Solution of the electroencephalography (EEG) forward problem using MFEM

Ben Zwick

Intelligent Systems for Medicine Laboratory
The University of Western Australia

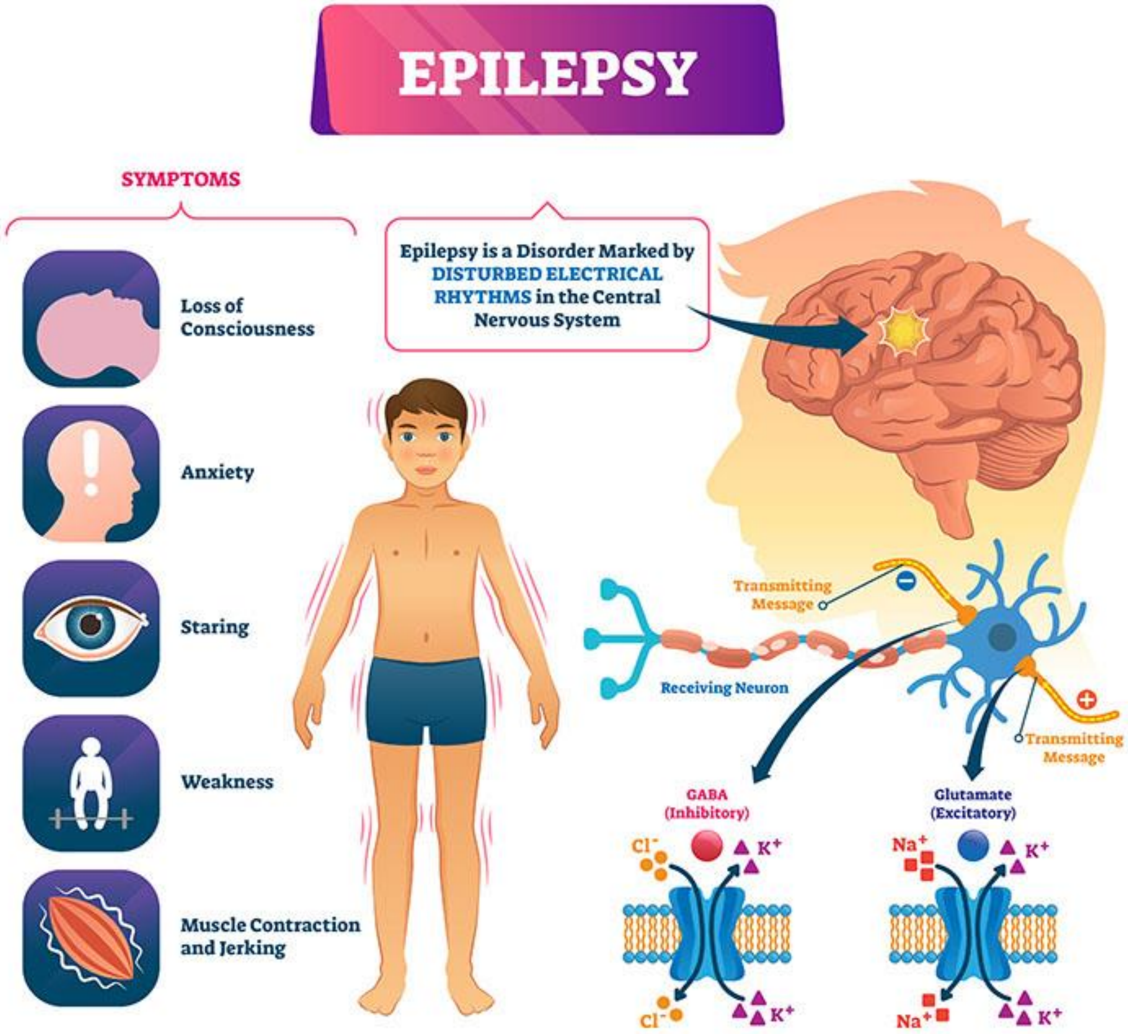
email: benjamin.zwick@uwa.edu.au

Motivation

> **Surgical Treatment for Epilepsy Is Underutilized**
>
> Surgical treatment for epilepsy is arguably the most underutilized of all proven effective therapeutic interventions in the field of medicine.

Jerome Engel (2003)

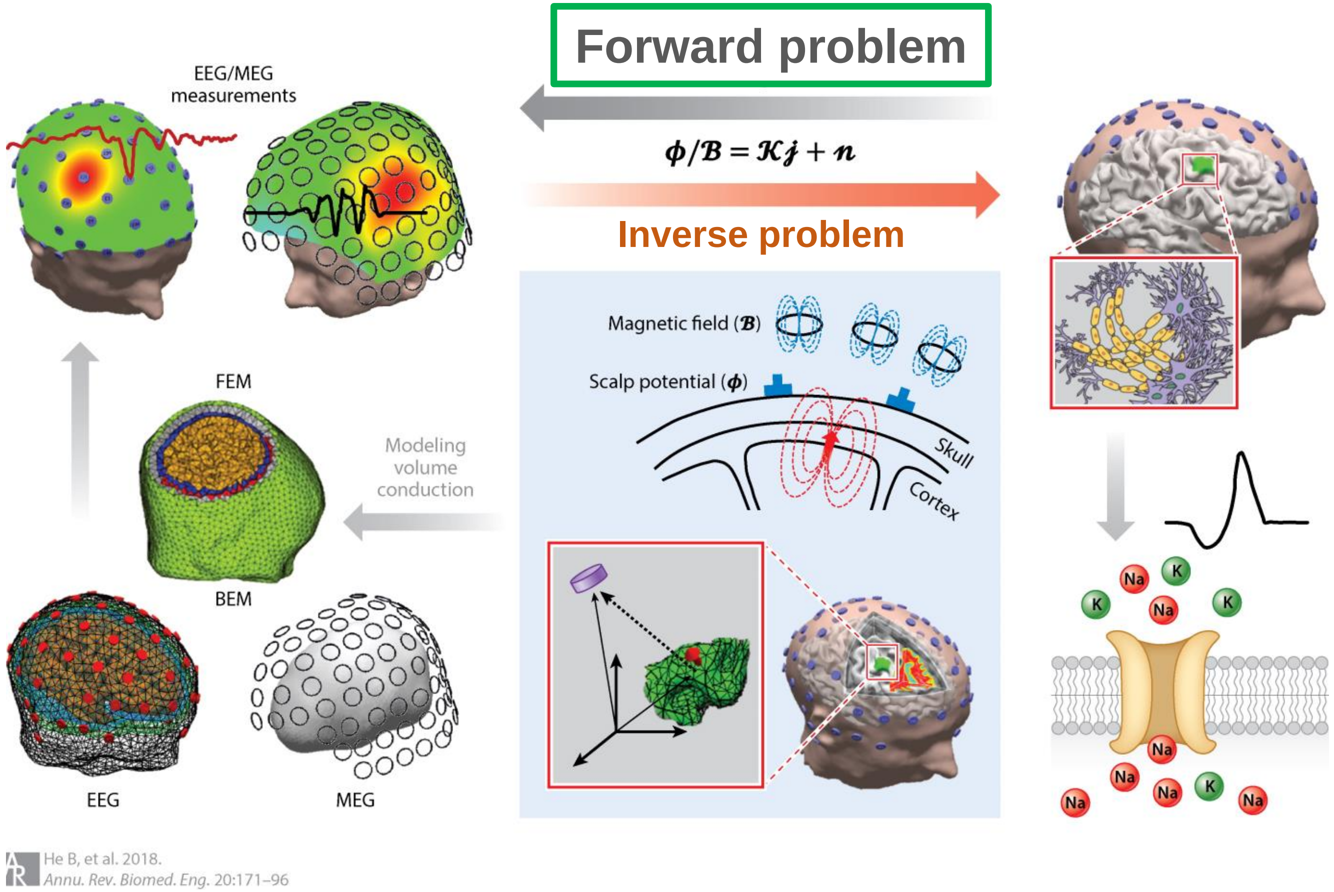
- Epilepsy is a common neurological disorder affecting about 50 million people of all ages
- Usual treatment is anti-epileptic drugs (AEDs) but these have side effects
- 40% of children with epilepsy develop drug-resistant refractory epilepsy
- Focal epilepsy can be cured by resection of the seizure-producing area of the brain
- **Major difficulty is to locate the seizure onset zone (SOZ)**



