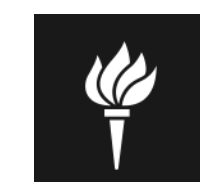


# Geometric Predicates for Unconditionally Robust Elastodynamics Simulation

Daniele Panozzo  
NYU Geometric Computing Lab



# Catastrophic Failures

- Wikipedia: "A catastrophic failure is a sudden and total failure from which recovery is impossible. *Catastrophic failures often lead to cascading systems failure.*"
- **Catastrophic failures** require user interaction to fix them.
- We usually do not tolerate catastrophic failures, but we usually accept them and think of them as unavoidable for meshing and simulation software.

# Many reasons, one is hidden in plain sight!

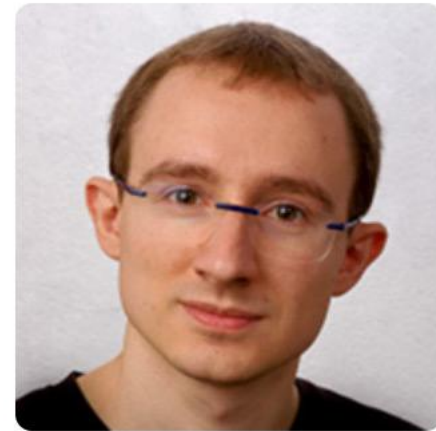
- We usually assume that floating point numbers are equivalent to real numbers
- This is however not the case, here is an example of statements/properties that are commonly used in numerical algorithms:
  - Multiplication distributes over addition
  - The gradient of a function is an ascend direction
  - The point at the intersection of two non-degenerate, non-parallel intersecting segments AB, CD is a point of both AB and CD
- The statements above are “mostly” true, so algorithms using them “often” work



# Geometric Computing Lab

University of Victoria

## Faculty



[Daniele Panozzo](#)



[Denis Zorin](#)

## Postdocs



Daniel Zint



Michael Tao



[Teseo Schneider](#)

## PhD Students



[Leyi Zhu](#)



[Siqi Wang](#)



[YunFan Zhou](#)



[Zizhou Huang](#)



[Arvi Gjoka](#)



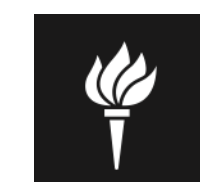
[Ryan Capouellez](#)



Max Paik



# Robust Forward Elastodynamics



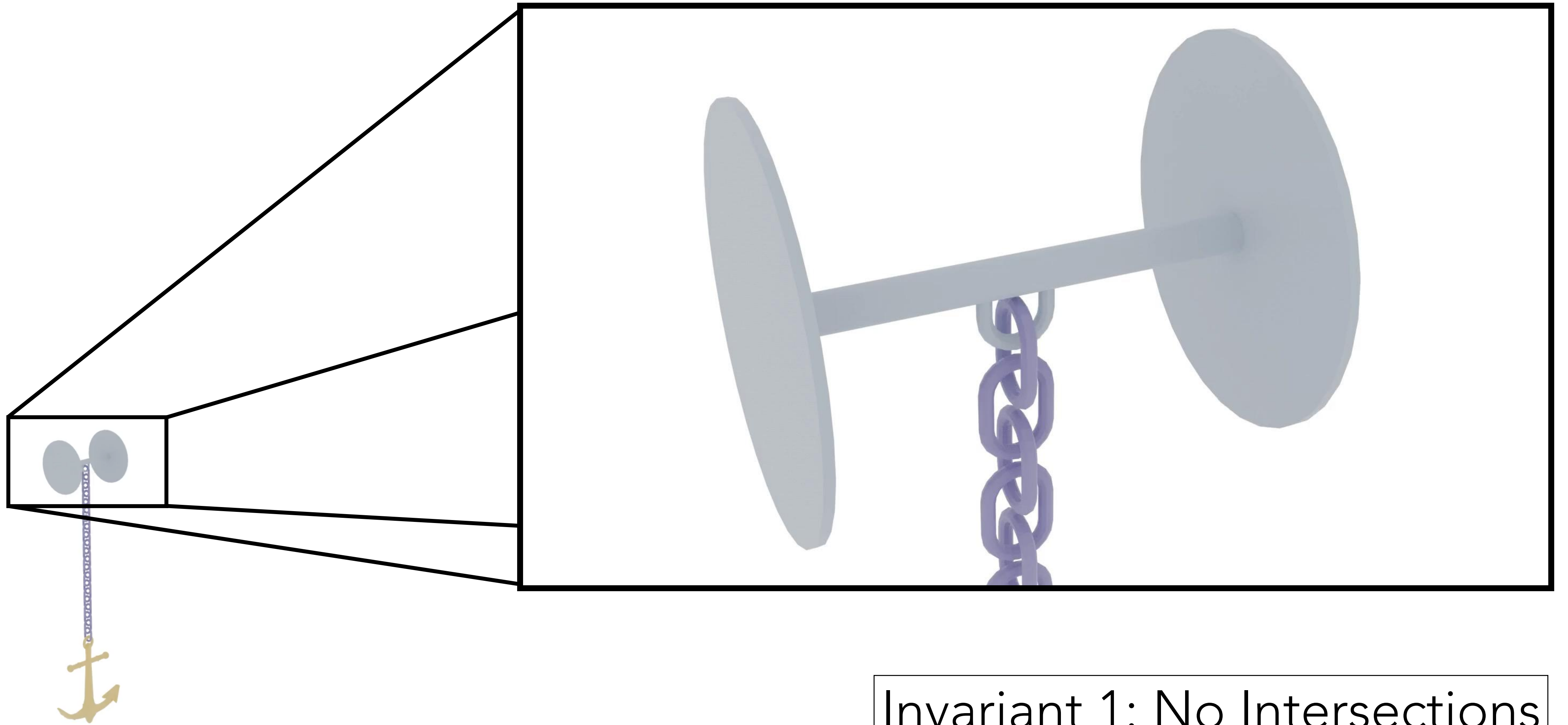


**Intersection-free Rigid Body Dynamics**

[Zachary Ferguson](#), [Minchen Li](#), [Teseo Schneider](#), [Francisca Gil Ureta](#), [Timothy Langlois](#), [Chenfanfu Jiang](#), [Denis Zorin](#),  
[Danny M. Kaufman](#), [Daniele Panozzo](#),

