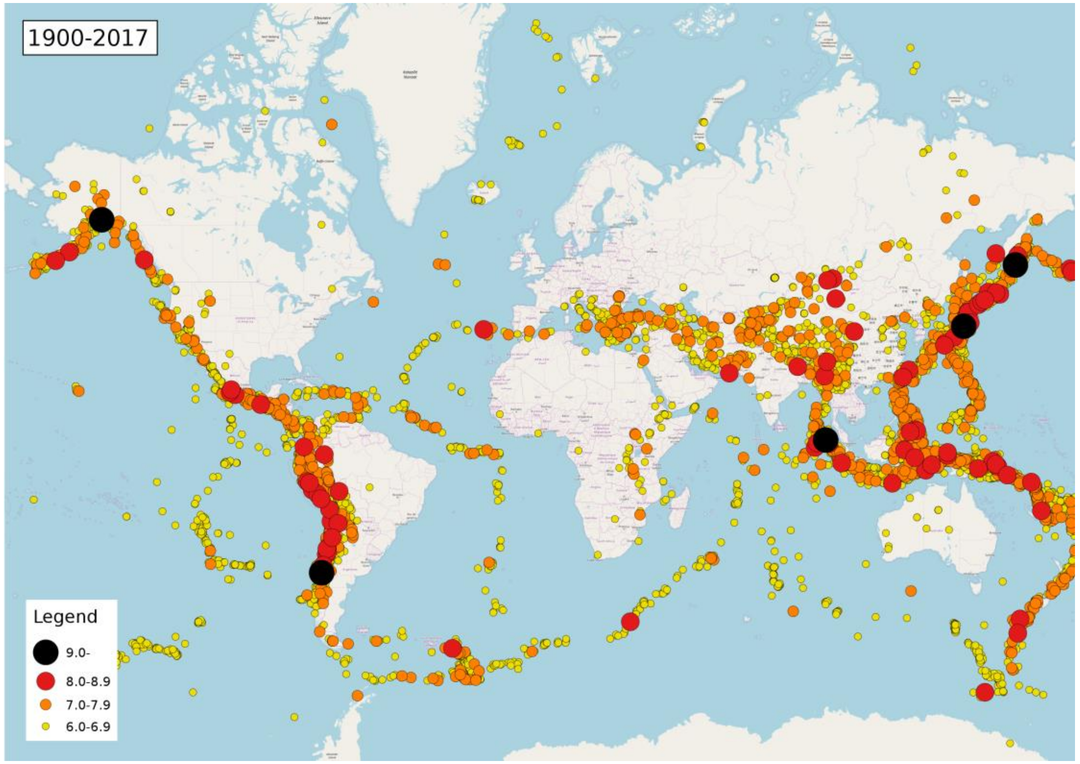






What seismologist is doing?

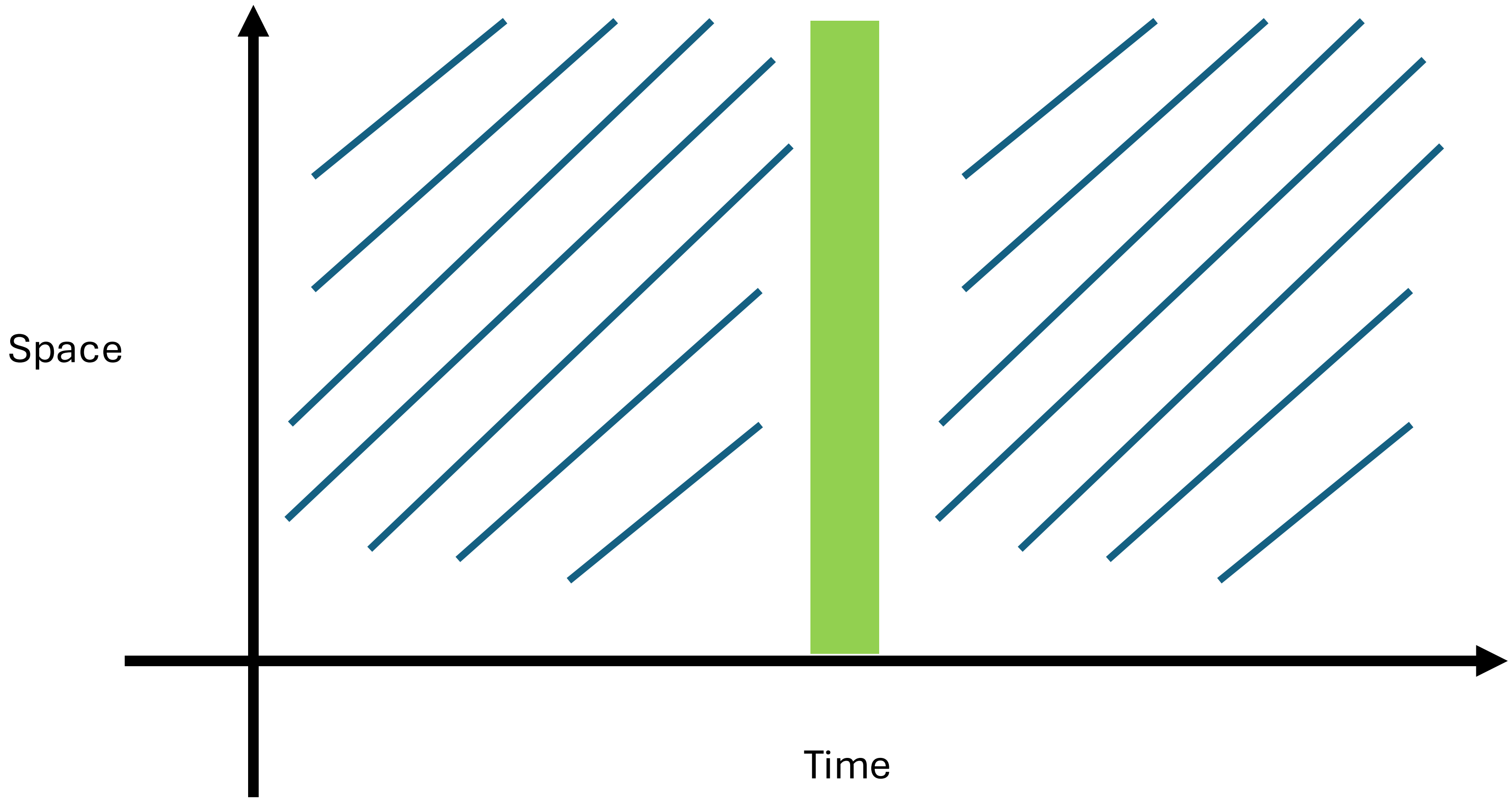




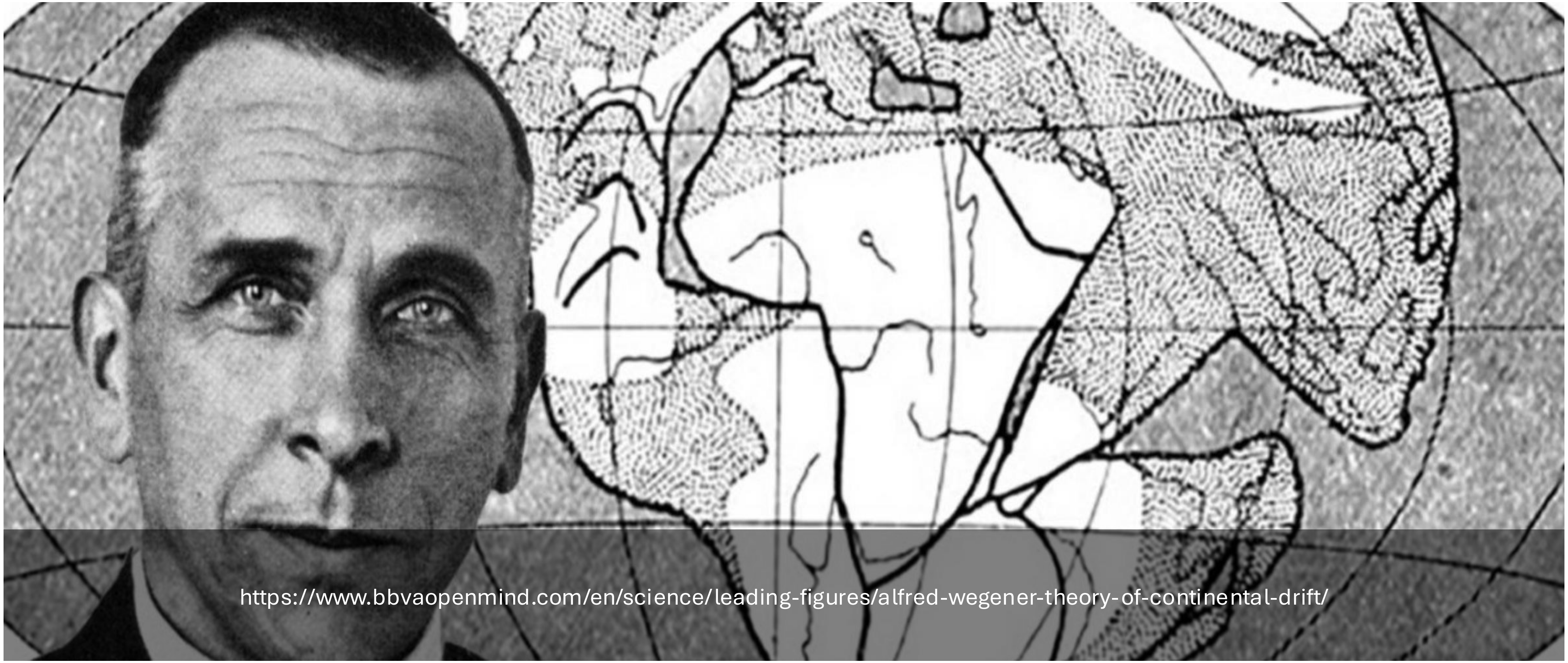
https://www.earth.ox.ac.uk/~smachine/cgi/index.php?page=tomo_depth

https://en.wikipedia.org/wiki/Lists_of_earthquakes

Why forward modeling is needed?

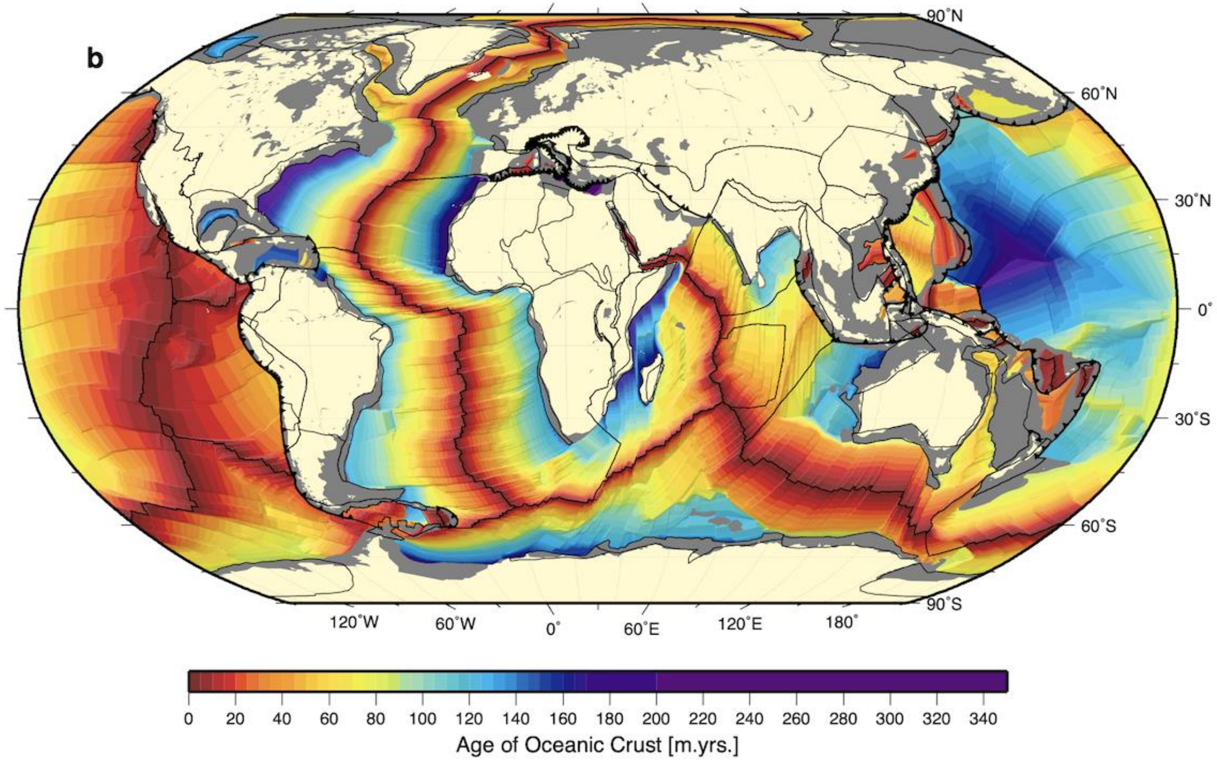


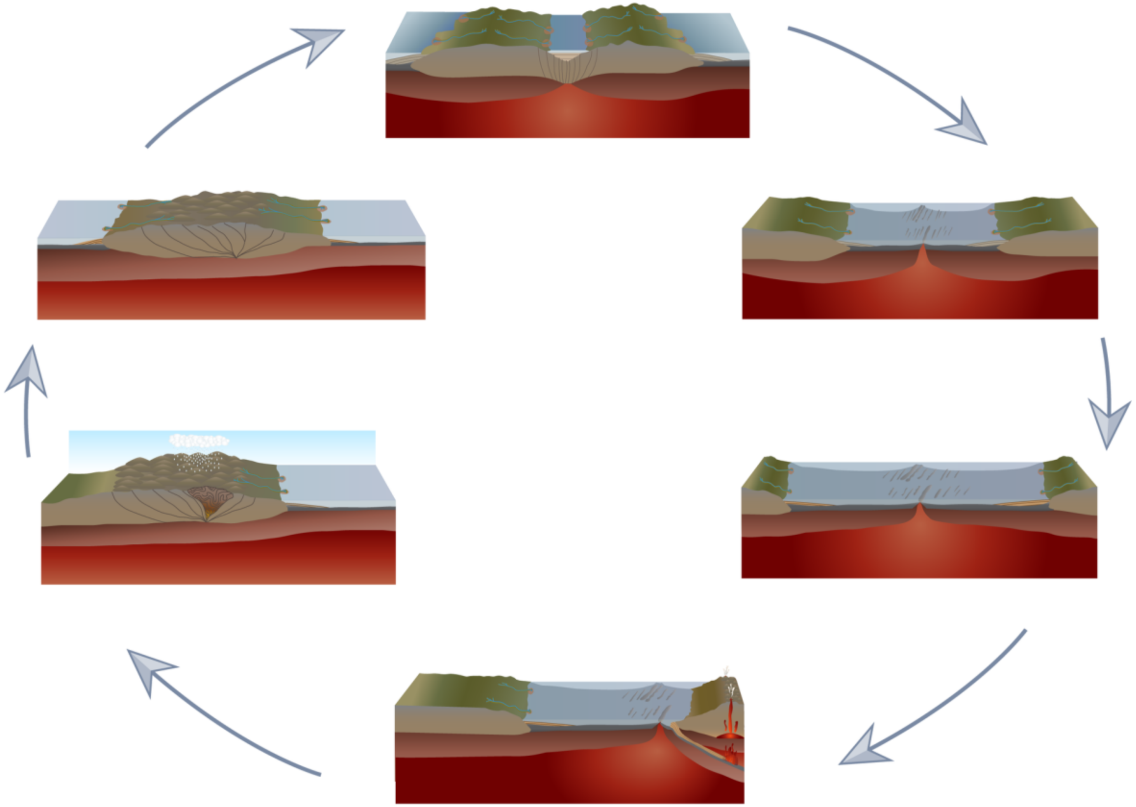
Father of Plate tectonics (Alfred Wegener)



https://www.bbvaopenmind.com/en/science/leading-figures/alfred-wegener-theory-of-continental-drift/