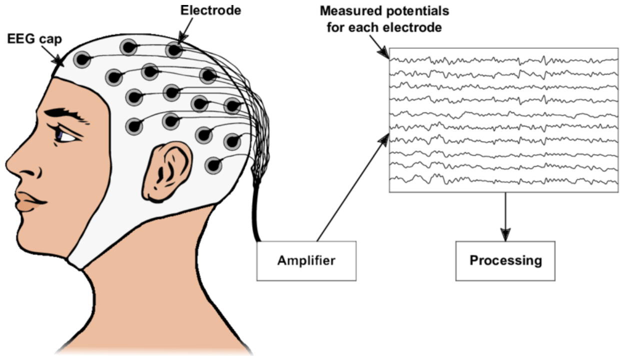
https://myacare.com/blog/pediatric-epilepsy-how-is-epilepsy-different-in-children

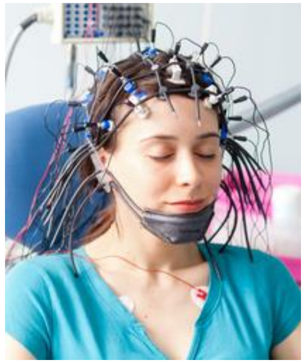
- Engel JJ. A Greater Role for Surgical Treatment of Epilepsy: Why and When? Epilepsy Currents. 2003;3(2):37–40.
- Engel JJ. The current place of epilepsy surgery. Current Opinion in Neurology. 2018;31(2):192–7.
- World Health Organization. Epilepsy: A Public Health Imperative. 2019 (https://www.who.int/publications/i/item/epilepsy-a-public-health-imperative)

Electrophysiological source imaging (ESI)

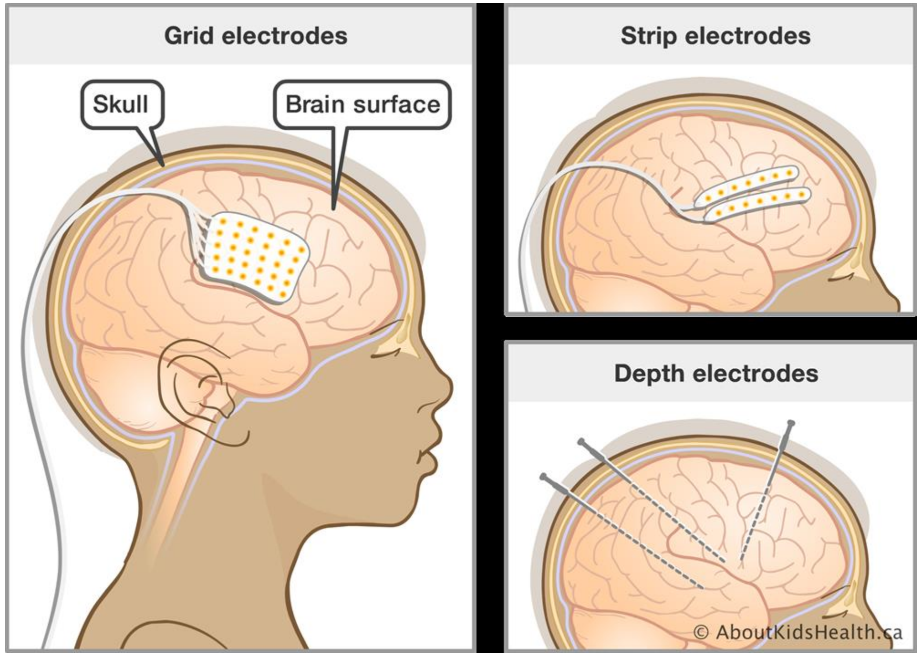


Electroencephalography (EEG)





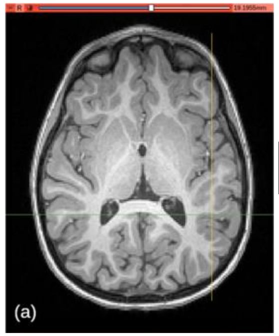
Invasive intracranial EEG (iEEG)



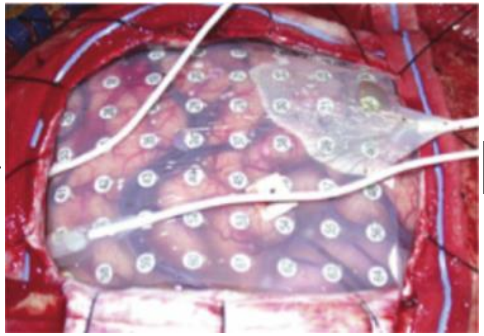
Electrocorticography (ECoG) SEEG

- He et al. Electrophysiological Source Imaging. Annu Rev Biomed Eng. 2018;20(1):171–96.