ACM Transaction on Graphics (SIGGRAPH), 2021

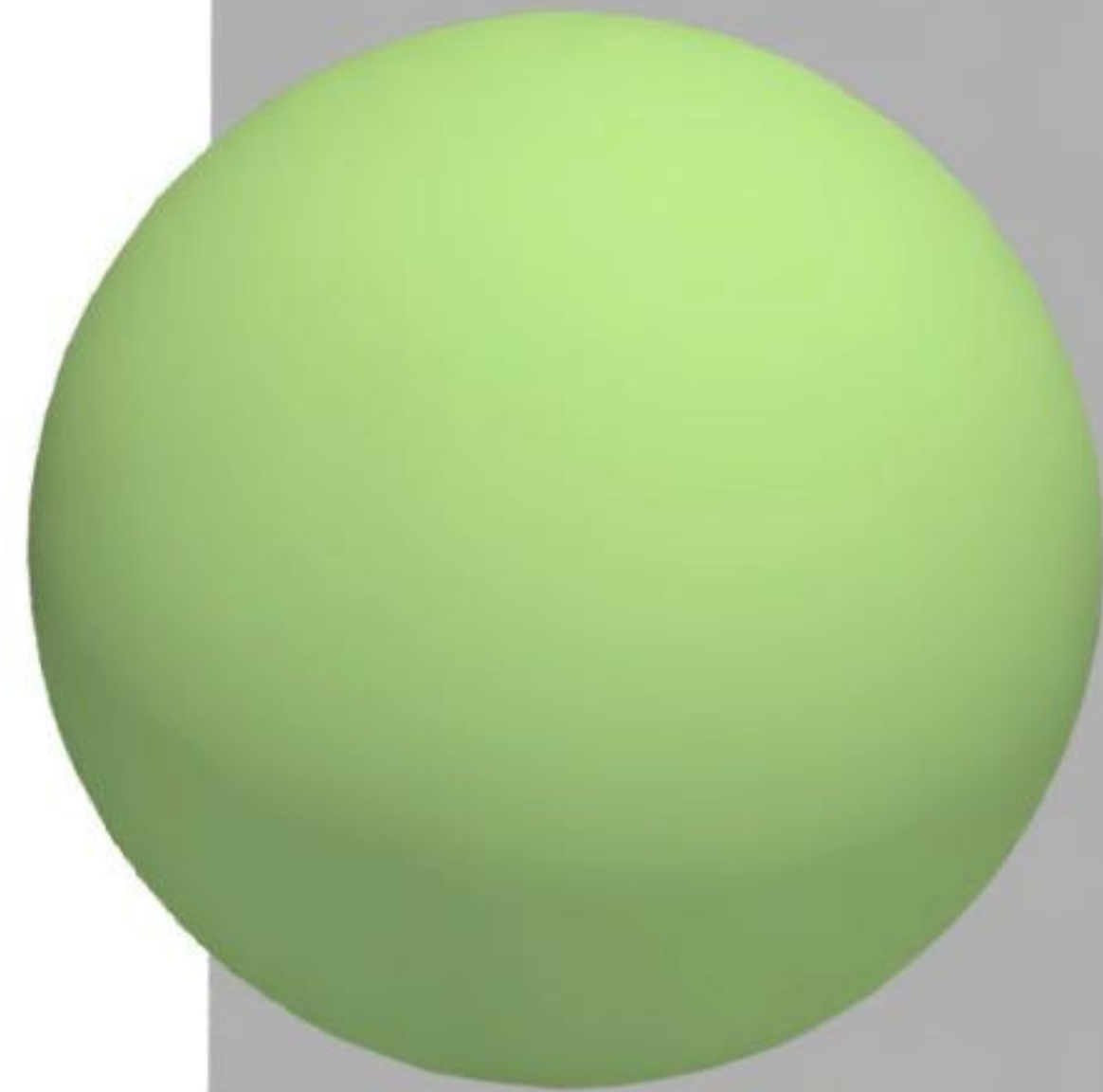
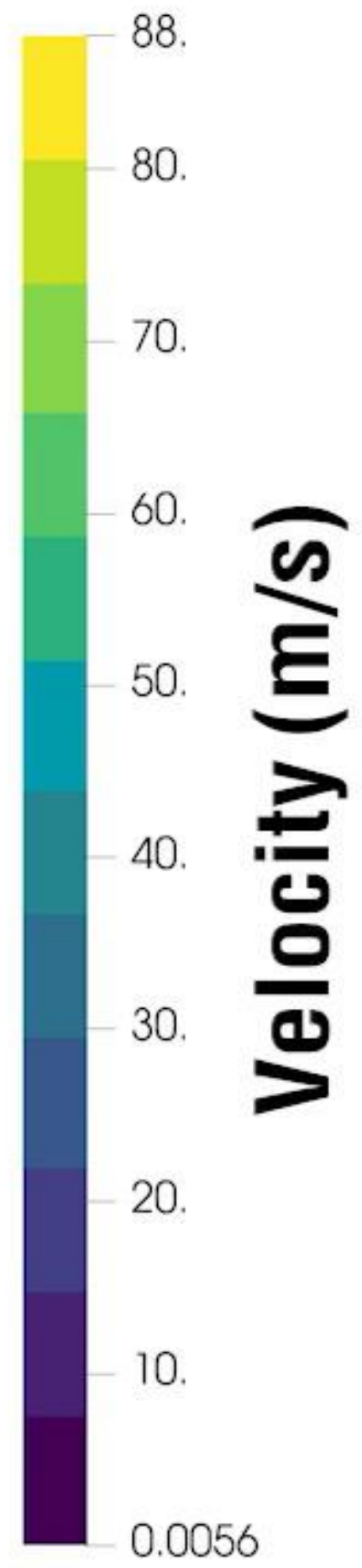
[\[Paper\]](#) [\[Video 1\]](#) [\[Video 2\]](#) [\[Website\]](#)