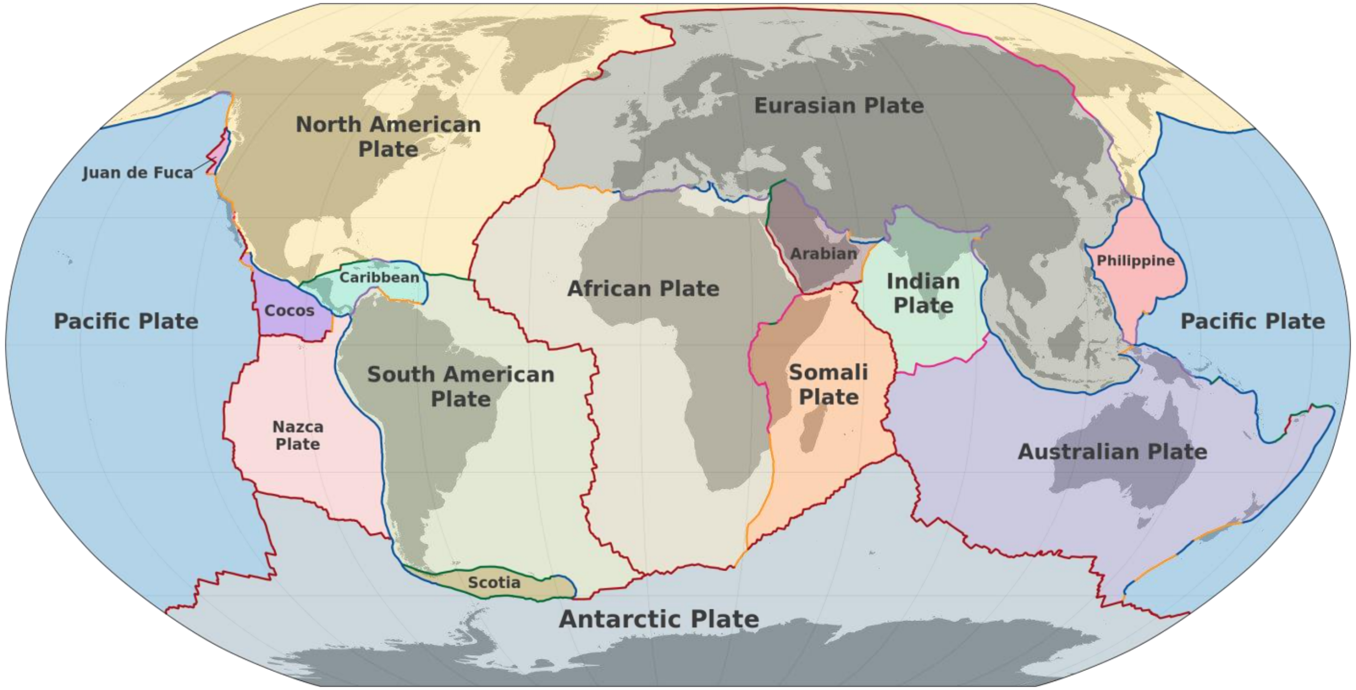
Sea floor spreading / Wilson cycle

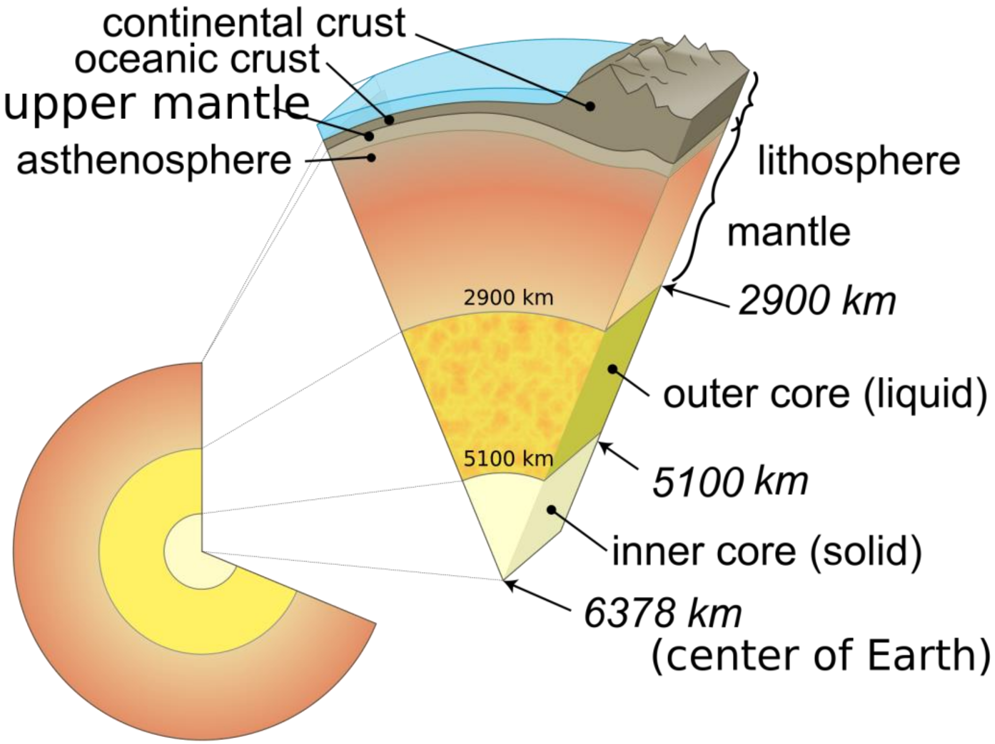




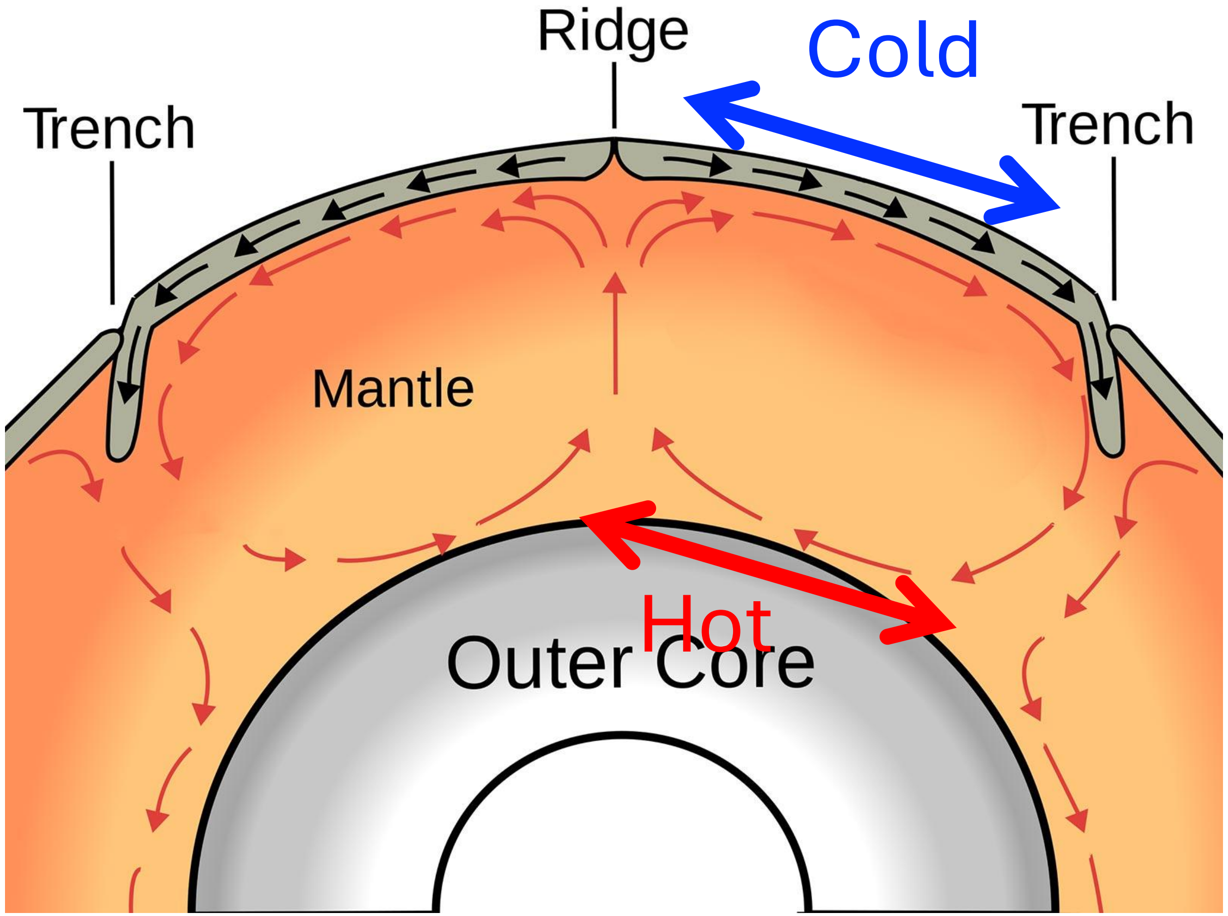
https://www.earthbyte.org/a-global-dataset-of-present-day-oceanic-crustal-age-and-seafloor-spreading-parameters/

Plate tectonics





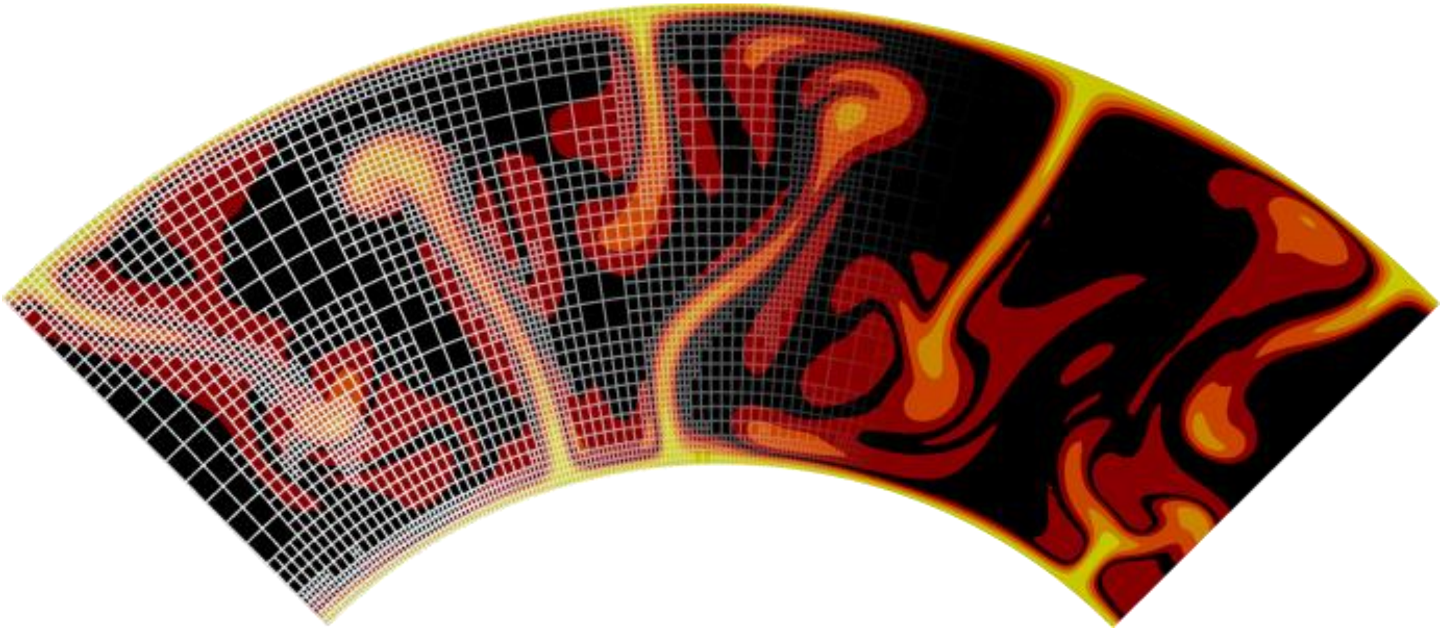
Mantle dynamics or geodyanmcis



https://www.earthbyte.org/a-global-dataset-of-present-day-oceanic-crustal-age-and-seafloor-spreading-parameters/

ASPECT: Advanced Solver for Planetary Evolution, Convection, and Tectonics

deal.II

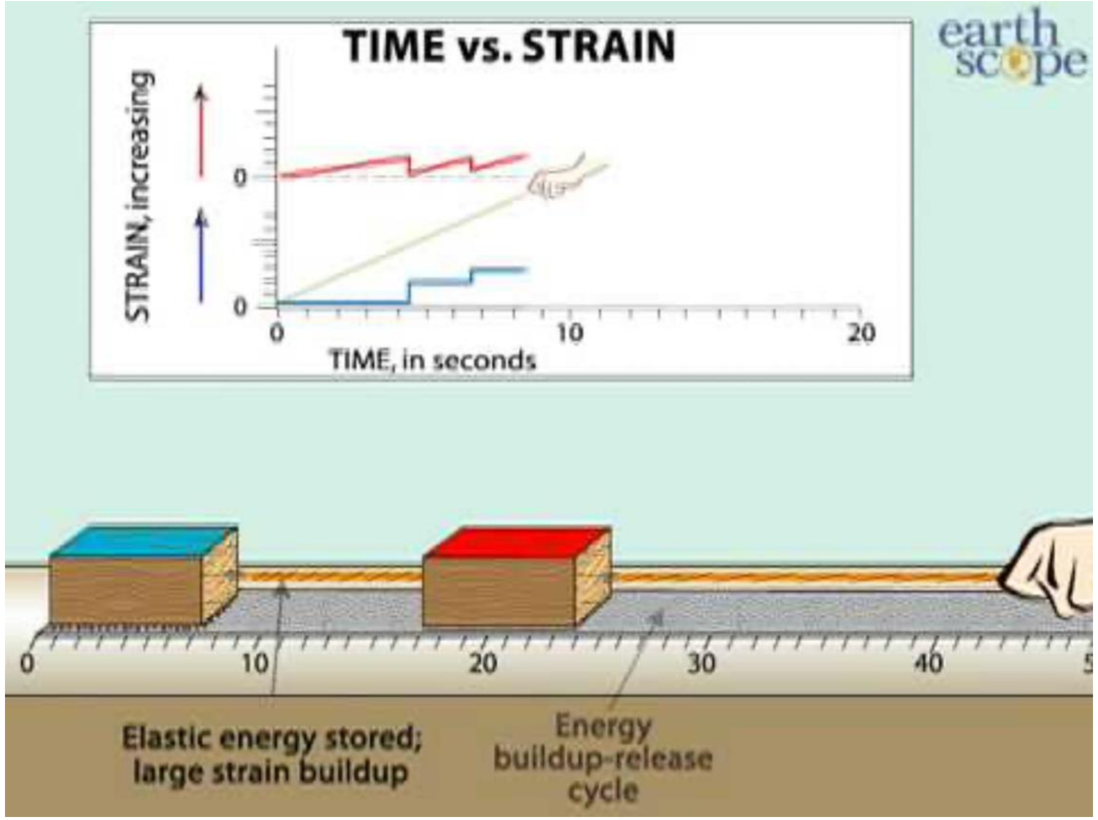


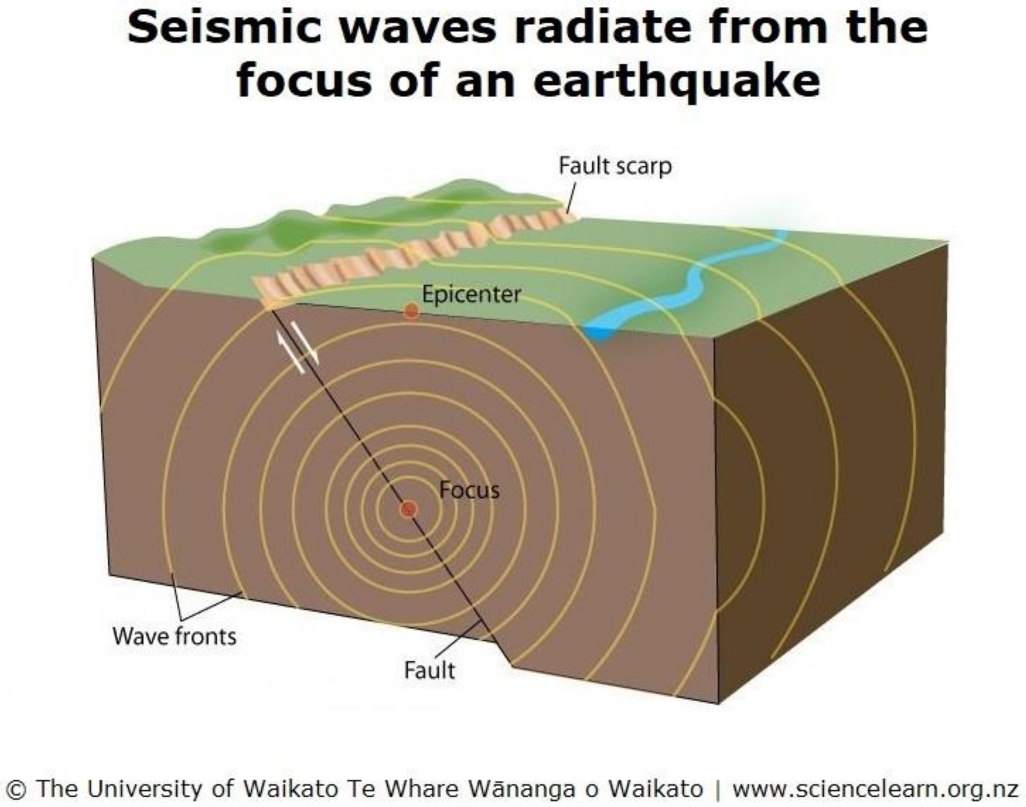
ASPECT assumes slow-moving and incompressibility flow.