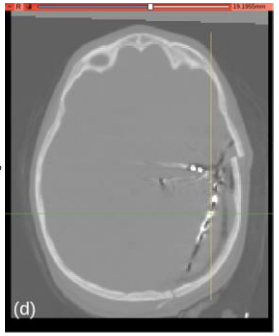
ECoG-induced brain shift



Preoperative MRI



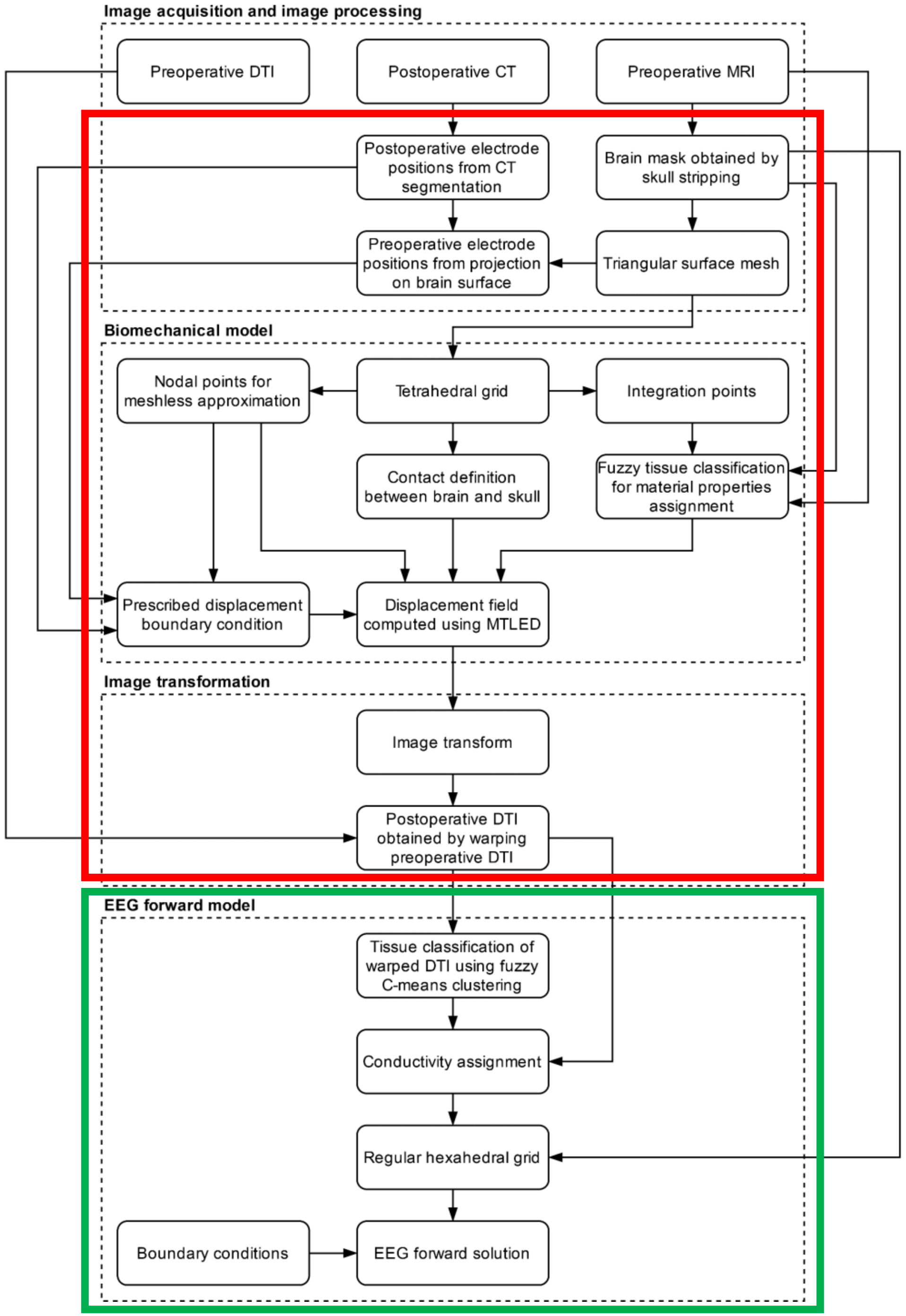
Electrodes implanted after craniotomy



Postoperative CT

Predicted postoperative geometry

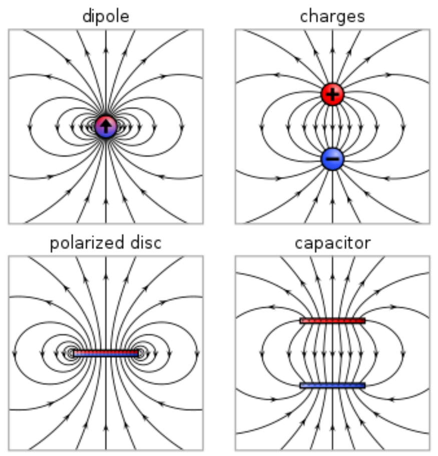
- Placement of subdural electrodes causes “brain shift”
- We use biomechanics-based image registration to register the preoperative MRI to the postoperative CT
- We solve the EEG/ECoG forward problem on the predicted postoperative (after electrode implantation) geometry



- Medical image data courtesy Boston Children’s Hospital
- Zwick BF et al. Patient-specific solution of the electrocorticography forward problem in deforming brain. NeuroImage. 2022;263:119649.