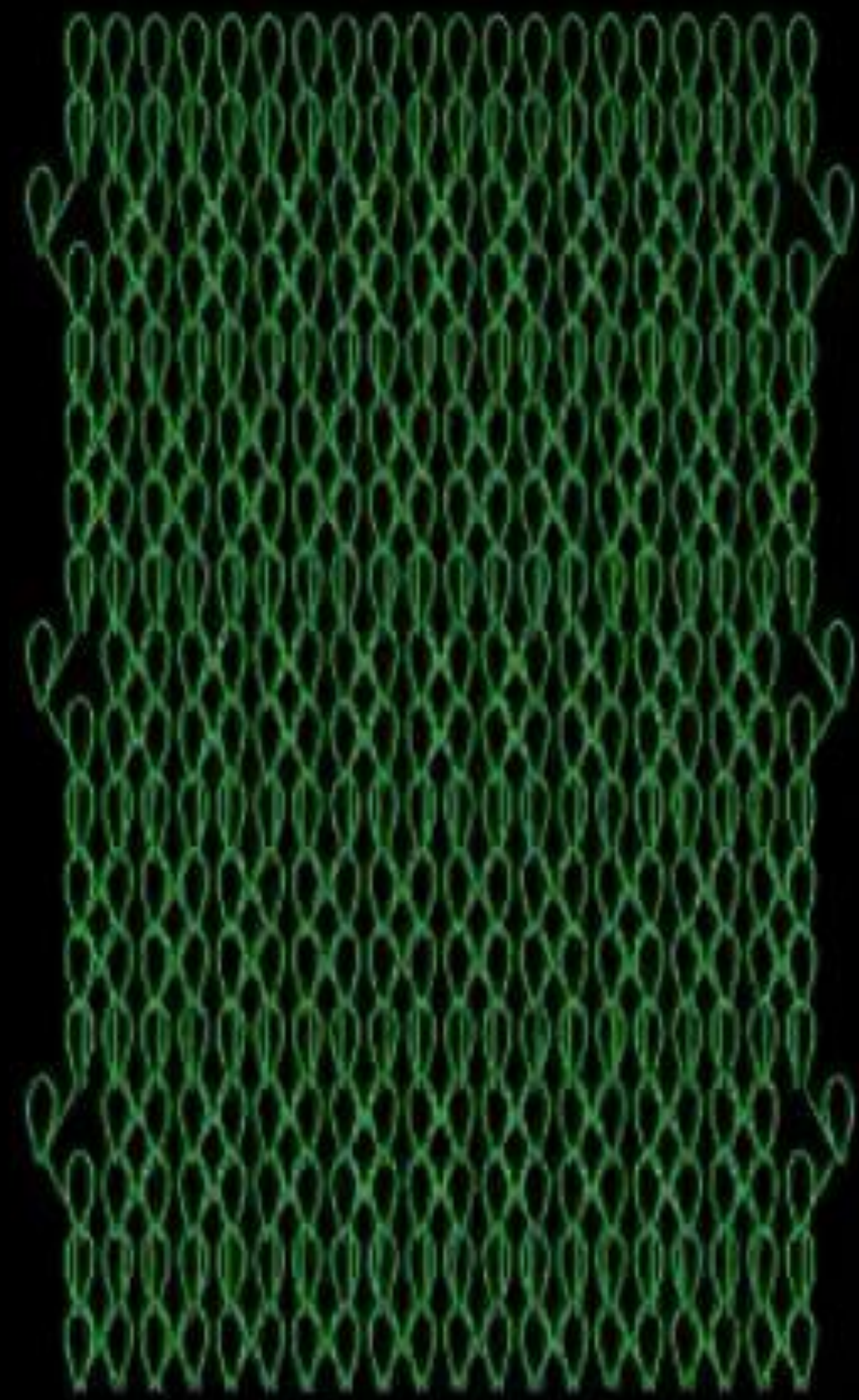
Invariant 1: No Intersections