The Earth's mantle has a very high viscosity in the upper mantle. This high viscosity implies that the mantle resists flow and deformation, resulting in very low accelerations

$$\nabla \cdot \sigma + \rho g = 0$$

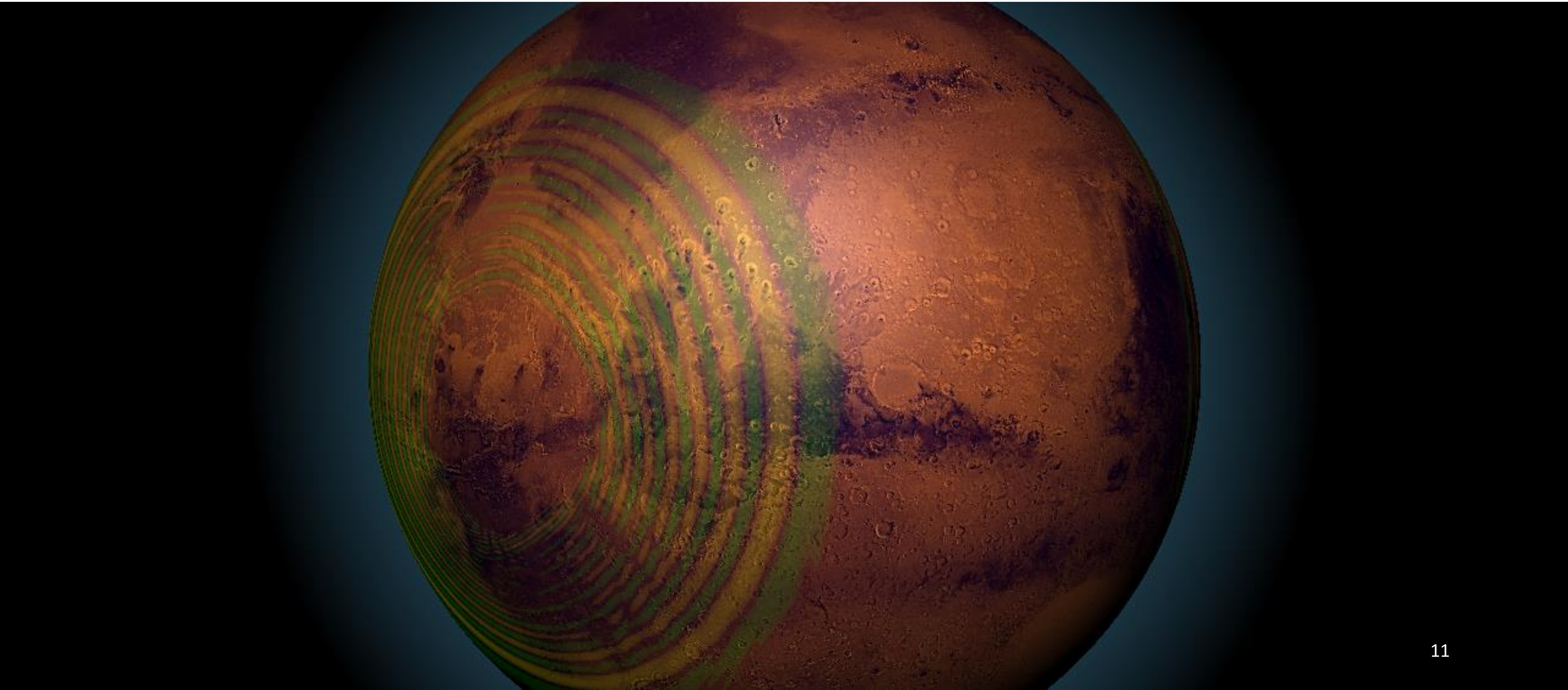
Earthquake mechanisms

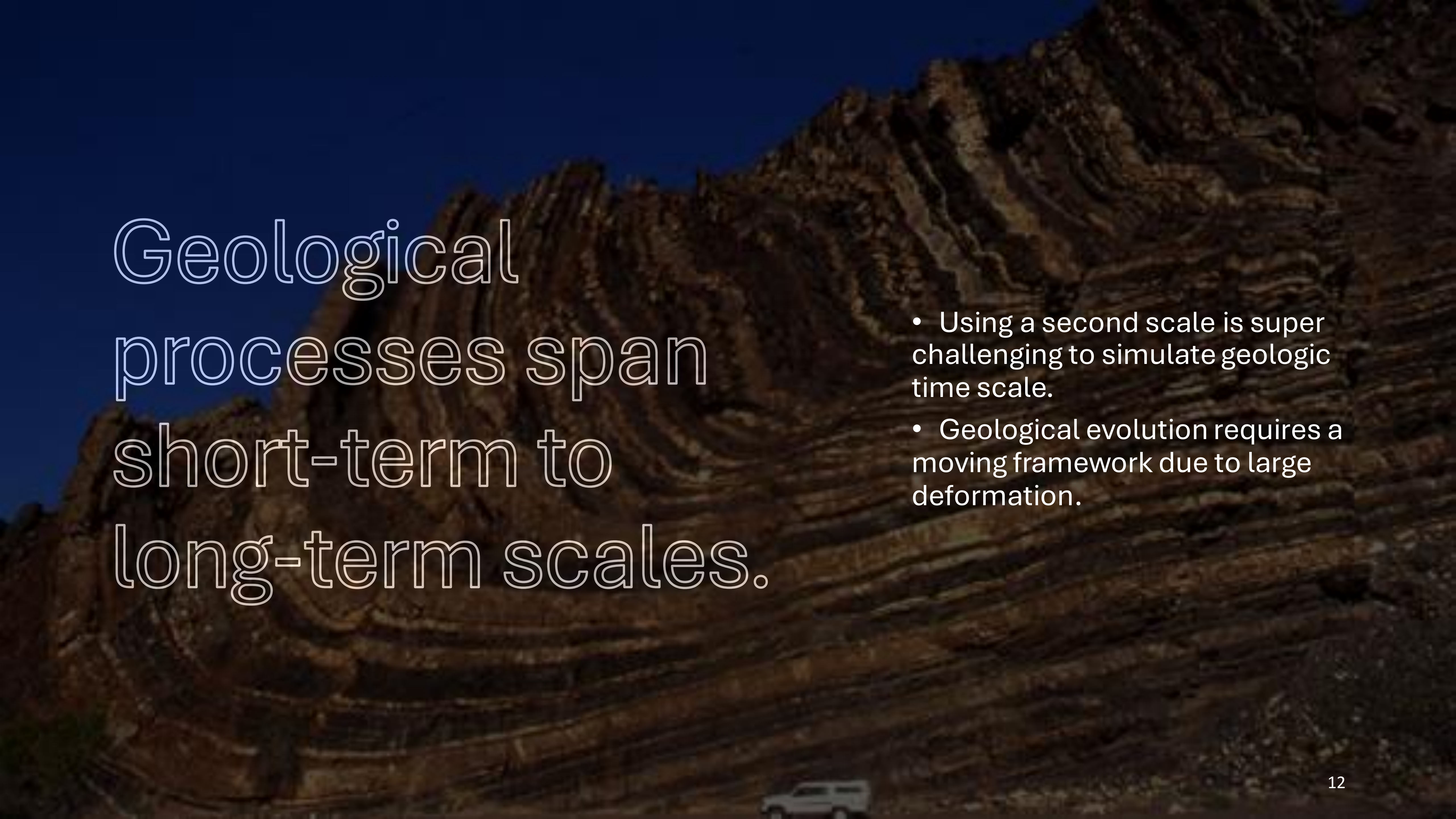




Earthquakes release accumulative elastic strain → elasticity is necessary.

SPECFEM





Geological processes span short-term to long-term scales.

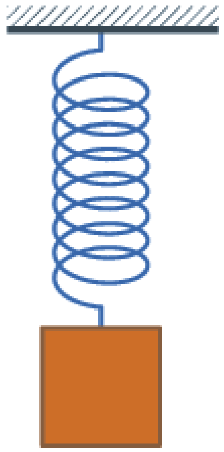
- Using a second scale is super challenging to simulate geologic time scale.
- Geological evolution requires a moving framework due to large deformation.

FLAC: Fast Lagrangian Analysis of Continua

- Around 1980s
  - **Dynamic relaxation** for (quasi-)static solutions
    - Local damping

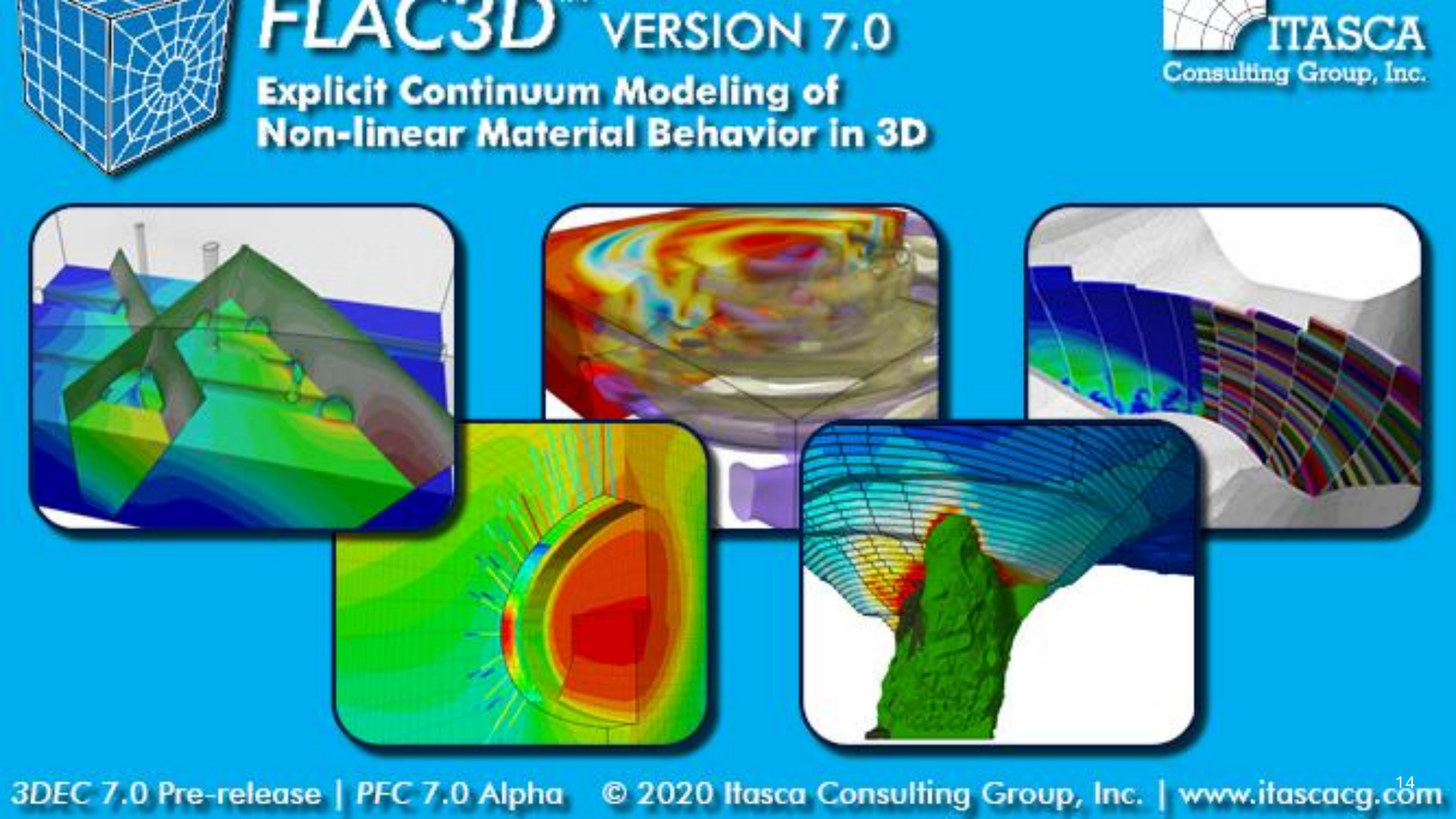
$$v_a^{i,(t+\Delta t/2)} = v_a^{i,(t-\Delta t/2)} + (F_a^{i,(t)} + F_d^i)\frac{\Delta t}{m_a},$$

$$\text{where} \quad F_d^i = -\alpha |F_a^{i,(t)}| \mathrm{sgn}(v_a^i), \text{ where } 0 \leq \alpha < 1.$$

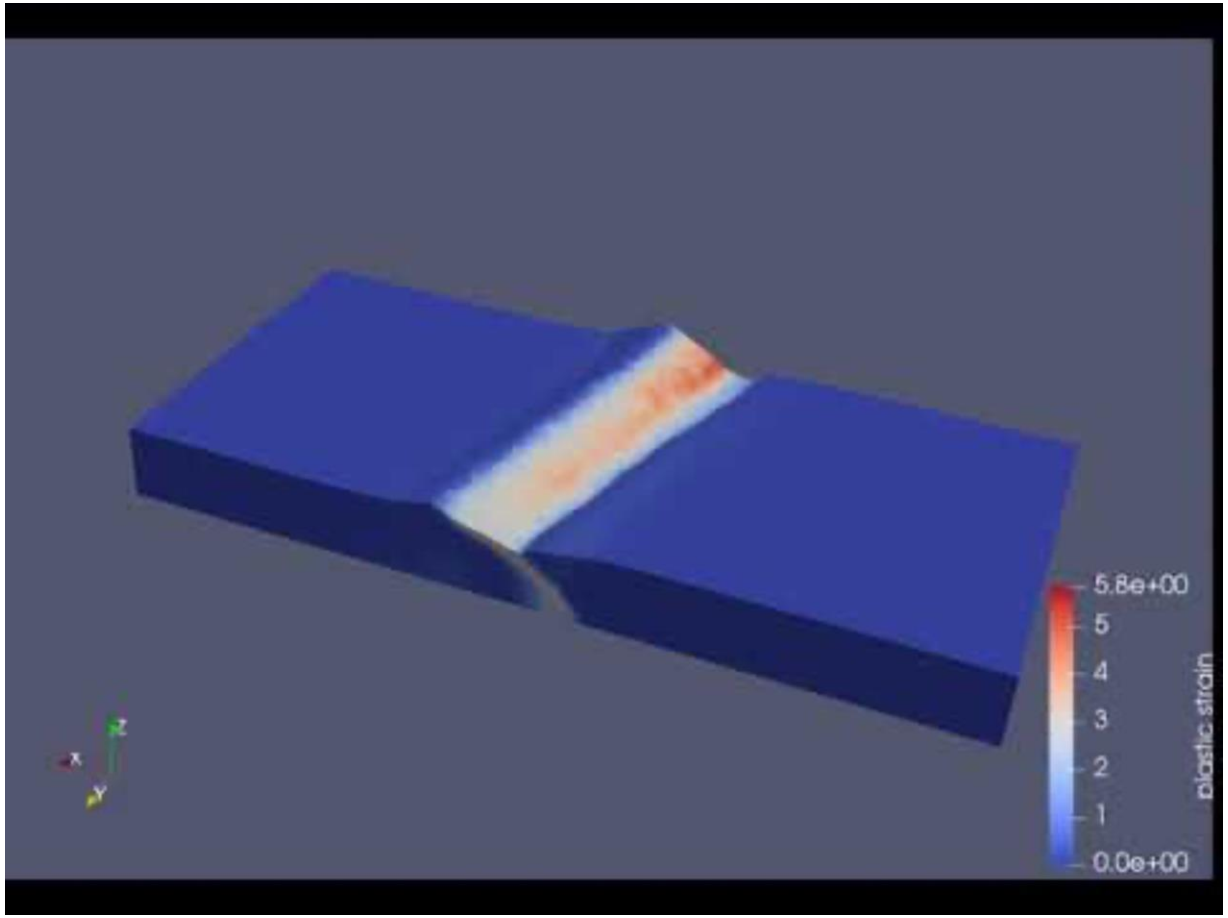


  - **Mass scaling** for a large and stable Δt

$$\Delta t < \frac{\Delta x}{v_p} \qquad v_p = \sqrt{\frac{K}{m_s}} \qquad m_s \gg m_g = \int \rho \, dV$$


FLAC3D VERSION 7.0
Explicit Continuum Modeling of
Non-linear Material Behavior in 3D
ITASCA
Consulting Group, Inc.
3DEC 7.0 Pre-release | PFC 7.0 Alpha
© 2020 Itasca Consulting Group, Inc. | www.itascacg.com

DynEarthSol (DES)