EEG forward problem

Current dipole $\vec{p}$ at x_0

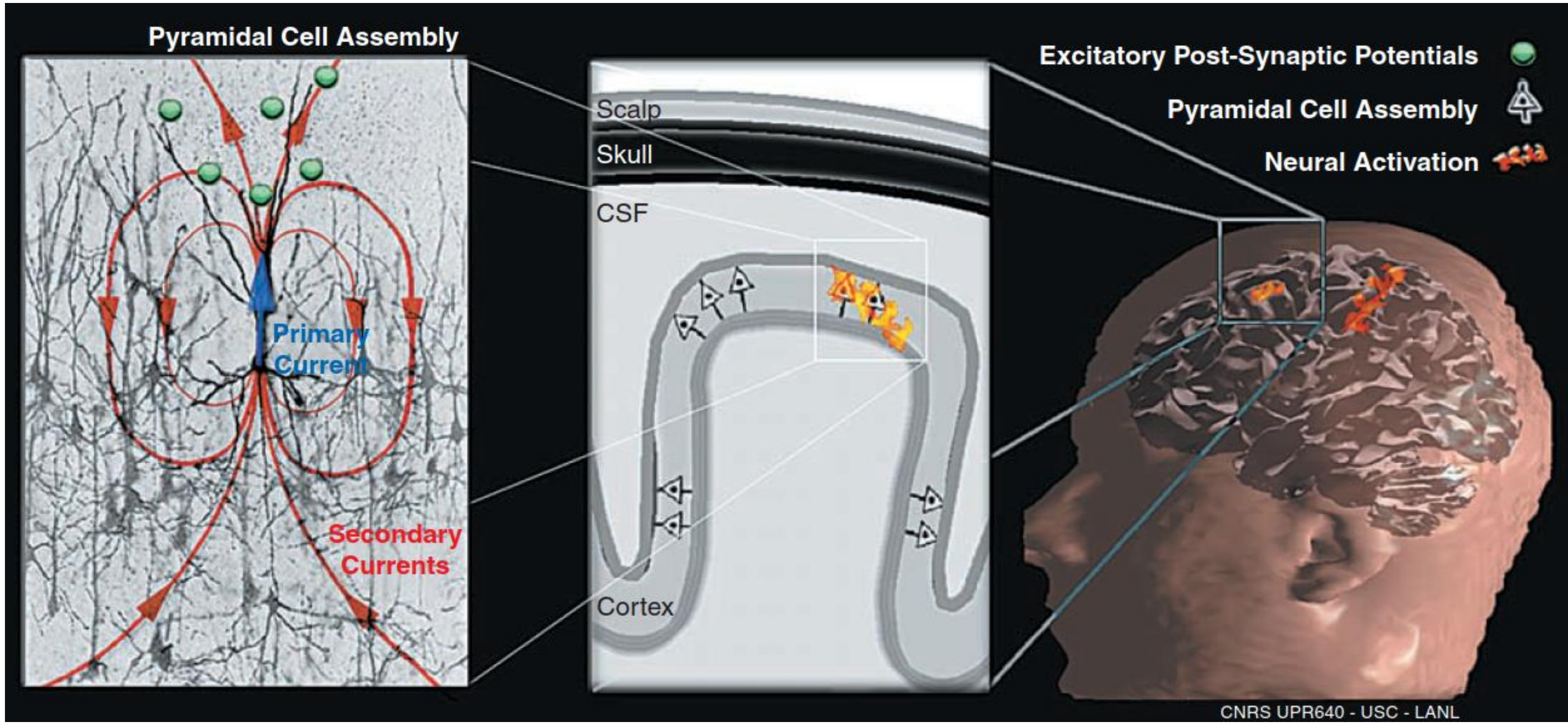


Mathematical model (electrostatics):

- Tensor-weighted Poisson's equation with homogeneous Neumann BC (no current flows across scalp into air)
- Equivalent current dipole (ECD) model: In the simplest case the activated part of the brain is so small that it can be considered as a single point x_0.

$$-\nabla \cdot \left(\sigma(\nabla u)\right) = f \text{ in } \Omega$$
$$n \cdot \left(\sigma(\nabla u)\right) = 0 \text{ on } \partial\Omega$$

$$f = \nabla \cdot \vec{J}$$
$$\vec{J} = \vec{p} \cdot \delta(x - x_0)$$



- Baillet, S., Mosher, J.C., and Leahy, R.M. (2001). Electromagnetic brain mapping. IEEE Signal Processing Magazine 18, 14-30.
- Brette & Destexhe (2012). Handbook of Neural Activity Measurement.

Software implementation using MFEM + PyMFEM

- Mesh
  - Regular hexahedral grid in MFEM format generated using MVox (https://github.com/benzwick/mvox)
- Anisotropic and inhomogenous conductivity $\sigma(x)$
  - MatrixArrayCoefficient with L2 vector GridFunction for 6 symmetric tensor components
- Poisson's equation
  - DiffusionIntegrator
- Dipole source (full subtraction approach, Drechsler et al. 2009)
  - $u(x) = u^{\mathrm{corr},y}(x) + u^{\infty,y}(x)$
  - NumPy + Numba to define PyMFEM VectorNumbaFunction
  - DomainLFGradIntegrator + BoundaryNormalLFIntegrator
- Solver
  - Hypre PCG solver with BoomerAMG preconditioner
- Miscellaneous
  - Reused assembled stiffness matrix for computation of lead field matrix with multiple RHS
  - Combined PyMFEM with NumPy to simplify postprocessing

$$\begin{aligned} div\ \sigma(x)\nabla\left(u^{\mathrm{corr},y}(x) + u^{\infty,y}(x)\right) &= J^p(x) \quad \text{for a.e. } x\in\Omega, \\ \langle\sigma(x)\nabla\left(u^{\mathrm{corr},y}(x) + u^{\infty,y}(x)\right), n(x)\rangle &= 0 \quad \text{for a.e. } x\in\Gamma, \\ \int_\Omega \left(u^{\mathrm{corr},y}(x) + u^{\infty,y}(x)\right)dx &= 0. \end{aligned}$$

Eq. (5) can be written in the form

$$div\ \sigma(x)\nabla u^{\mathrm{corr},y}(x) = f(x) \quad \text{for a.e. } x\in\Omega,$$

$$\langle\sigma(x)\nabla u^{\mathrm{corr},y}(x), n(x)\rangle = g(x) \quad \text{for a.e. } x\in\Gamma,$$

$$\int_\Omega u^{\mathrm{corr},y}(x)dx = -\int_\Omega u^{\infty,y}(x)dx,$$

with the right-hand side functions

$$f(x) := div\ (\sigma(y) - \sigma(x))\nabla u^{\infty,y}(x) \quad \text{for } x\in\Omega,$$