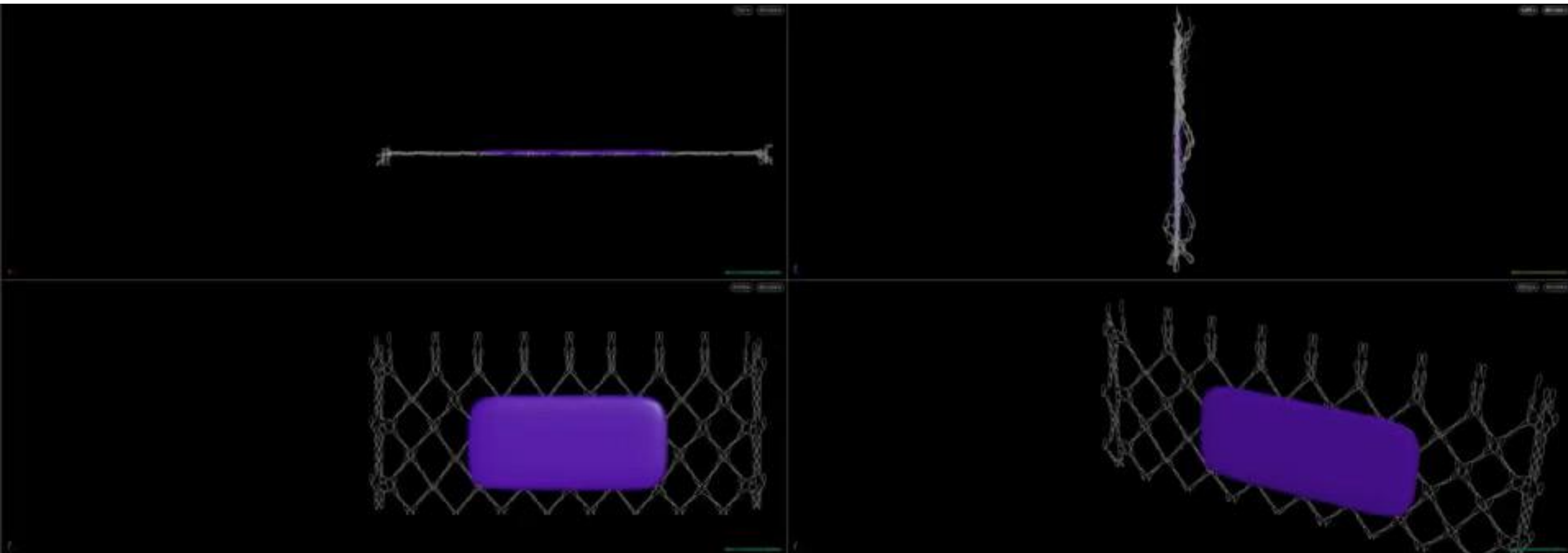


dt: 2e-5s



Invariant 2: Positive Volume