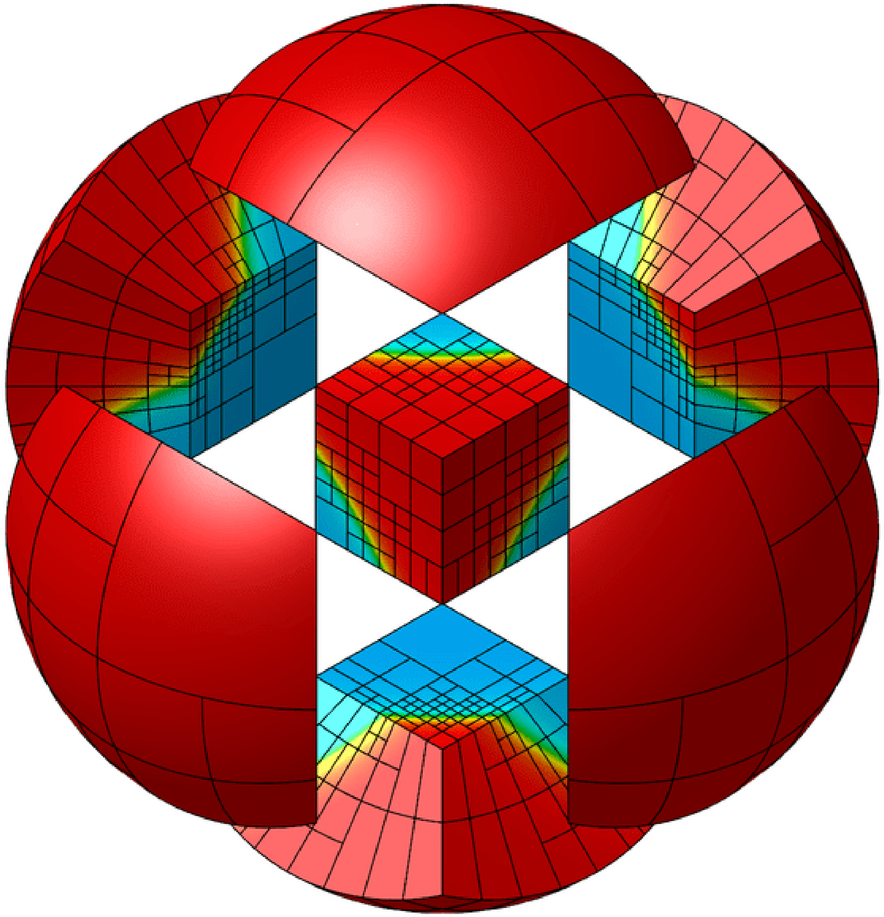


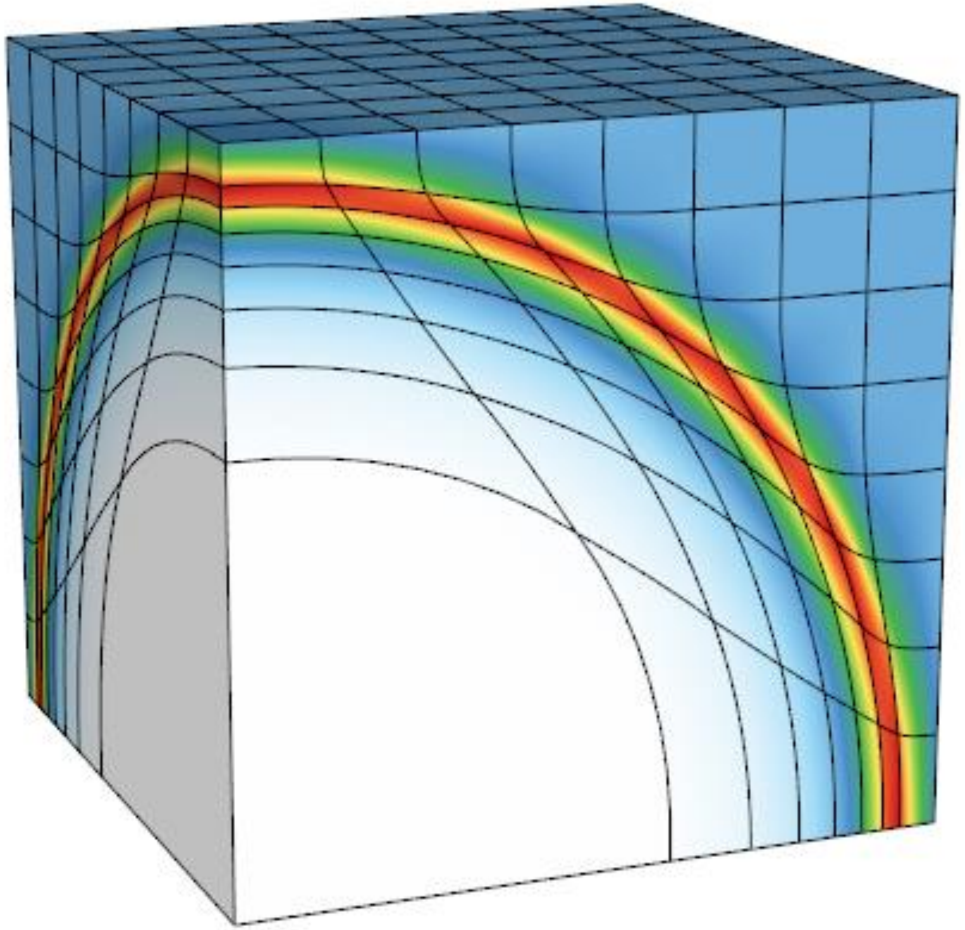
Main characteristics as a numerical method:

- dynamic form of the momentum balance equation
- finite element code with triangular (2D) or tetrahedral (3D) element
- explicit time integration
- Lagrangian description of motion
- OpenMP and GPU acceleration

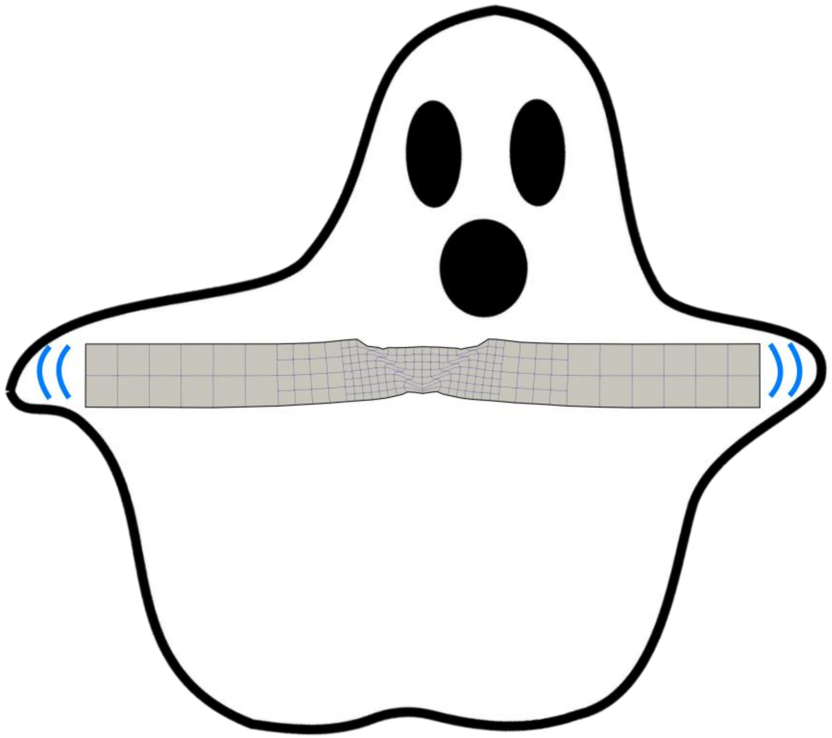
- DES is very hard-wired with triangular and tetrahedral.
- We need more flexibility in the FLAC algorithm.

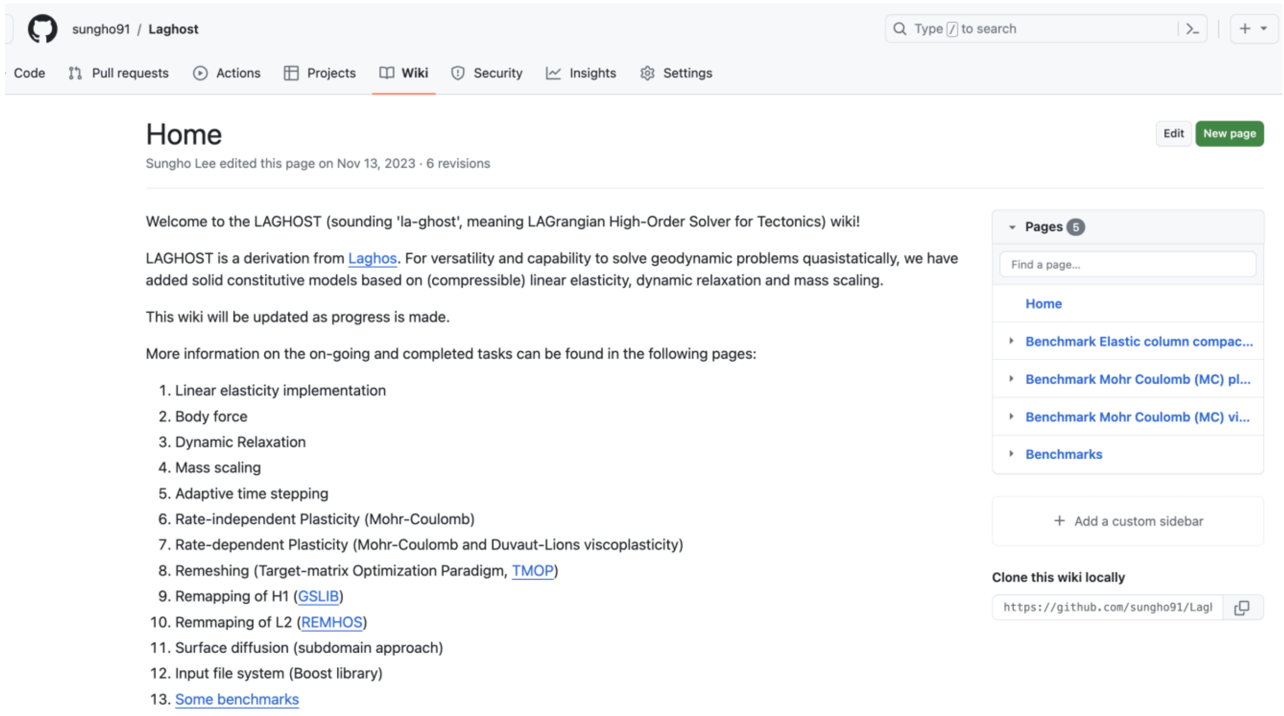
MFEM and Laghos





Laghost (LAGrangian High-Order Solver for Tectonics)





sungho91 / Laghost

Code | Pull requests | Actions | Projects | Wiki | Security | Insights | Settings

Home

Sungho Lee edited this page on Nov 13, 2023 · 6 revisions

Welcome to the LAGHOST (sounding 'la-ghost', meaning LAGrangian High-Order Solver for Tectonics) wiki!

LAGHOST is a derivation from Laghos. For versatility and capability to solve geodynamic problems quasistatically, we have added solid constitutive models based on (compressible) linear elasticity, dynamic relaxation and mass scaling.

This wiki will be updated as progress is made.

More information on the on-going and completed tasks can be found in the following pages:

1. Linear elasticity implementation
2. Body force
3. Dynamic Relaxation
4. Mass scaling
5. Adaptive time stepping
6. Rate-independent Plasticity (Mohr-Coulomb)
7. Rate-dependent Plasticity (Mohr-Coulomb and Duvaut-Lions viscoplasticity)
8. Remeshing (Target-matrix Optimization Paradigm, TMOP)
9. Remapping of H1 (GSLIB)
10. Remmaping of L2 (REMHOS)
11. Surface diffusion (subdomain approach)
12. Input file system (Boost library)
13. Some benchmarks

Pages 5

- Home
- Benchmark Elastic column compac...
- Benchmark Mohr Coulomb (MC) pl...
- Benchmark Mohr Coulomb (MC) vi...
- Benchmarks

Clone this wiki locally

https://github.com/sungho91/Lagl

https://github.com/sungho91/Laghost

Implementation of elasticity

$$\rho \frac{d\boldsymbol{v}}{dt} = -(\nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g})$$

I modified the stress tensor to account for elasticity and added a body force term.

$$\boldsymbol{\sigma}_n = \boldsymbol{\sigma}_{n-1} + \Delta \overset{\Delta}{\boldsymbol{\sigma}}$$

$$\overset{\Delta}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} - \dot{\omega} \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot \dot{\omega}$$

Introducing the Jaumann stress rate; for frame-independence.

$$\dot{\boldsymbol{\sigma}} = \lambda \dot{\epsilon_{kk}} + 2G \dot{\epsilon_{ij}}$$

$$\dot{\epsilon} = \frac{1}{2}(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T) \qquad \dot{\omega} = \frac{1}{2}(\nabla \boldsymbol{v} - \nabla \boldsymbol{v}^T)$$

Governing equations

$$\rho \frac{d\boldsymbol{v}}{dt} = -(\nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g})$$

Momentum Conservation

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\nabla \cdot \boldsymbol{v}$$

Mass conservation

$$\rho \frac{de}{dt} = \boldsymbol{\sigma} : \nabla \boldsymbol{v}$$

Energy conservation

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

Equation of Motion

Discrete formulation

$$\boldsymbol{M}_v = \int \rho\, w_i w_j \qquad \boldsymbol{M}_e = \int \rho\, \phi_i \phi_j \qquad F_{i,j} = \int (-\boldsymbol{\sigma} : \nabla w_i)\phi_j$$

Basis for H1
(Gauss-Lobatto)

Basis for L2
(Gauss-Lobatto or Bernstein)

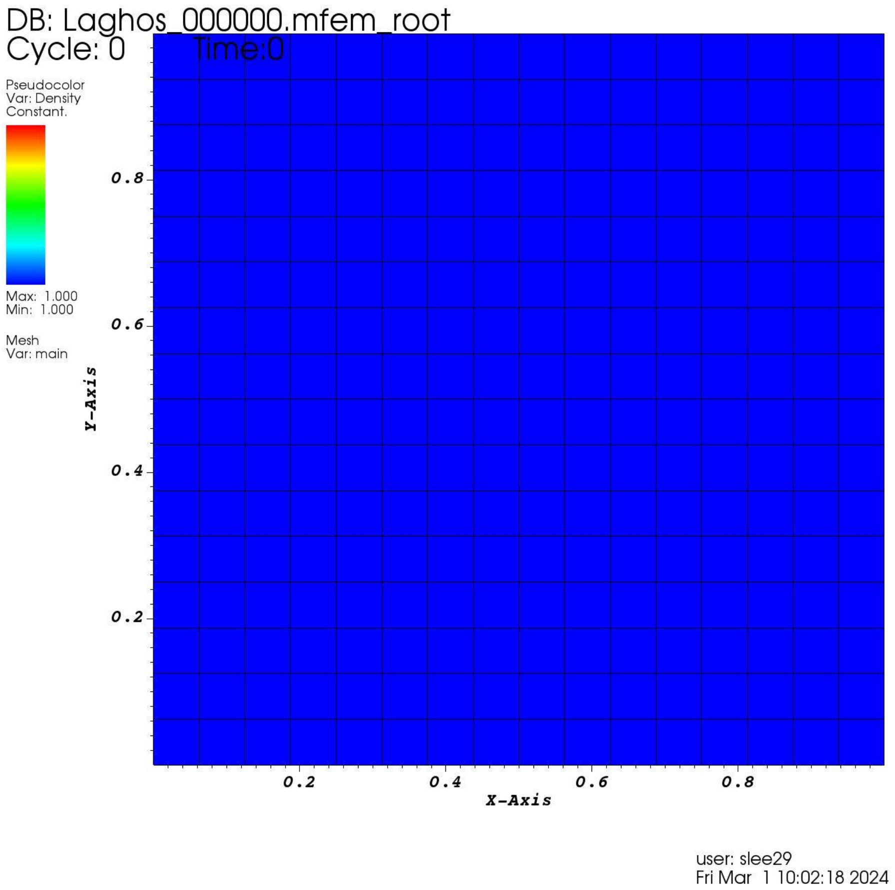
$$\boldsymbol{M}_v \frac{d\boldsymbol{v}}{dt} = \boldsymbol{F} \cdot \boldsymbol{1} \qquad \boldsymbol{M}_e \frac{d\boldsymbol{e}}{dt} = -\boldsymbol{F}^T \cdot \boldsymbol{v}$$

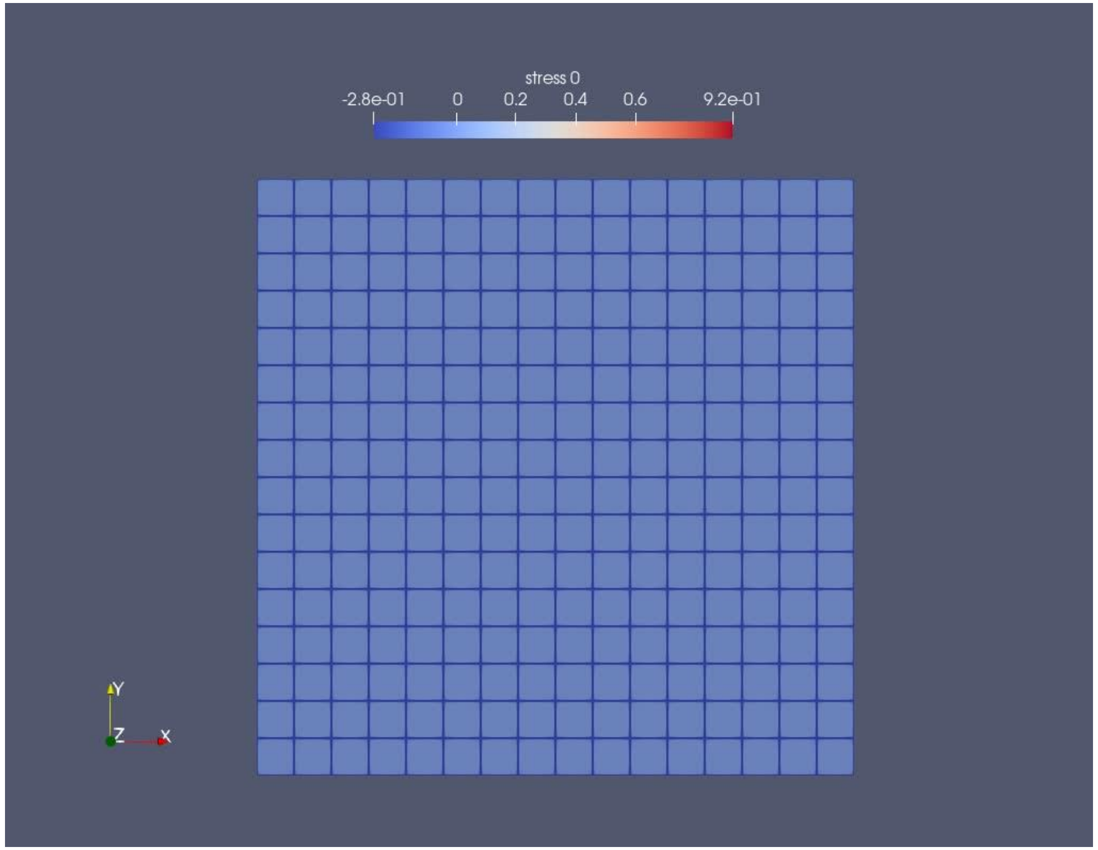
$$\boldsymbol{M}_e \frac{d\overset{\Delta}{\boldsymbol{\sigma}}}{dt} = (\rho\overset{\Delta}{\boldsymbol{\sigma}})\phi_k$$

stress rate (new added feature)