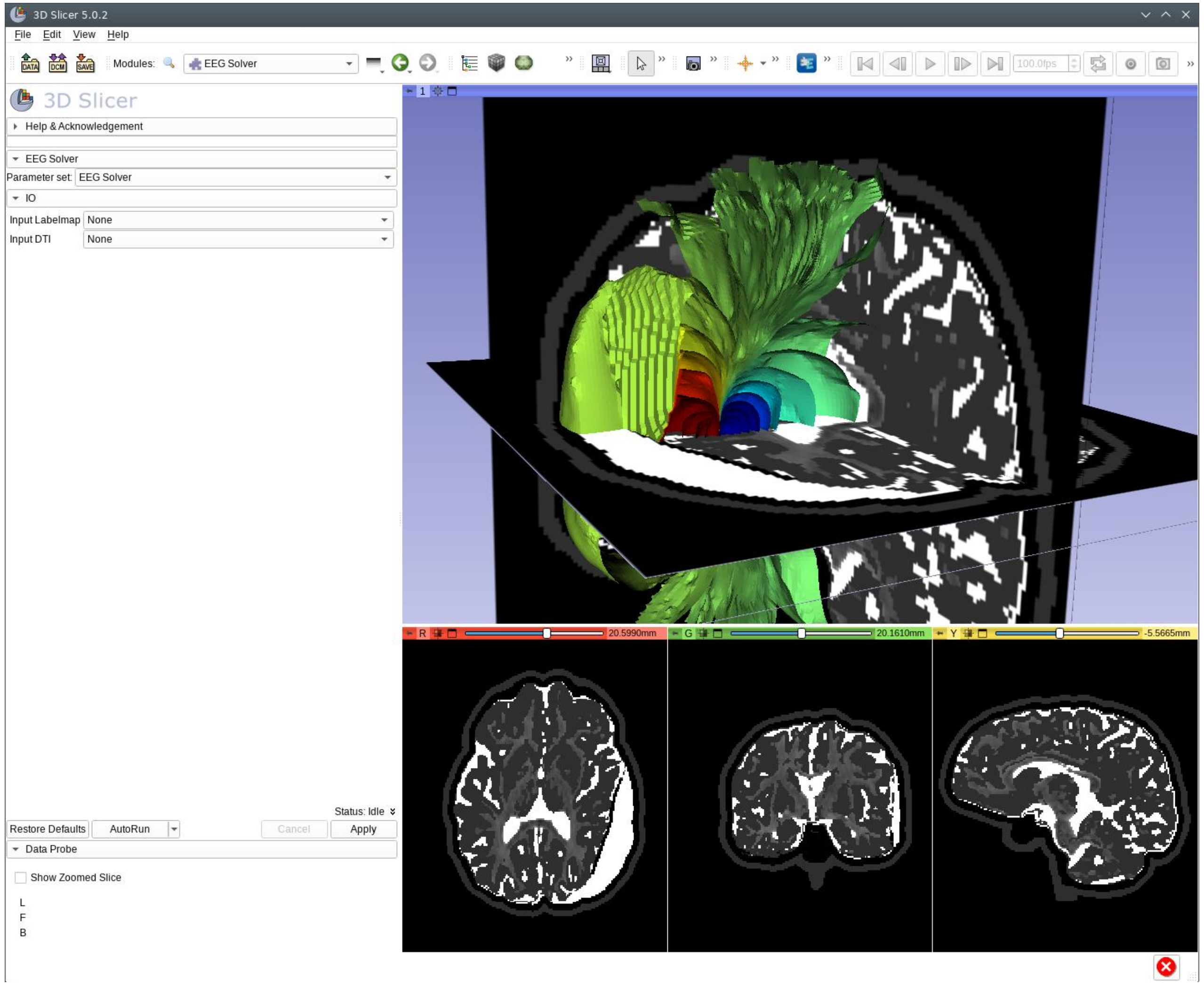
$$g(x) := -\langle\sigma(x)\nabla u^{\infty,y}(x), n(x)\rangle \quad \text{for } x\in\Gamma.$$

- Drechsler et al. A full subtraction approach for finite element method based source analysis using constrained Delaunay tetrahedralisation. NeuroImage. 2009;46(4):1055–65.

SlicerCBM: Computational biophysics for medicine in 3D Slicer

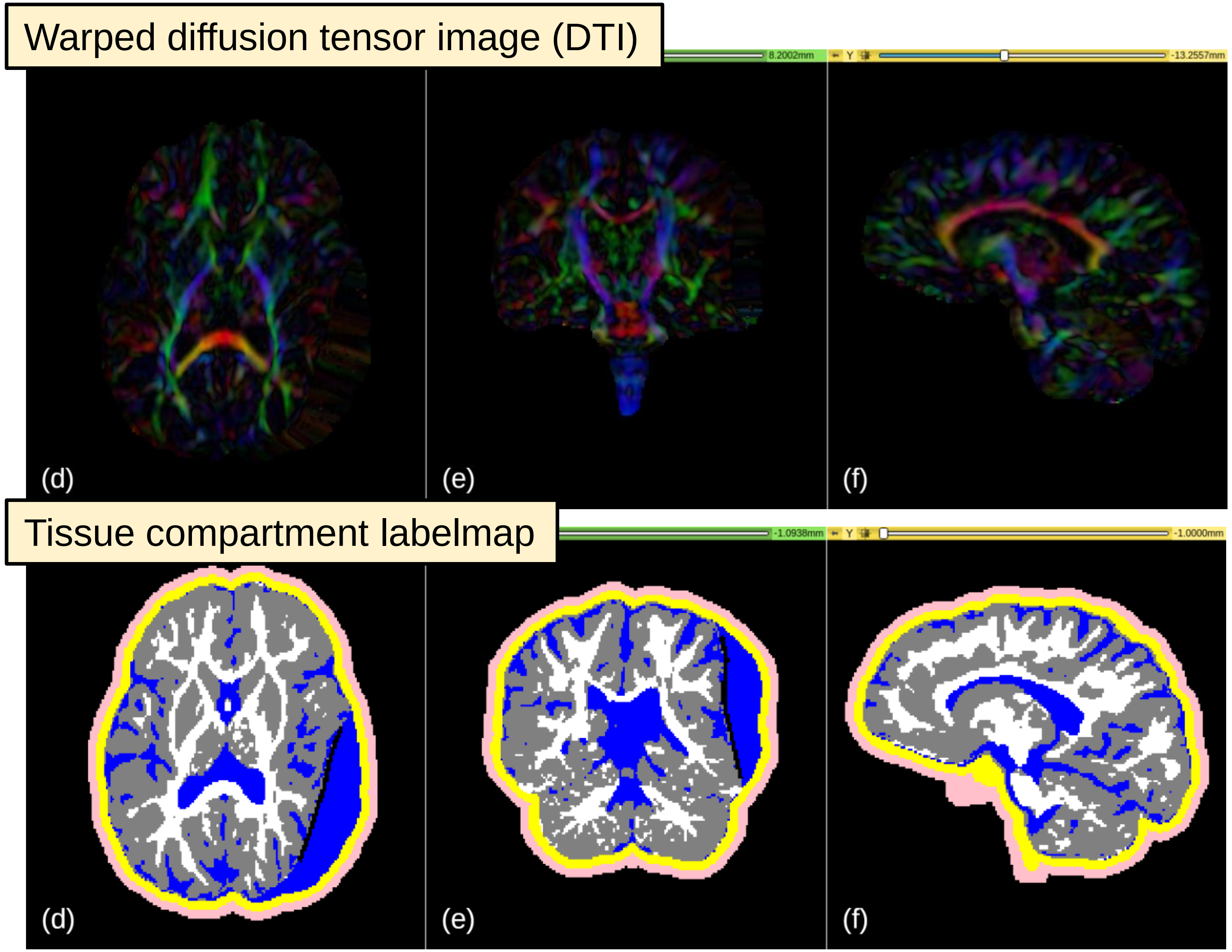
- 3D Slicer is a free, open source and multi-platform software package widely used for medical, biomedical, and related imaging research
- 3D Slicer can be extended using C++ or Python
- We are developing the SlicerCBM extension for 3D Slicer to include modules for biomechanical and electrical modeling