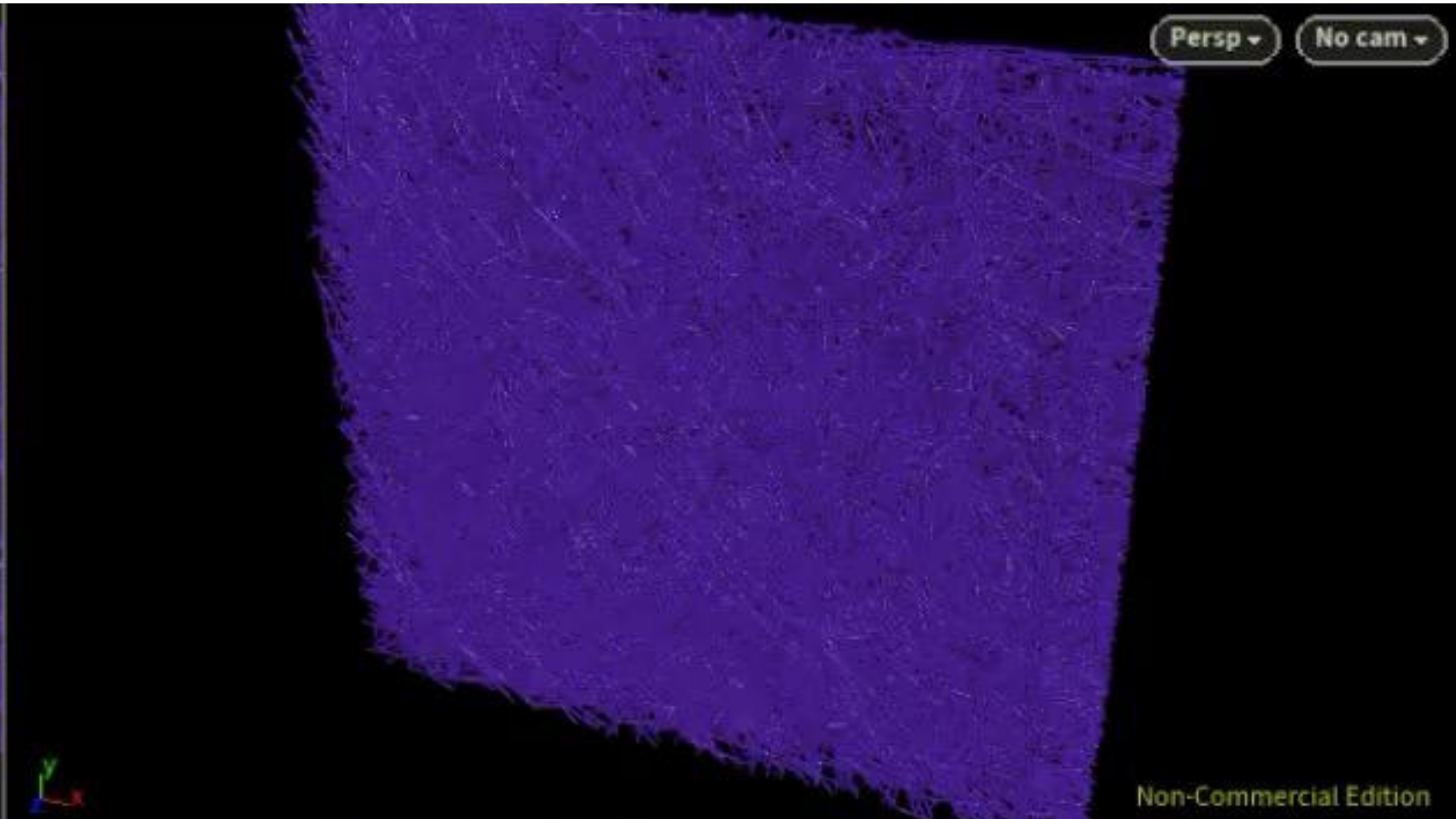
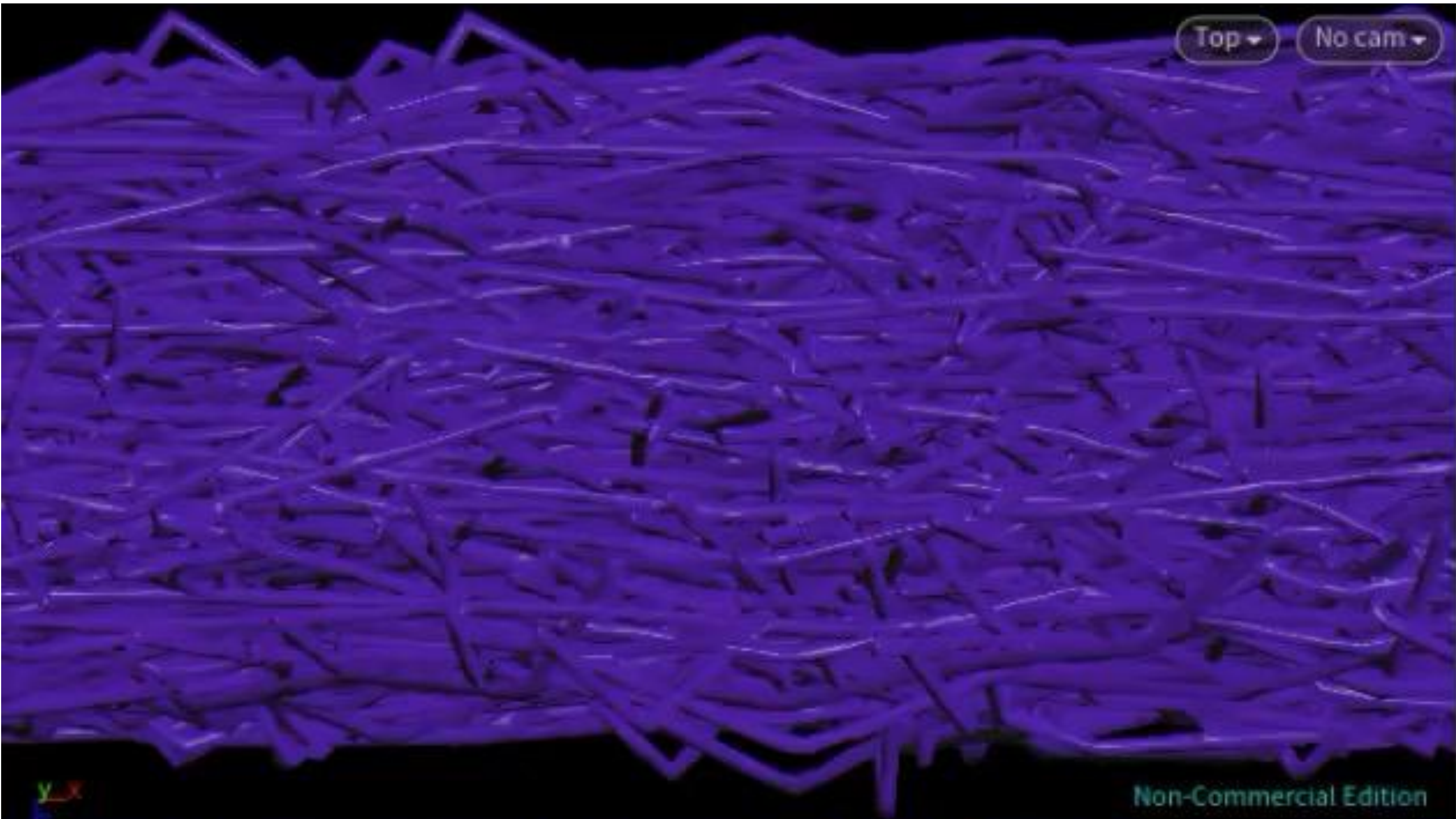