Blast in Elastic box

Hamering on ealstic plane

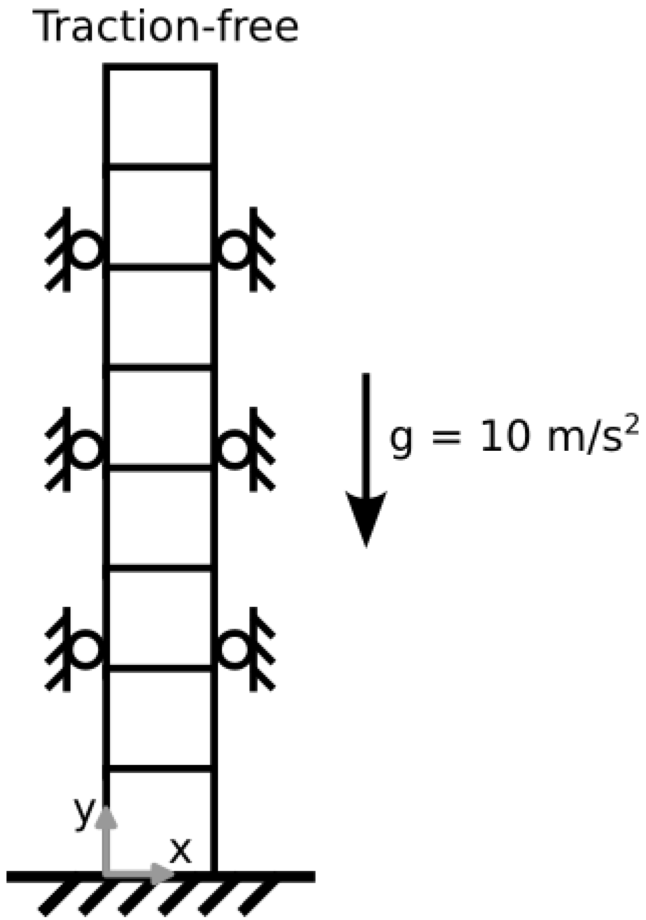


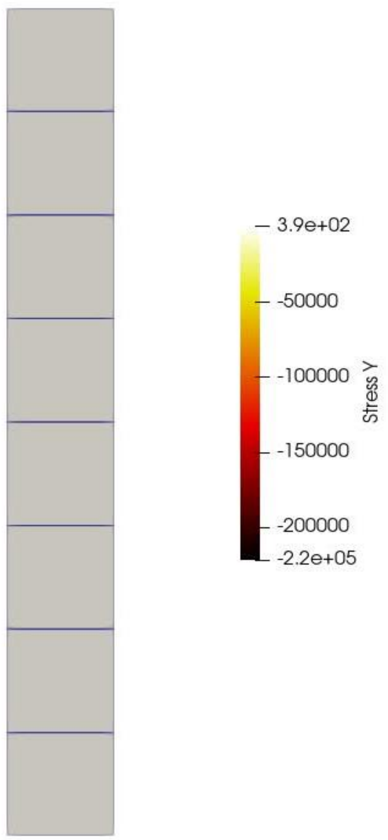


Laghos

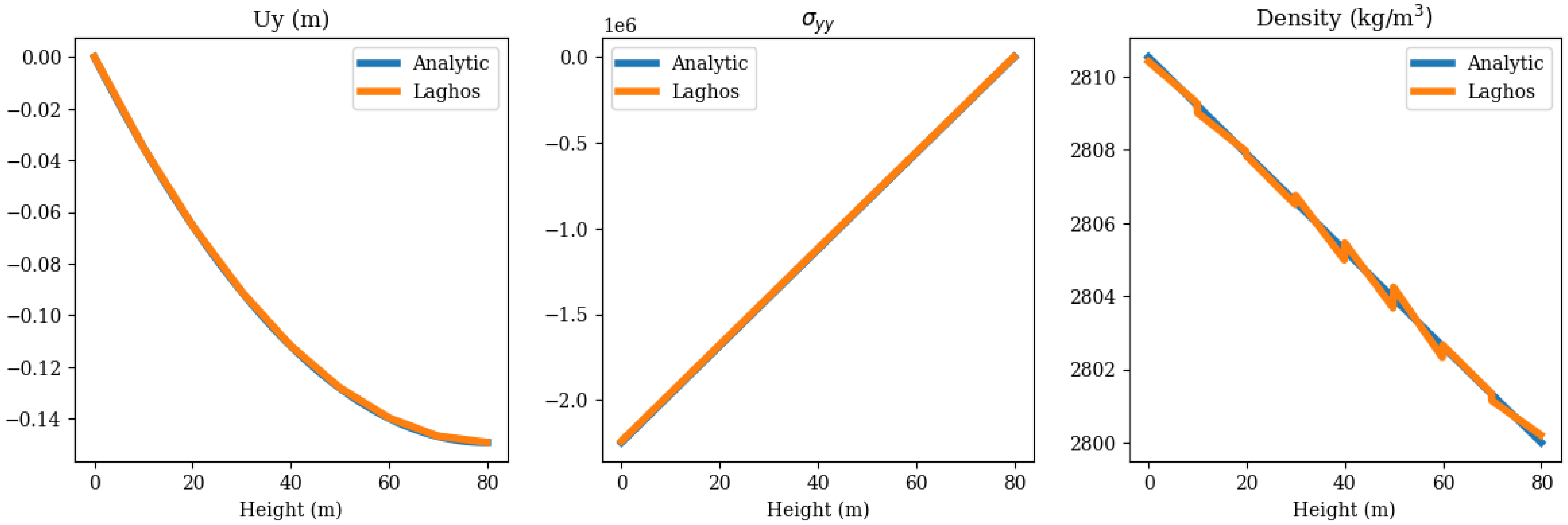
Laghost

Benchmark 1: Elastic column consolidated by self-weight





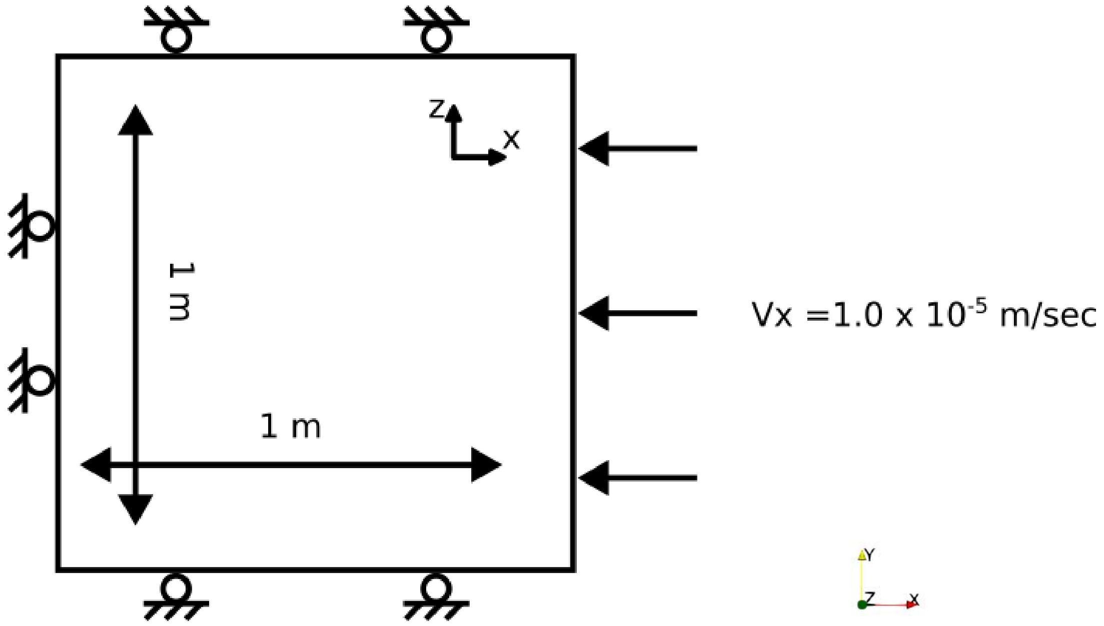
Benchmark 1:
Elastic column compacting by self-weight

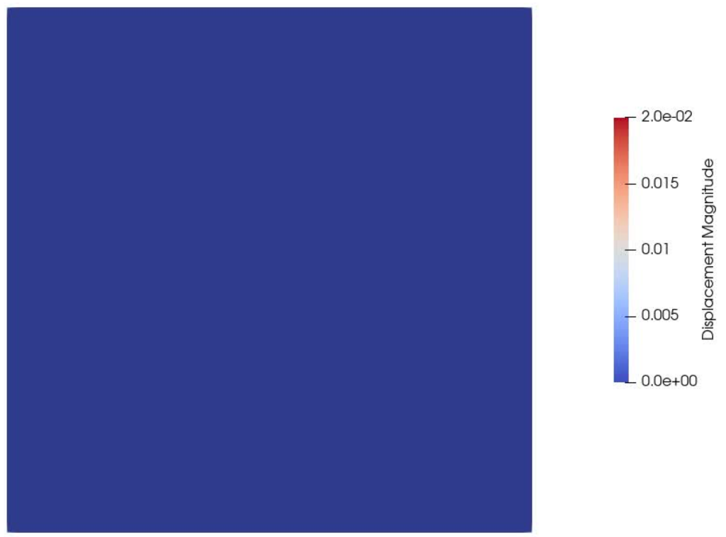


See the details:
https://github.com/sungho91/Laghost/wiki/Benchmark---Elastic-column-compacted-by-self-weight

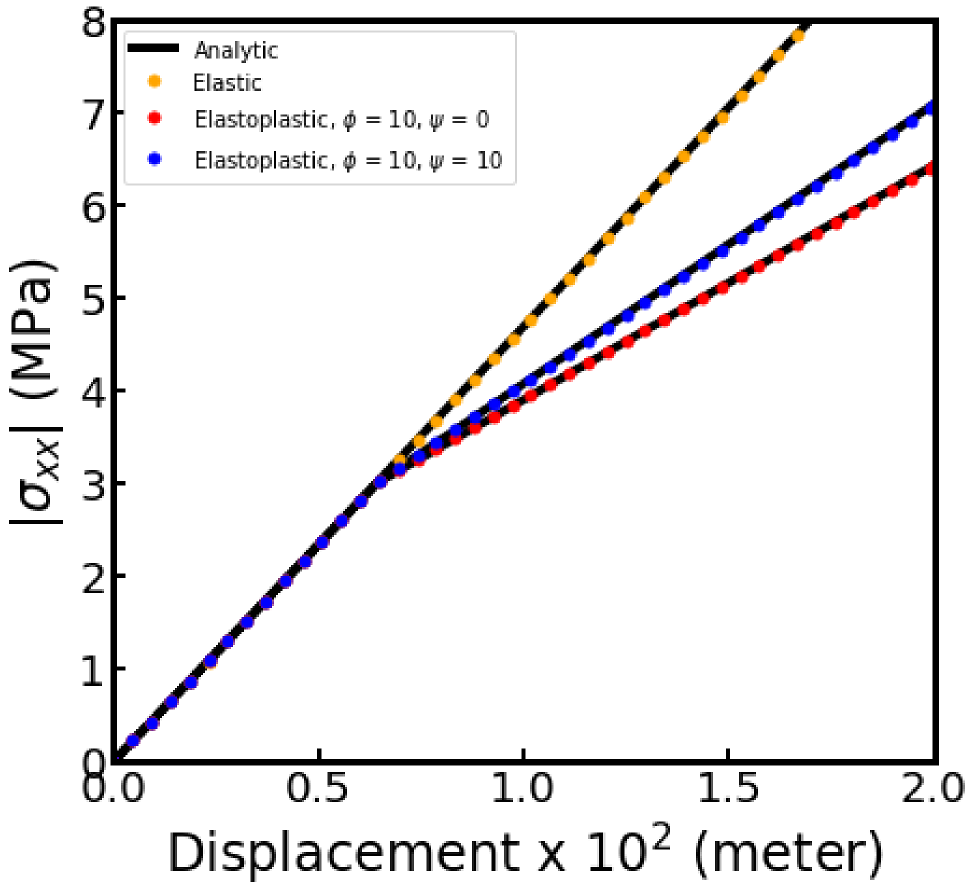
Rel. error for uy: 4.484e-03
Rel. error for syy: 4.391e-03
Rel. error for density: 3.935e-05