mfem



- 3D Slicer: https://www.slicer.org
- SlicerCBM: https://github.com/SlicerCBM

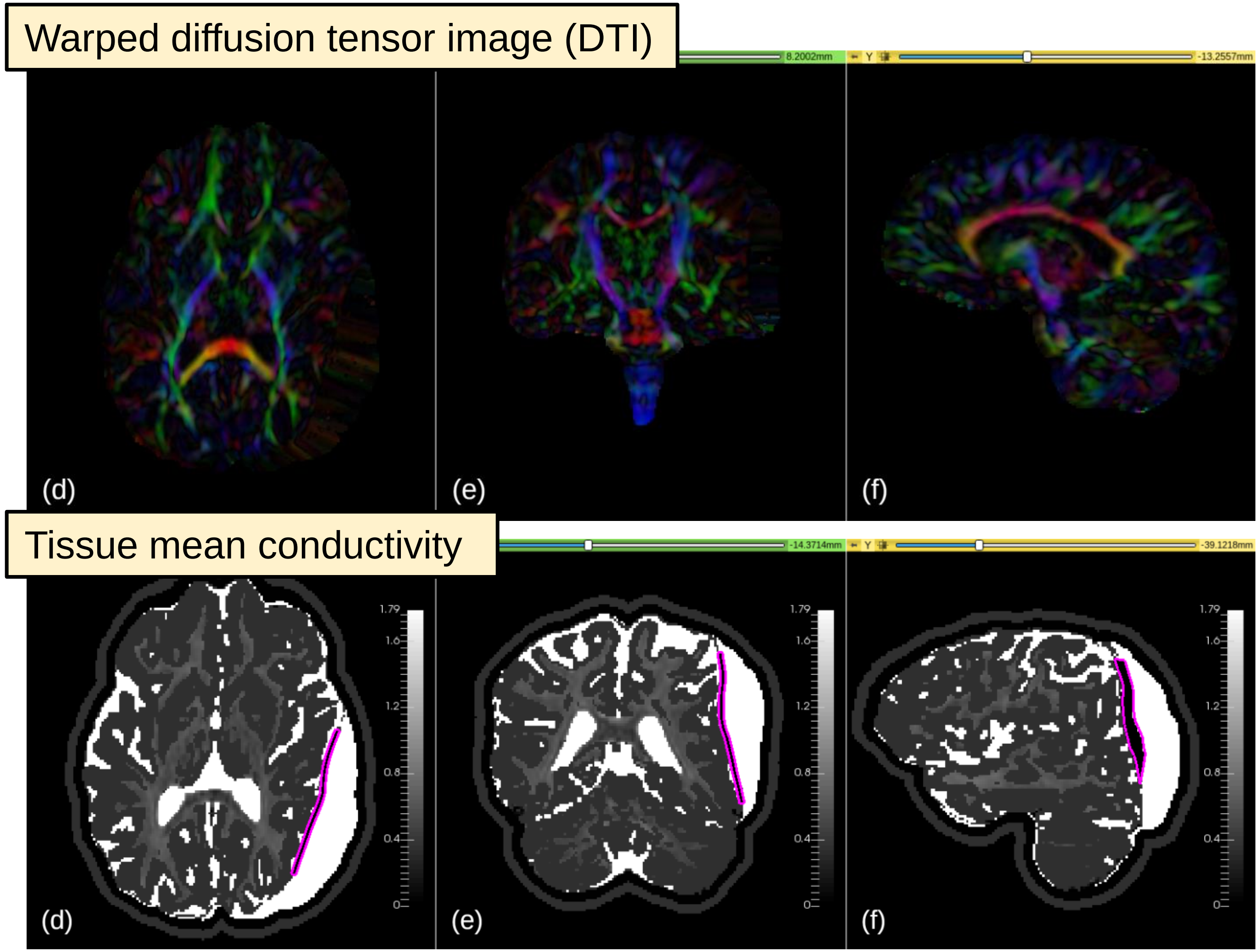
DTI-based soft tissue classification



- 5 tissue compartments:
  - Scalp
  - Skull
  - CSF
  - Gray matter
  - White matter
- Fuzzy C-means clustering to classify CSF, gray matter and white matter in two steps:
  1. Mean diffusivity used to separate CSF and brain tissue
  2. Fractional anisotropy used to separate gray matter and white matter

- Pierpaoli C et al. Diffusion tensor MR imaging of the human brain. Radiology. 1996;201(3):637–48.
- Zwick BF et al. Patient-specific solution of the electrocorticography forward problem in deforming brain. NeuroImage. 2022;263:119649.

DTI-based conductivity assignment



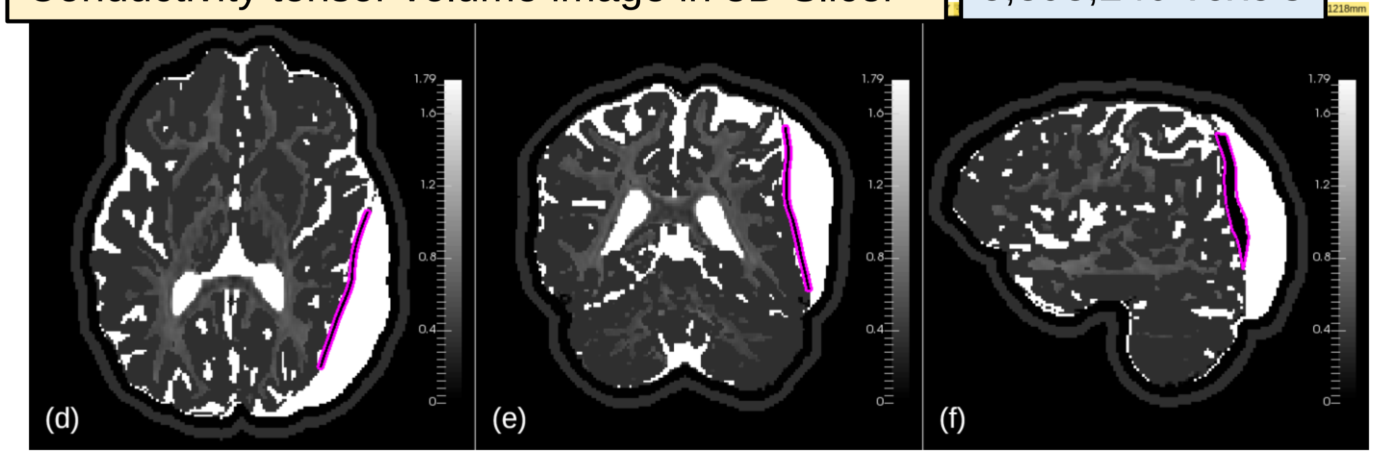
- Isotropic conductivity for all tissue compartments except white matter
- Anisotropic conductivity for white matter using diffusion tensor MRI (Tuch et al. 2001)