Copyright Steve Abramowitch

Daniele Panozzo





Copyright Ian Sigal & Steve Abramowitch

**Houdini** 

Non-Commercial Edition Launch flipbook render

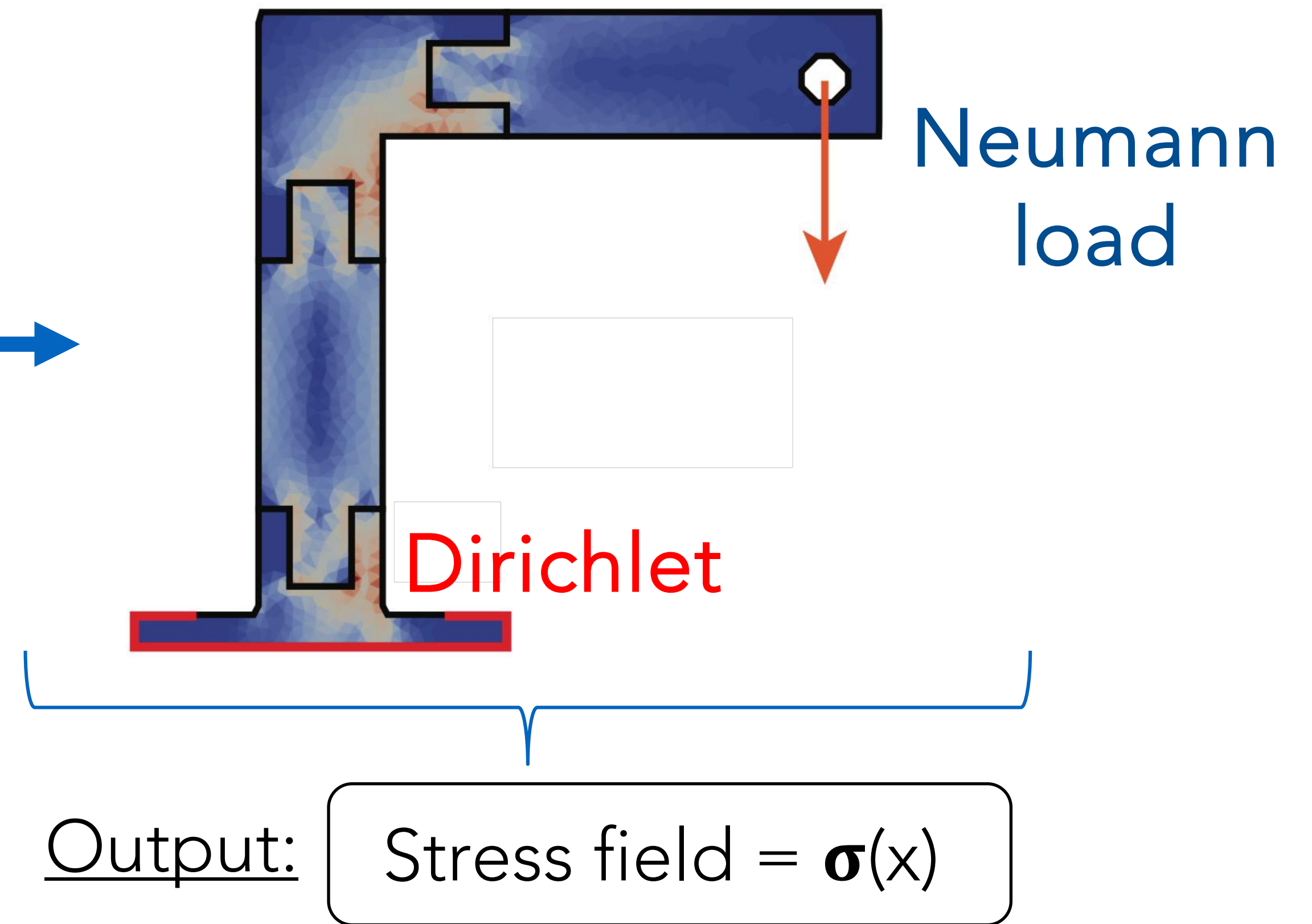
Non-Commercial Edition



# Inverse Problems

# Forward Simulation

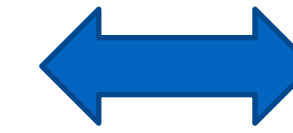
Input:  
Domain:  
Object (Mesh)  
Boundary conditions:  
Forces  
Displacements