Benchmark 2:
material compressed by a constant velocity



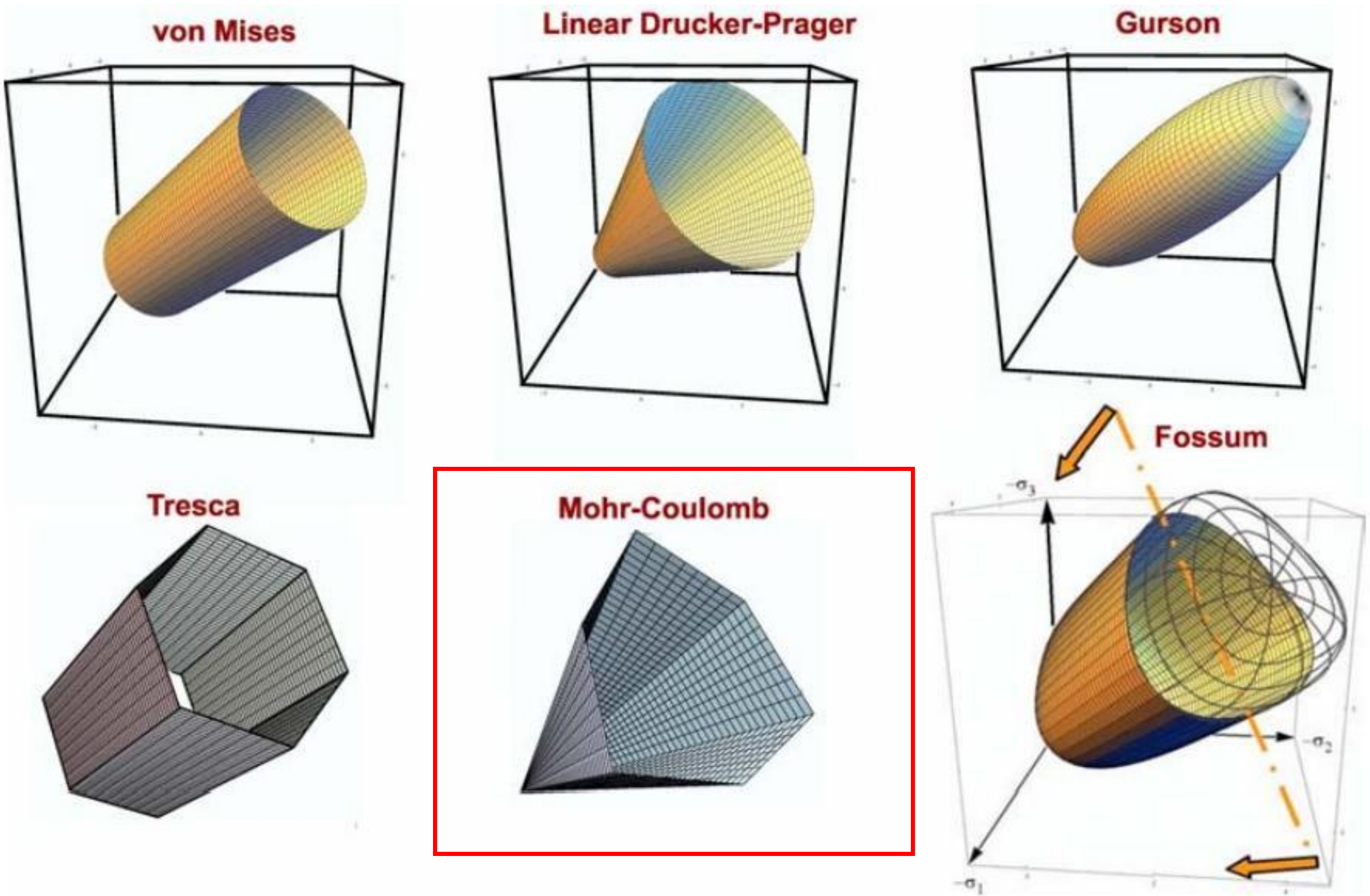


Benchmark 2:
material compressed by a constant velocity



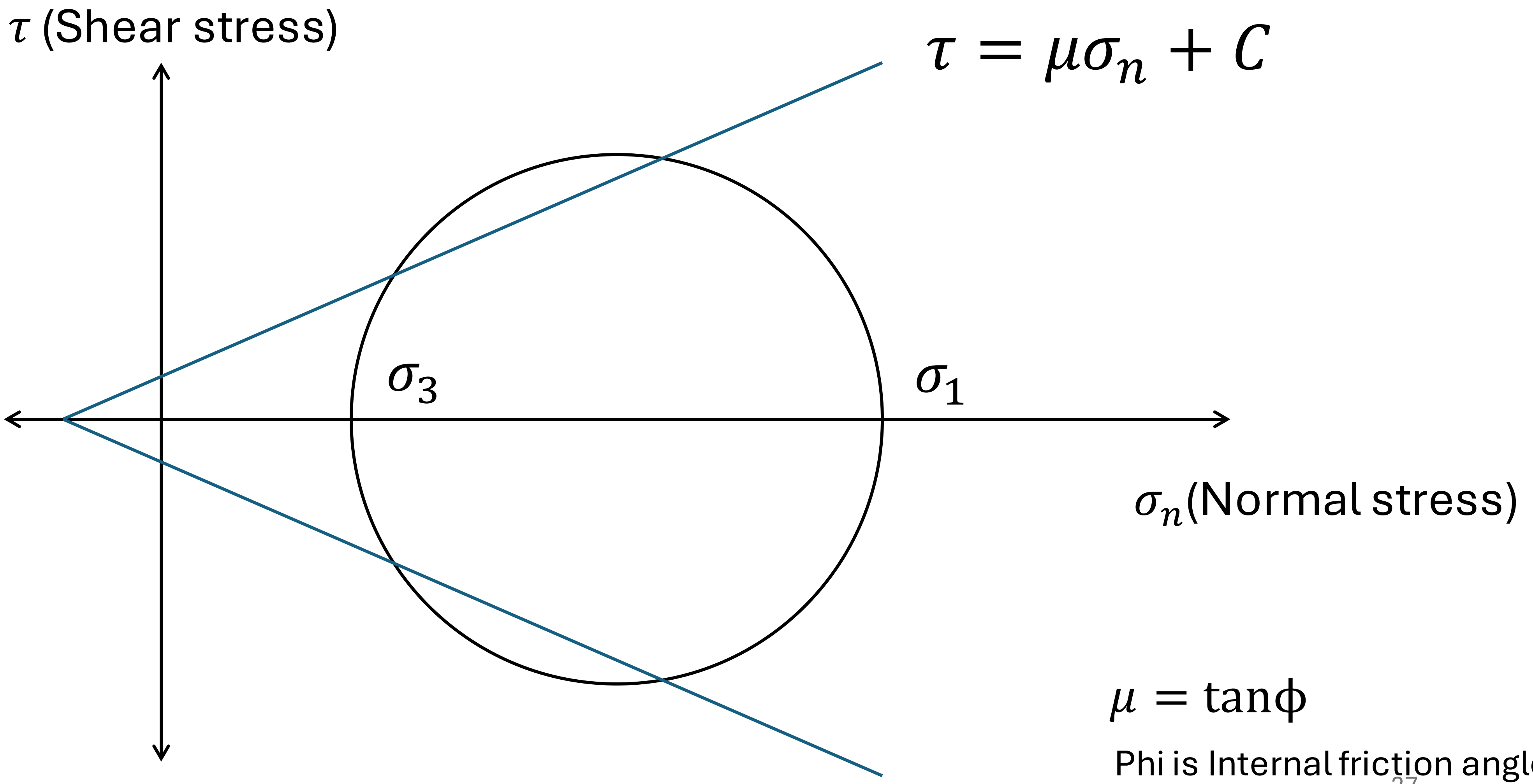
Rel. error for elastic: 4.946e-04

See the details: https://github.com/sungho91/Laghost/wiki/Benchmark-Mohr-Coulomb-(MC)-plastic-material-compressed-by-a-constant-velocity



https://csmbrannon.net/2011/06/22/publication-experimental-assessment-of-unvalidated-assumptions-in-classical-plasticity-theory/

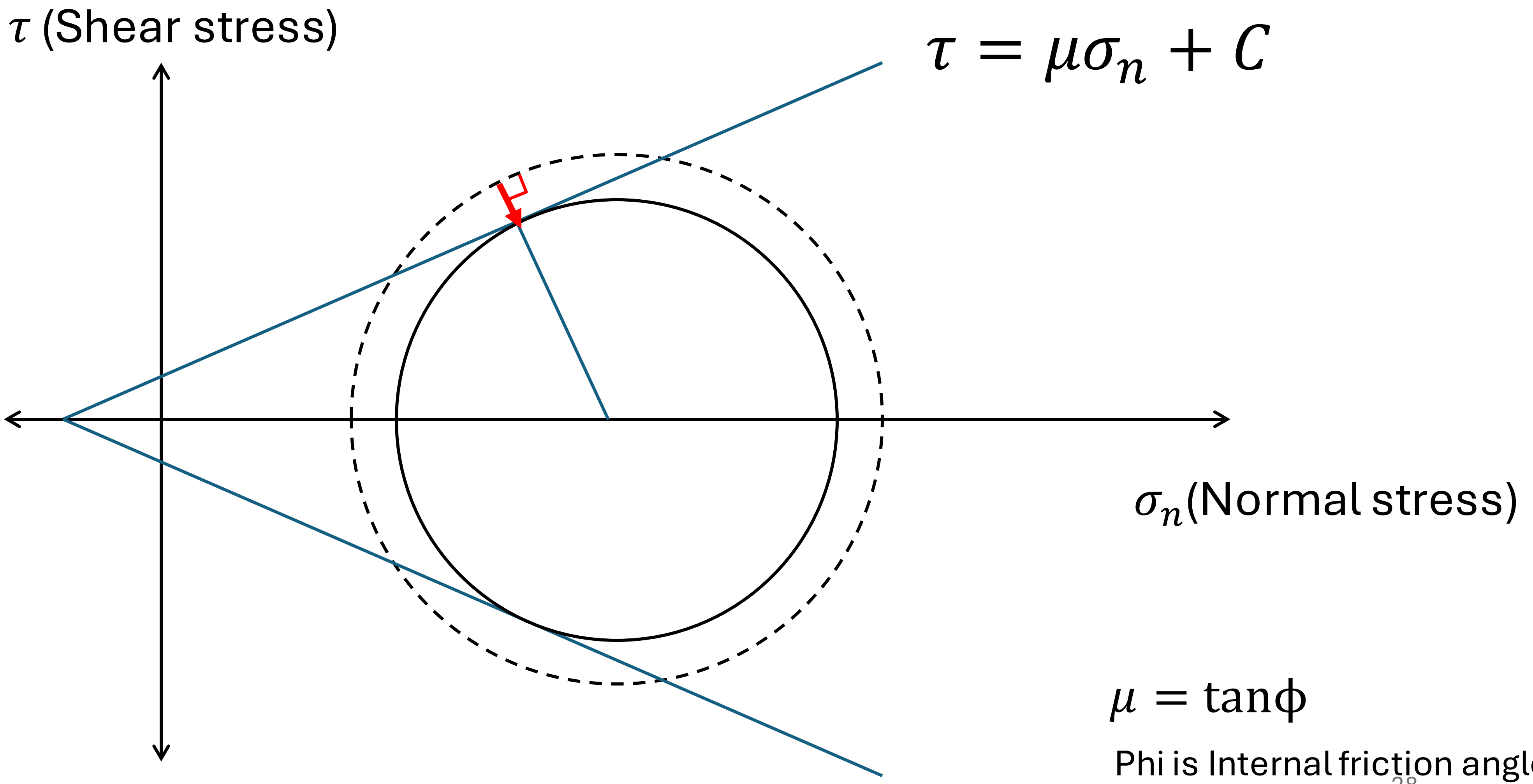
Return-mapping



$\mu = \tan\phi$

Phi is Internal friction angle

Return-mapping



$\mu = \tan\phi$

Phi is Internal friction angle

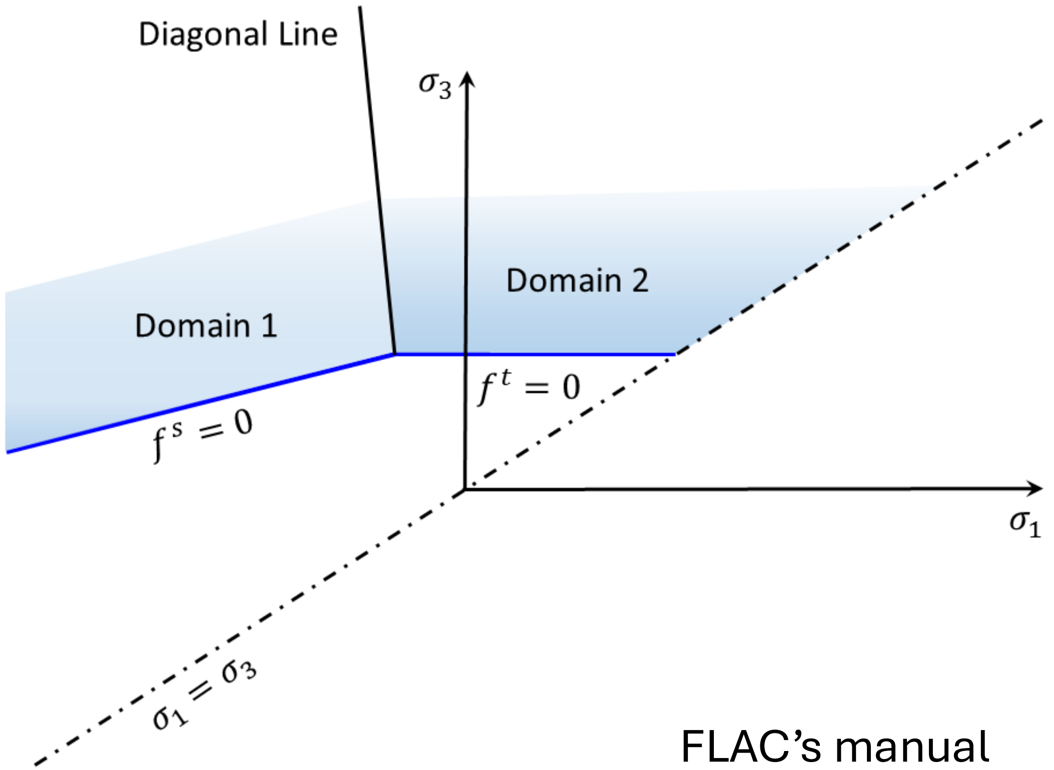
Failure function

Shear failure function

$$f_s = \sigma_1 - \frac{1+sin\phi}{1-sin\phi}\sigma_3 + 2C\sqrt{\frac{1+sin\phi}{1-sin\phi}}$$

Tensile failure

$$f_t = \sigma_3 - \sigma_t$$



FLAC’s manual

Bisects the obtuse angle made by two yield functions

$$f_h = (\sigma_3 - \sigma_t) + \left(\sqrt{\frac{1+sin\phi}{1-sin\phi}^2 + 1} + \frac{1+sin\phi}{1-sin\phi}\right)\left(\sigma_1 - \frac{1+sin\phi}{1-sin\phi}\sigma_t + 2C\sqrt{\frac{1+sin\phi}{1-sin\phi}}\right)$$

Plastic flow potential

Plastic flow potential for shear failure

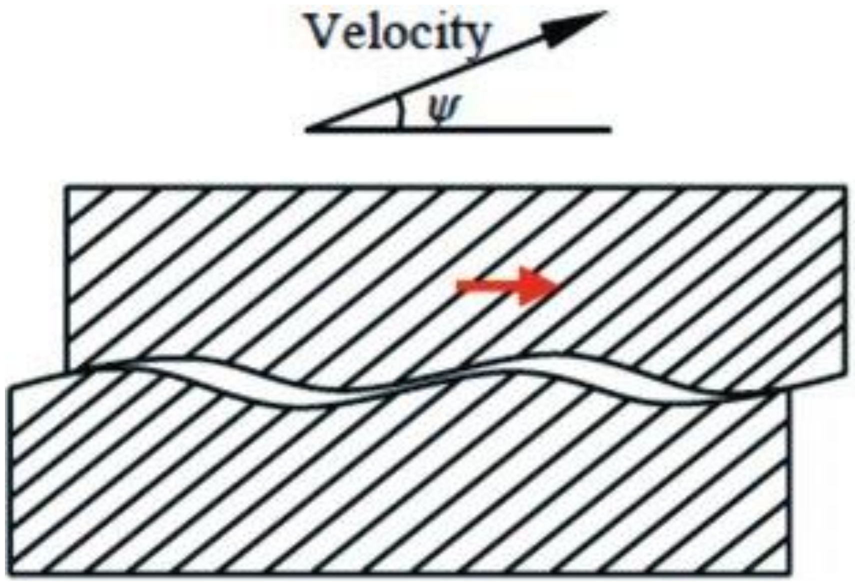
$$g_s = \sigma_1 - \frac{1 + sin\psi}{1 - sin\psi}\sigma_3$$

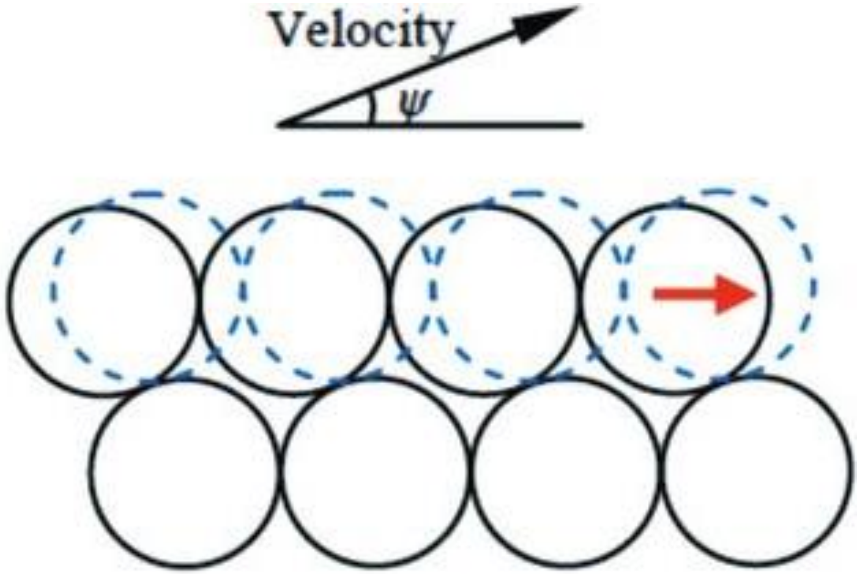
Plastic flow potential for tension failure