Compartment	Conductivity (S/m)
Scalp	0.33
Skull	0.012
Cerebrospinal fluid (CSF)	1.79
Electrode grid array	10e-6
Gray matter	0.33
White matter	anisotropic and inhomogeneous

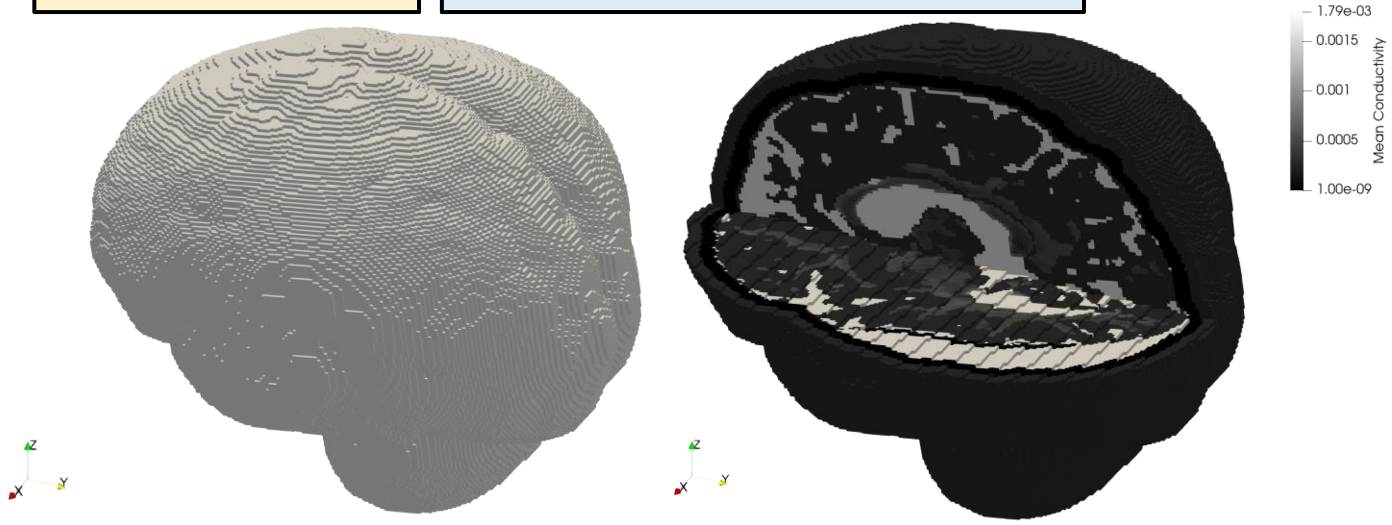
- Tuch DS et al. Conductivity tensor mapping of the human brain using diffusion tensor MRI. PNAS. 2001;98(20):11697–701.
- Zwick BF et al. Patient-specific solution of the electrocorticography forward problem in deforming brain. NeuroImage. 2022;263:119649.

Mesh generation

Conductivity tensor volume image in 3D Slicer | 5,898,240 voxels



Mesh in ParaView | 1,565,095 hexahedral elements

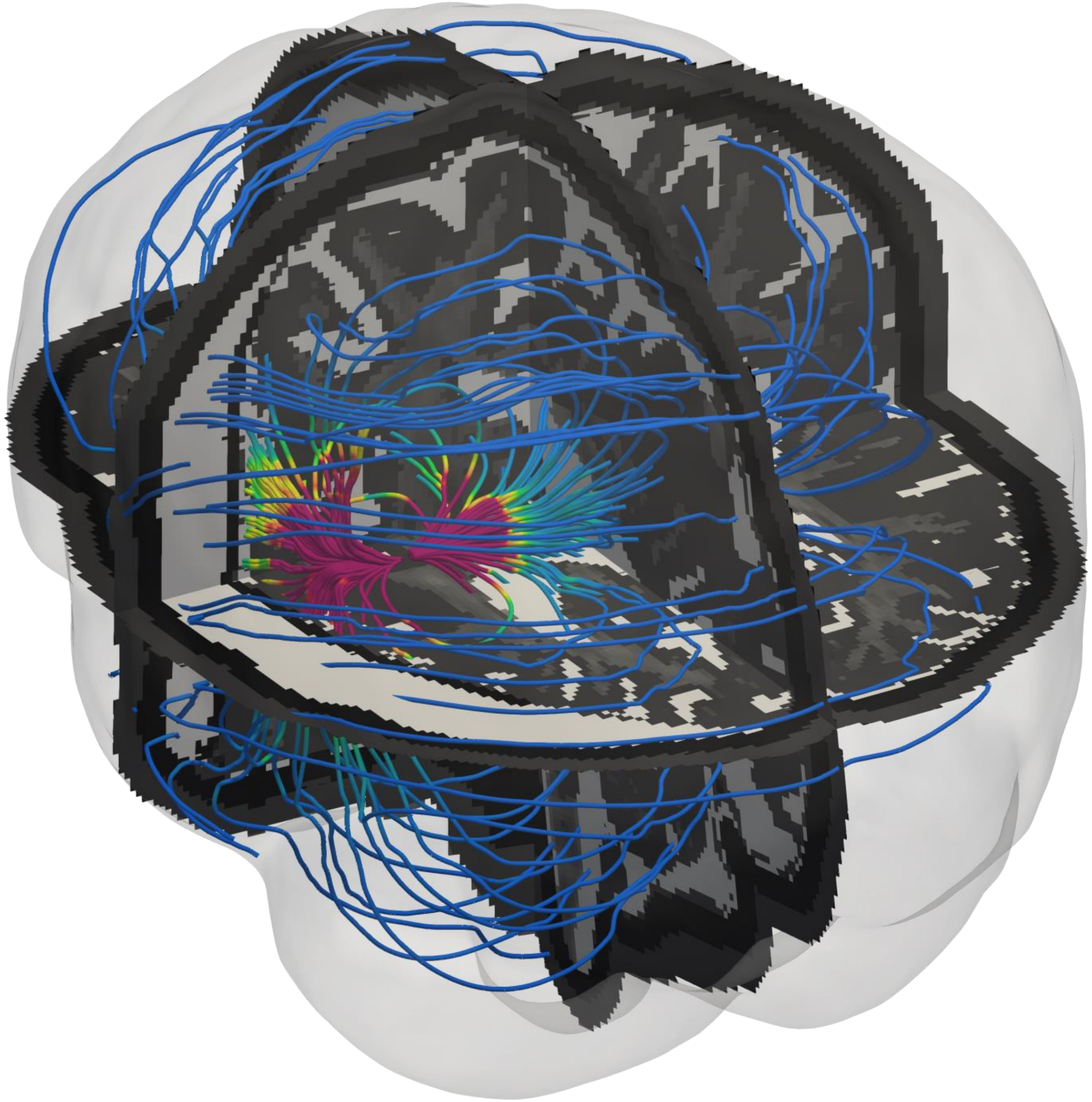


- "Image-as-a-model" concept where image is used to generate mesh without surface reconstruction
- Regular hexahedral grid with elements corresponding to image voxels
- MVox is an MFEM-based mesh generator that converts volume (3D) images to regular hexahedral meshes
- Inputs:
  - Mask
  - Labelmap (attributes)
  - Conductivity tensor image
- Outputs:
  - Mesh (MFEM formats)
  - Grid functions (conductivity)