maximize strength



minimize max stress

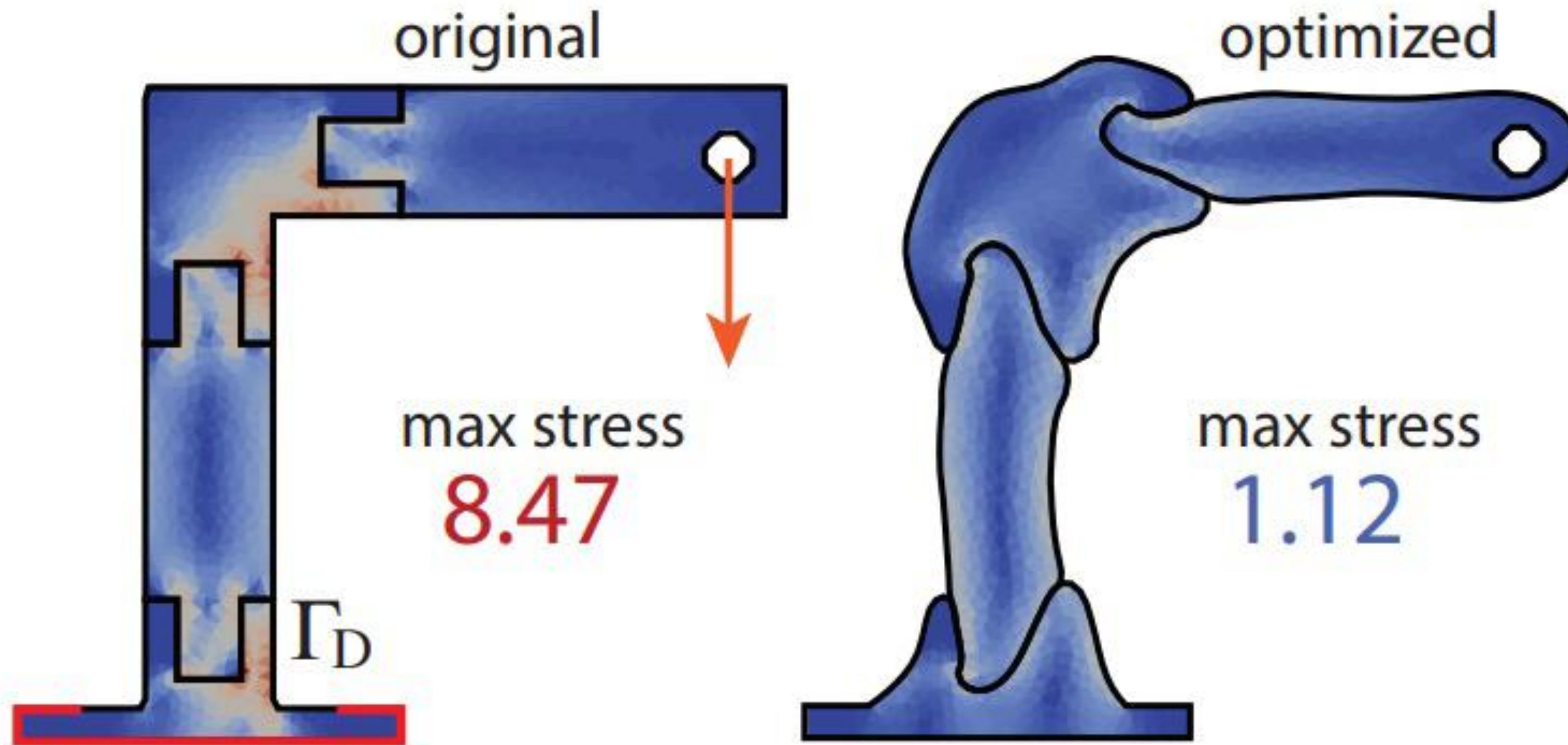


$$\min J(\Omega) = \left( \int_{\Omega} (\|\sigma\|_F)^p dx \right)^{1/p}$$

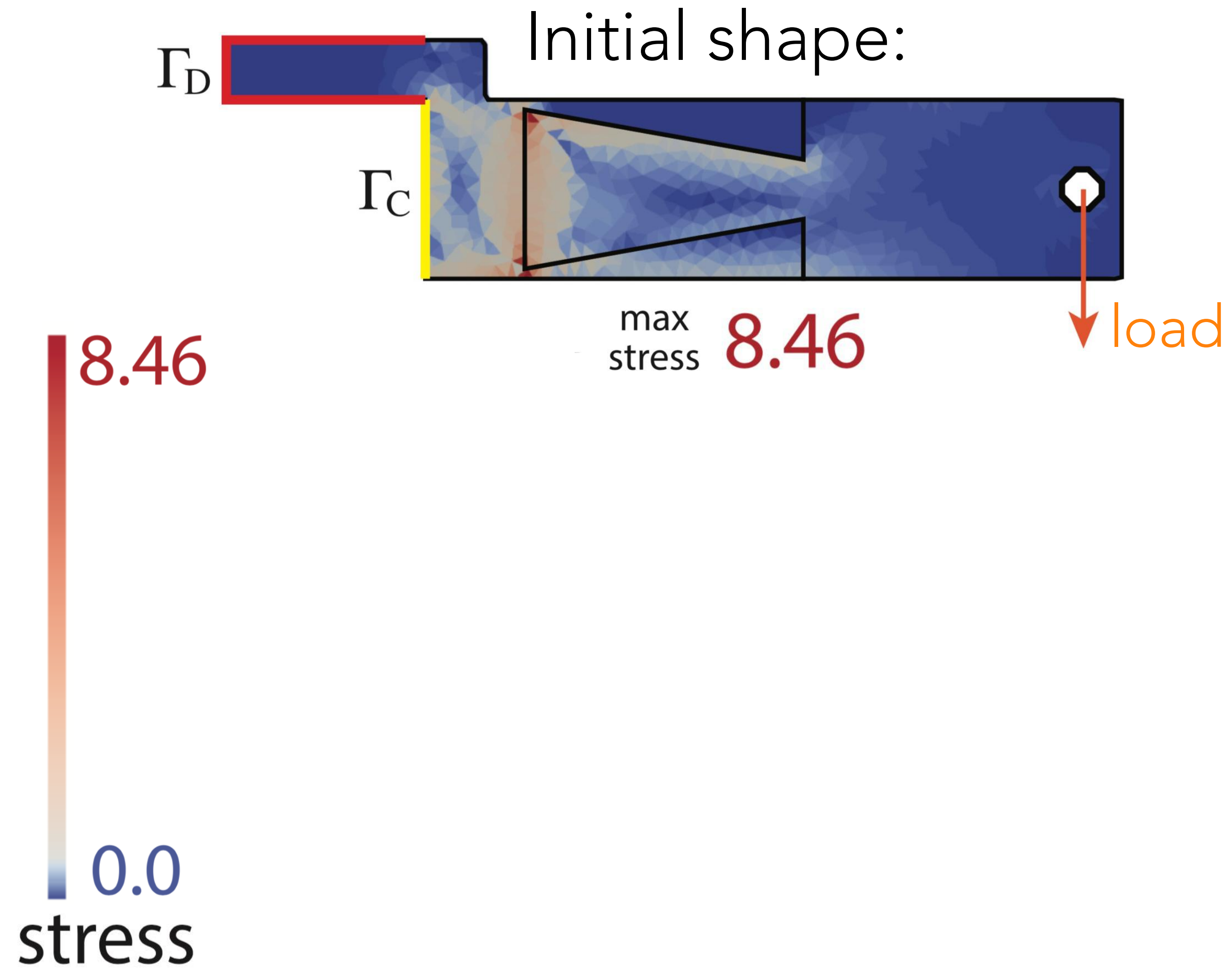