$$g_t = \sigma_3 - \sigma_t$$

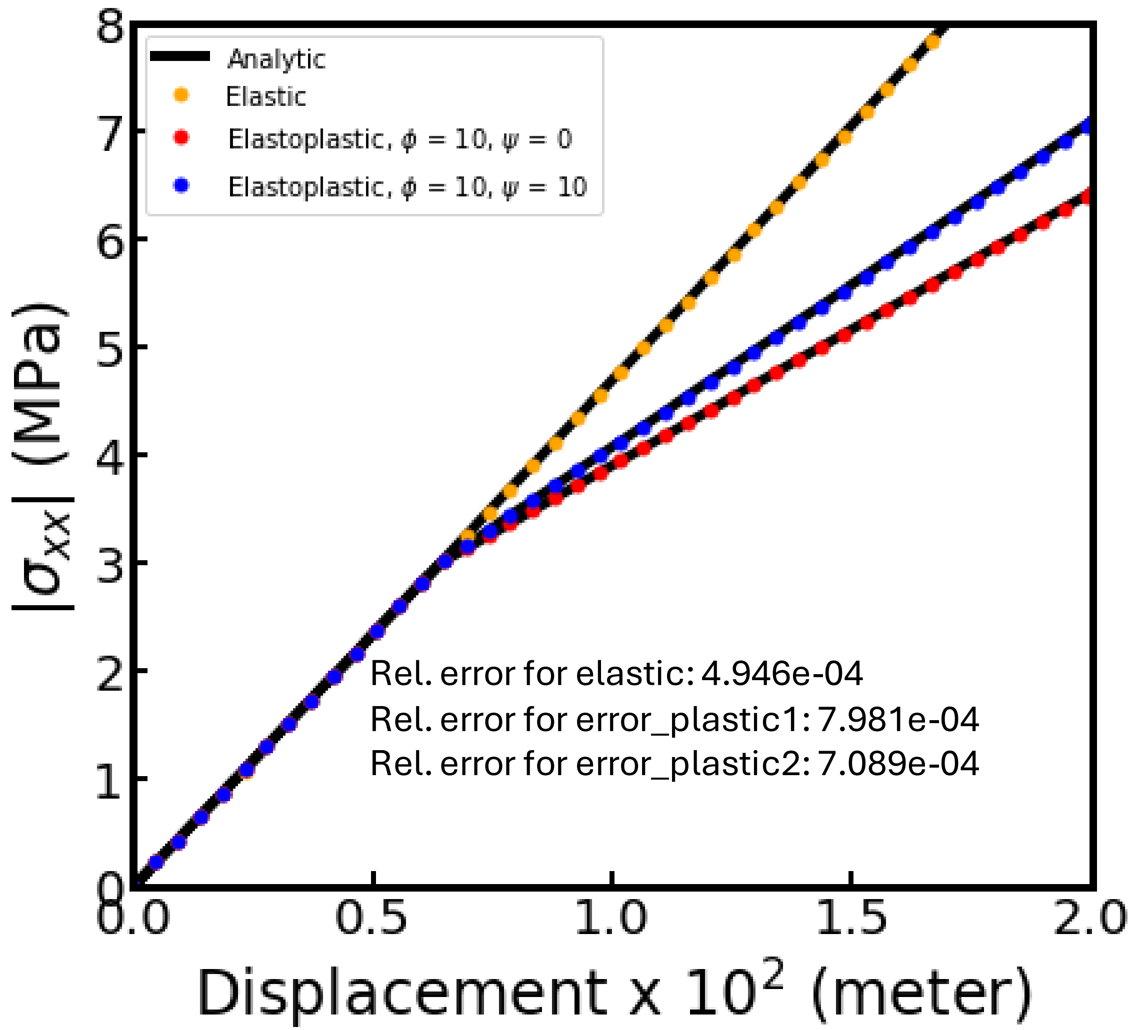
Plastic strain

$$\Delta\varepsilon_{pl} = \beta\frac{\partial g}{\partial\sigma}$$

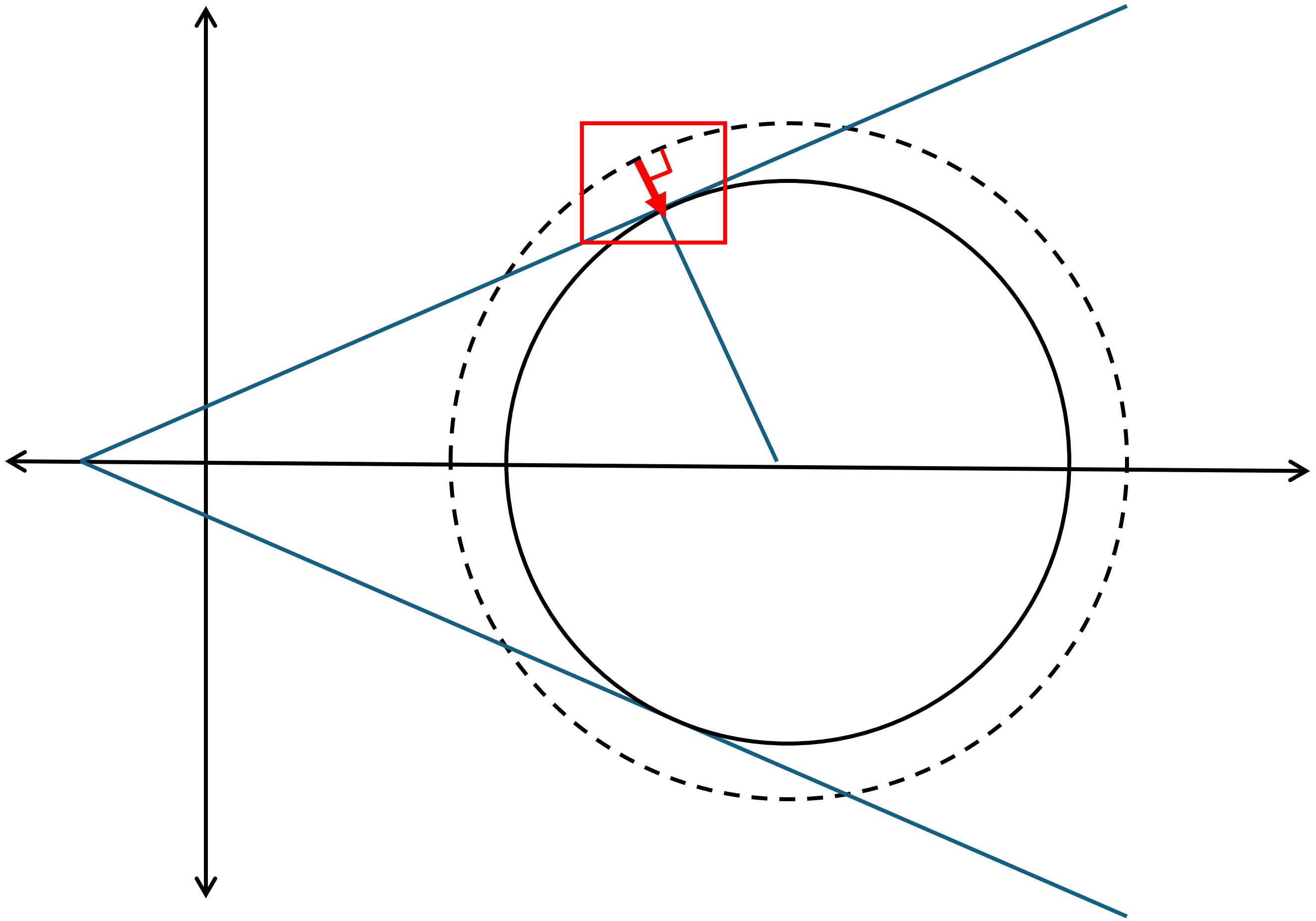




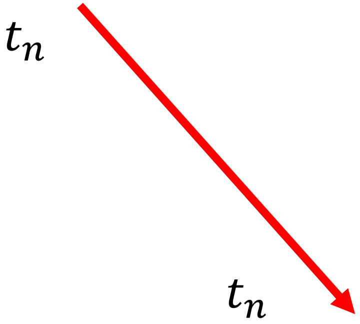
Benchmark 3: Mohr Coulomb (MC) plastic material compressed by a constant velocity



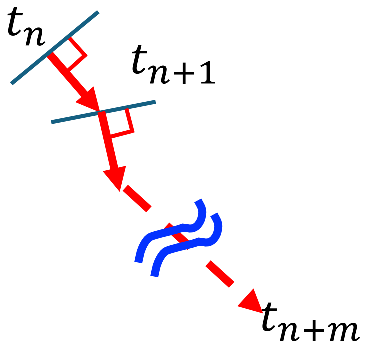
Rate-dependent Return-mapping



Model1 : Rate-independent
(Instantaneous, viscosity = 0)

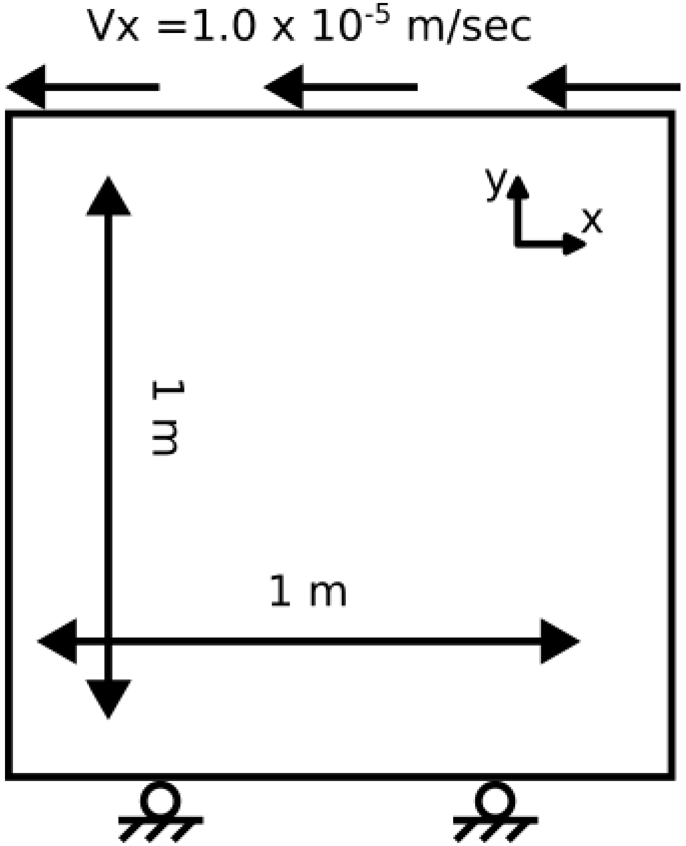


Model2 : Rate-dependent
(viscosity =/ 0)



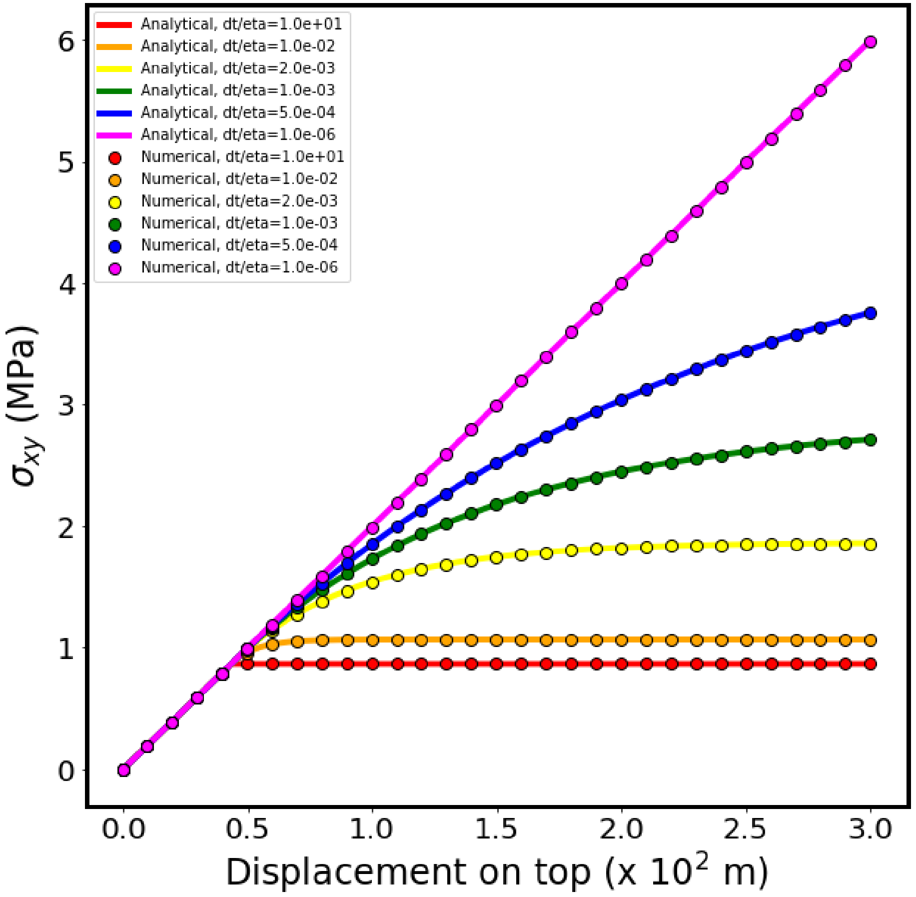
Special case:
Viscosity = ∞ (elasticity)

Benchmark 4: Mohr Coulomb (MC) viscosplastic material is sheared by a constant velocity



Benchmark 3: Mohr Coulomb (MC) plastic material compressed by a constant velocity

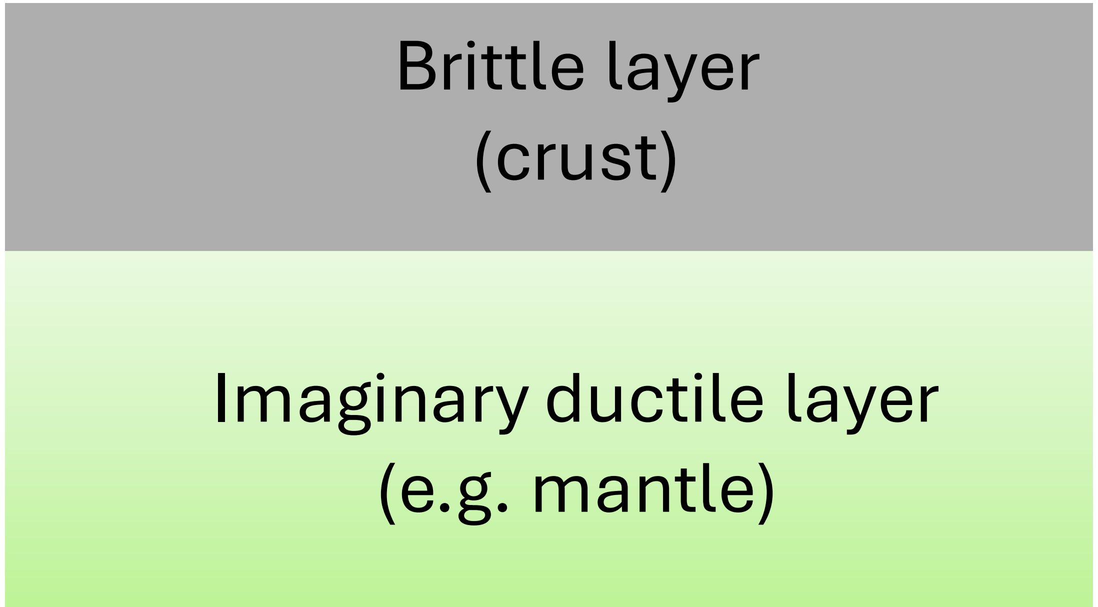
Rel. error for dt/eta = 10.0: 6.076e-03
Rel. error for dt/eta = 1e-2: 2.350e-03
Rel. error for dt/eta = 2e-3: 8.188e-04
Rel. error for dt/eta = 1e-3: 5.447e-04
Rel. error for dt/eta = 5e-4: 4.168e-04
Rel. error for dt/eta = 1e-6: 2.920e-04

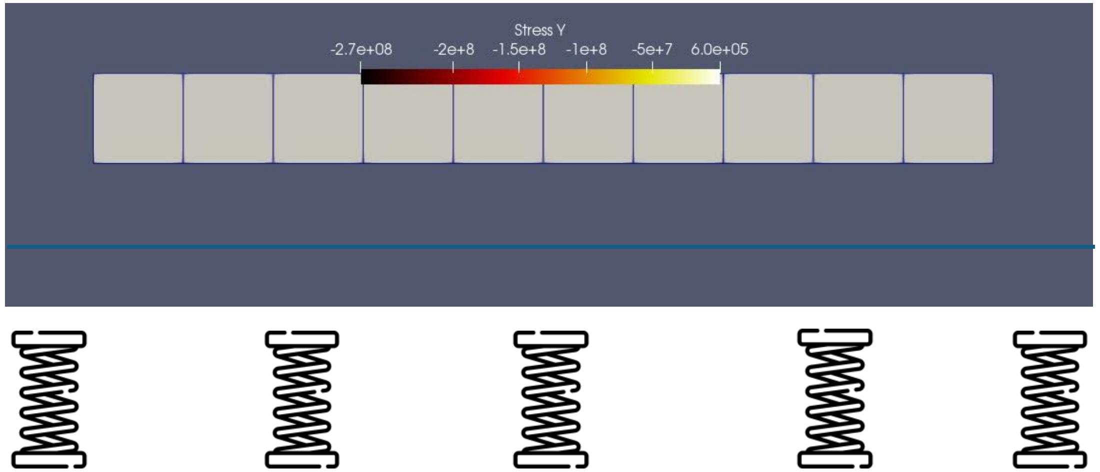


Elasticity

Perfect plasticity

Winker foundation (spring boundary)

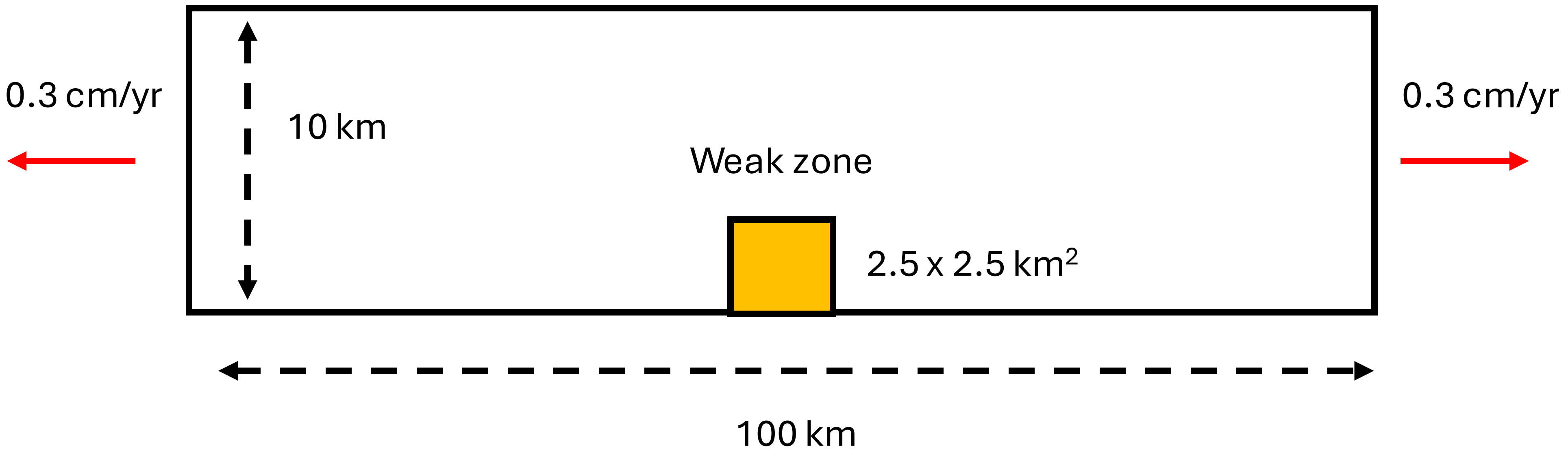




$f = -\int \rho(mantle) g \Delta h n_z dS_z$. on a bottom boudary.