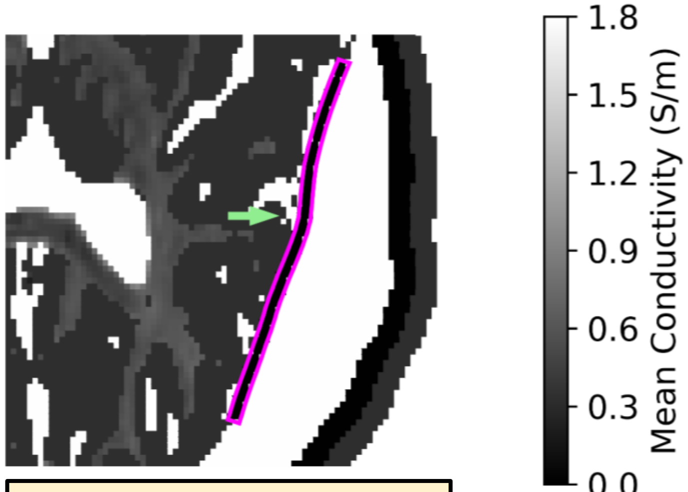
- MVox mesh generator: https://github.com/benzwick/mvox

Results

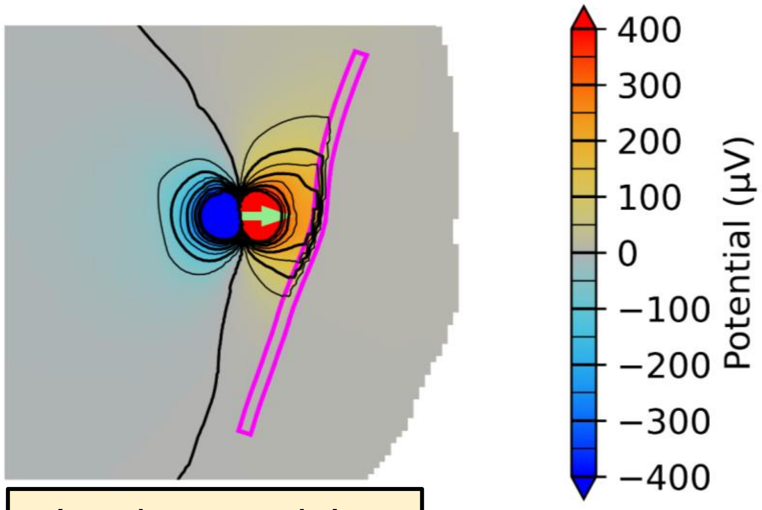
- Dipole source located within temporal lobe
- 1,618,745 nodes
- 1,565,095 elements
- Laptop computer with Intel Core i7 4.10 GHz CPU and 32 GB RAM
- Total run time:
  - Serial: 224 s
  - MPI with 4 proc: 87 s 2.6x faster
- Assembly and solution:
  - Serial: 140 s
  - MPI with 4 proc: 53 s 2.6x faster



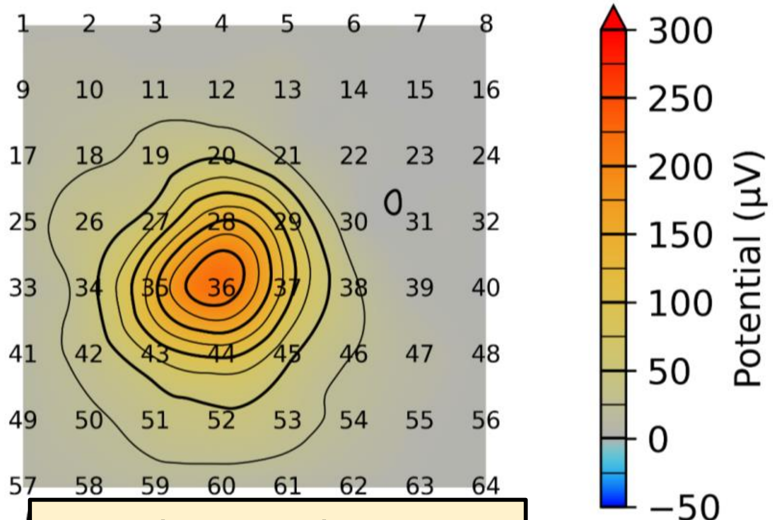
Results



Mean conductivity on axial slice of brain



Electric potential on axial slice of brain



Electric potential on 8x8 electrode grid array

- Green arrow: dipole source located within temporal lobe
- Purple outline: electrode grid array

- Zwick BF et al. Patient-specific solution of the electrocorticography forward problem in deforming brain. NeuroImage. 2022;263:119649.

Future work

- Implement additional source models (Saint-Venant, Whitney, etc.)
- Implement discontinuous Galerkin (DG) method
- Improve parallelization
- Compare performance with existing software (DUNEuro, SimBio, etc.)
- Continue SlicerCBM extension development (https://slicercbm.github.io)
- Solve the EEG inverse problem for source localization

Conclusions

- MFEM + PyMFEM provides rapid development environment
- PyMFEM + Numba is very fast (comparable to C++)
- PyMFEM + NumPy + PyVista, etc. simplifies postprocessing of results
- MFEM makes it easy to implement subtraction method for dipole source
- Fast computations on commodity hardware using hypre preconditioners and solvers

Acknowledgments

- Intelligent Systems for Medicine Laboratory, The University of Western Australia (https://isml.ecm.uwa.edu.au):
  - Dr George Bourantas
  - Ms Saima Safdar
  - Dr Grand Joldes
  - Prof. Adam Wittek
  - Prof. Karol Miller
- Computational Radiology Laboratory, Boston Children's Hospital and Harvard Medical School:
  - Dr Damon Hyde
  - Prof. Simon Warfield
- Funding:
  - National Health and Medical Research Project Grant no. APP1162030
- MFEM and PyMFEM community