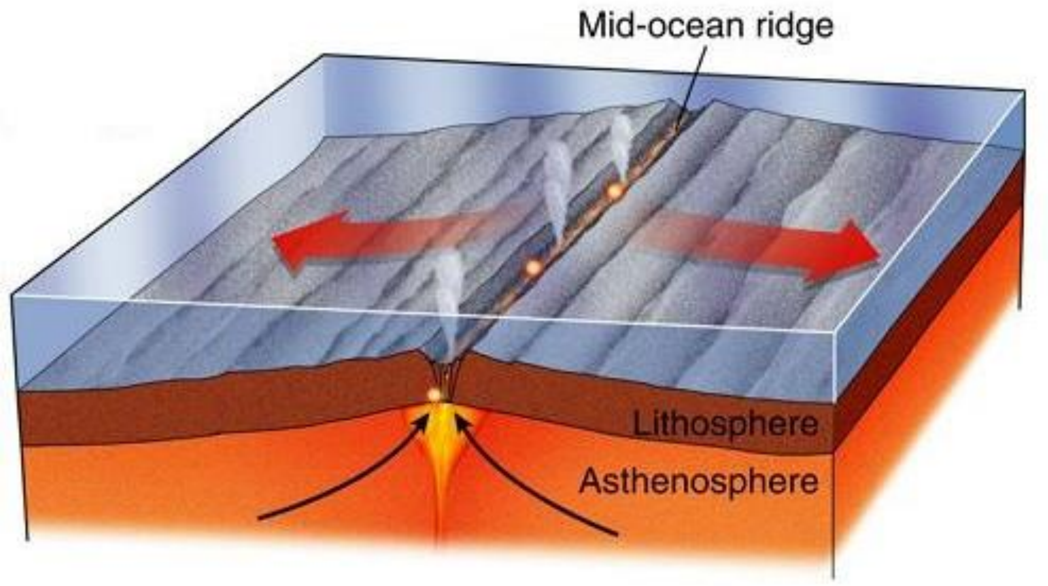
Δh is the height or thickness beneath a brittle layer.

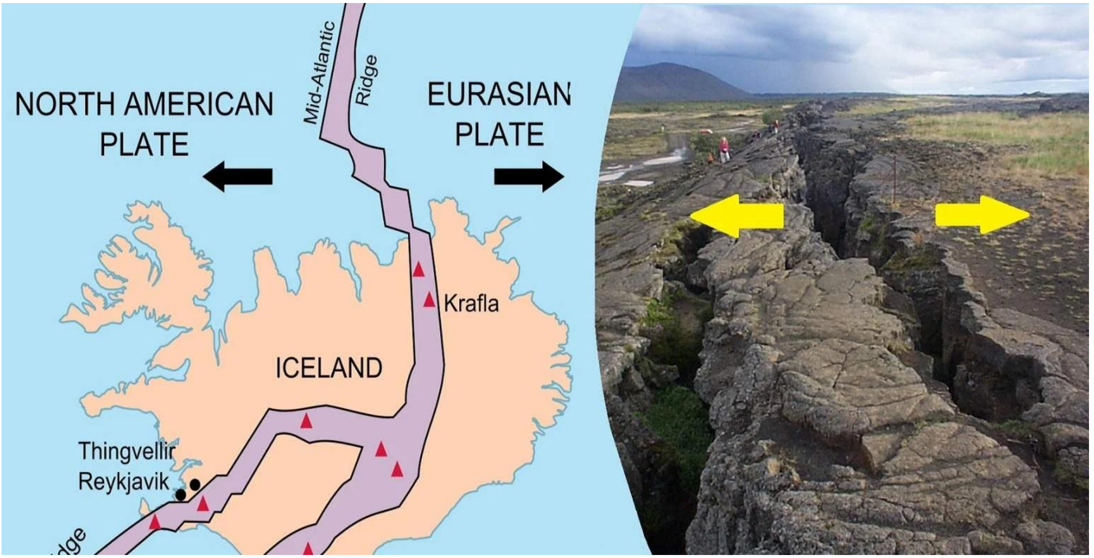
Geological application: normal fault evolution



Mid-Ocean ridge

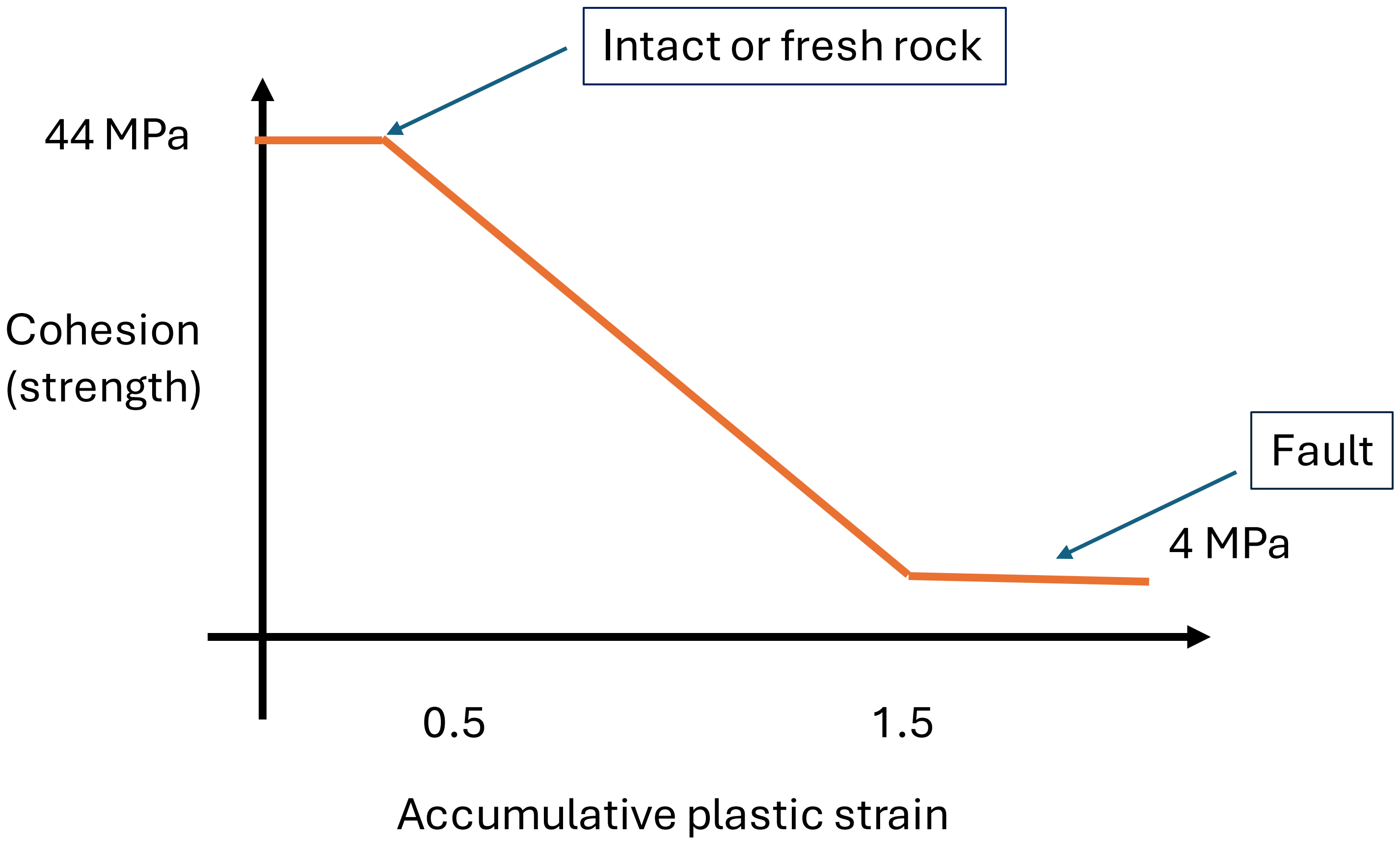
Due to tectonic stretching, a ridge is formed





https://www.geologyin.com/2017/07/why-iceland-is-being-torn-apart.html

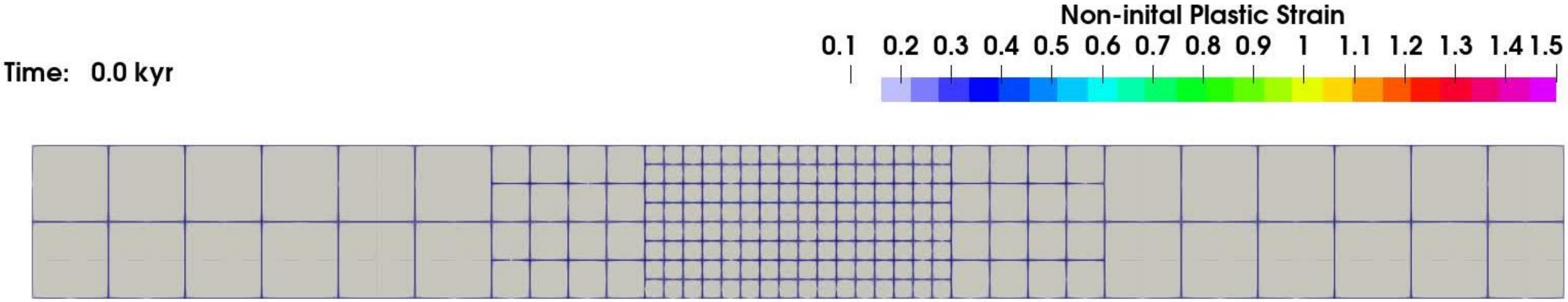
Plastic weakening mechanisms



Material properties

property	value
Density (kg/m^3)	2700.0
Lambda (GPa)	30.0
Mu (GPa)	30.0
Internal friction angle (deg)	31.0
Dilation angle (deg)	0.0
Surface diffusion (m^2/sec)	10^{-7}
Damping factor	0.8
Mass scaling factor	10^{6}

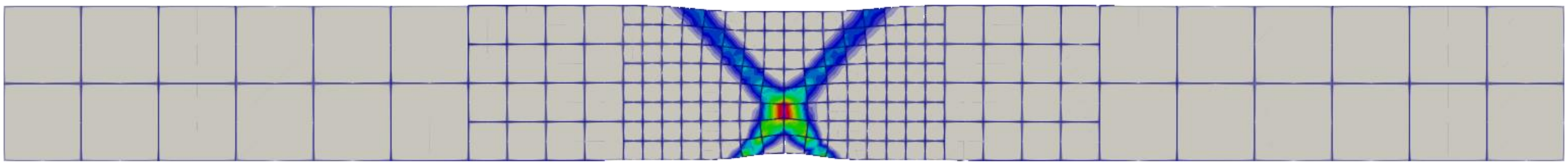
Non-initial plastic strain

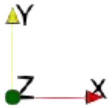


The minimum length is 625 m at zero → The minimum length is 1.61 m at 1 Myr. Remeshing and remapping are required.

Remeshing and remapping

- Laghost can conduct remesh using TMOP.
- Laghost interpolates (remap) H1 (GSLIB) and L2 (Remhos) onto the new mesh.

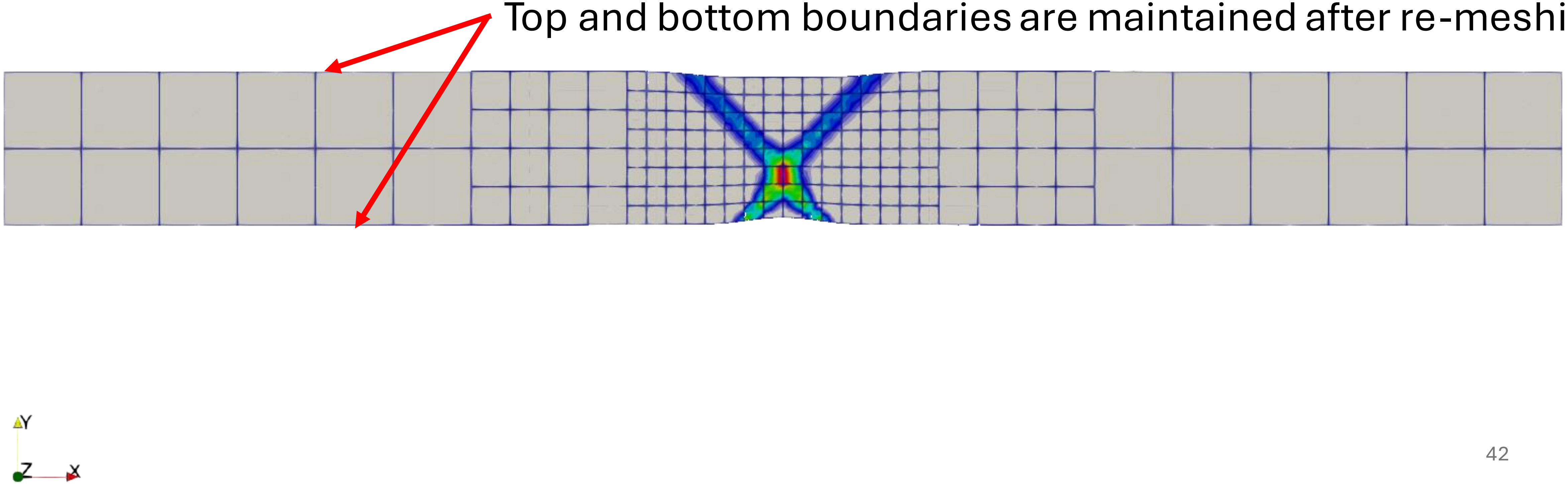




Remeshing and remapping

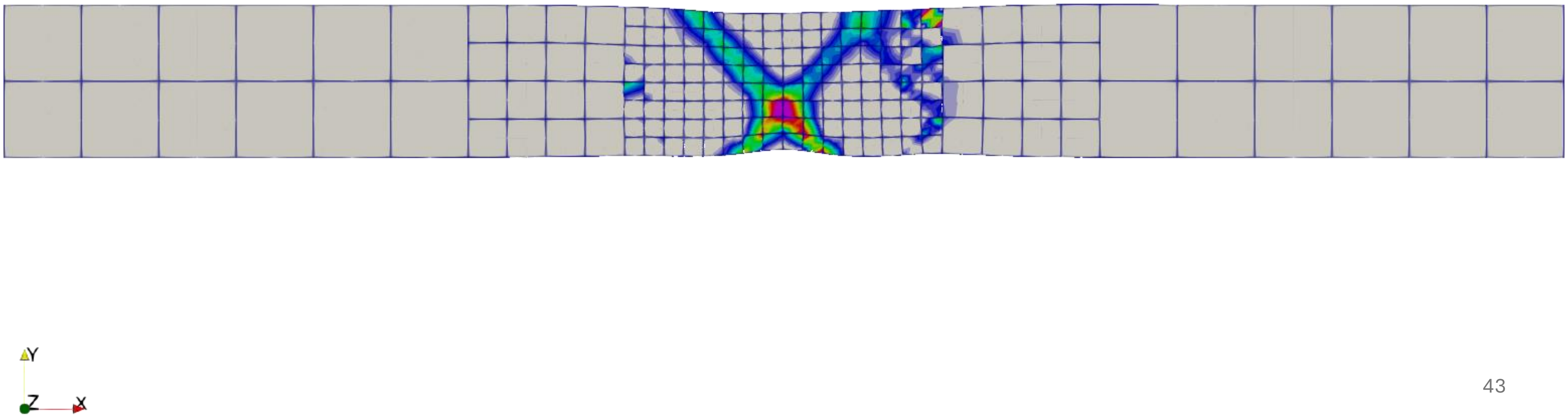
- Laghost can conduct remesh using TMOP.
- Laghost interpolates (remap) H1 (GSLIB) and L2 (remhos) onto the new mesh.

Top and bottom boundaries are maintained after re-meshing.



Remeshing and remapping

- Numerical instability has evolved on the NC boundaries.
- Remapping L2 by Remhos is not working correctly on NC mesh.



Future plans

- Enabling calculation in CUDA
  - Some r.h.s terms in the moment equation are not working correctly with GPU in the current version of Laghost.
- Implementation of thermal energy.
- Other rheology models

Thank you!