Requirement: Differentiability



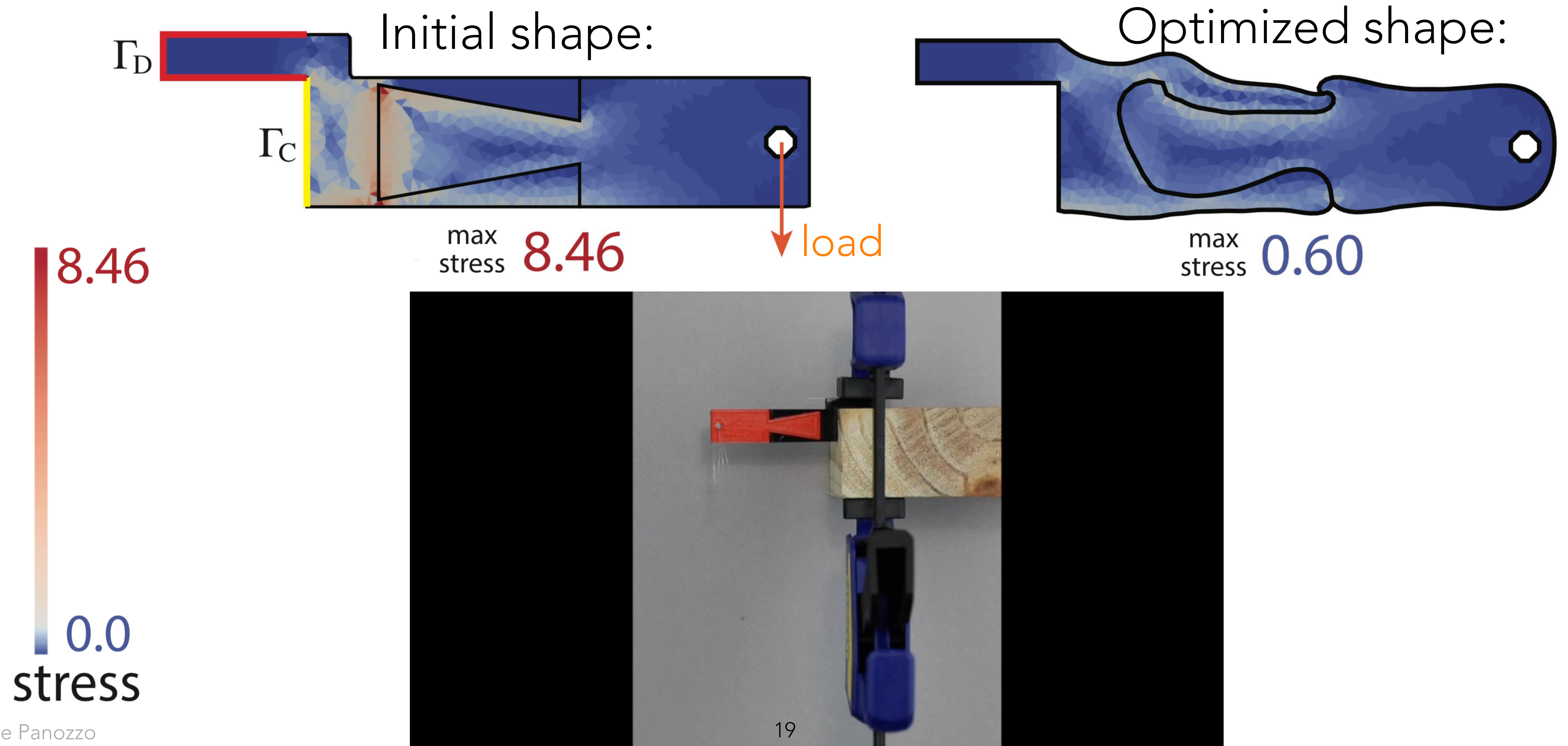
# PDE-constrained Optimization



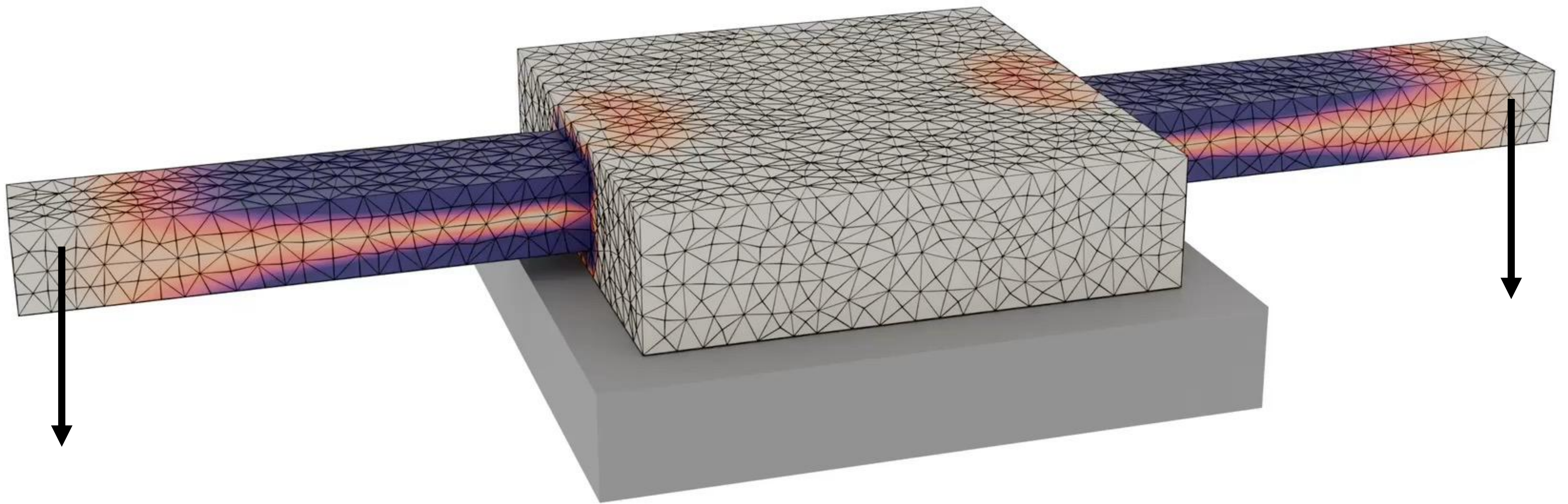
# Deformable-Deformable (RD)



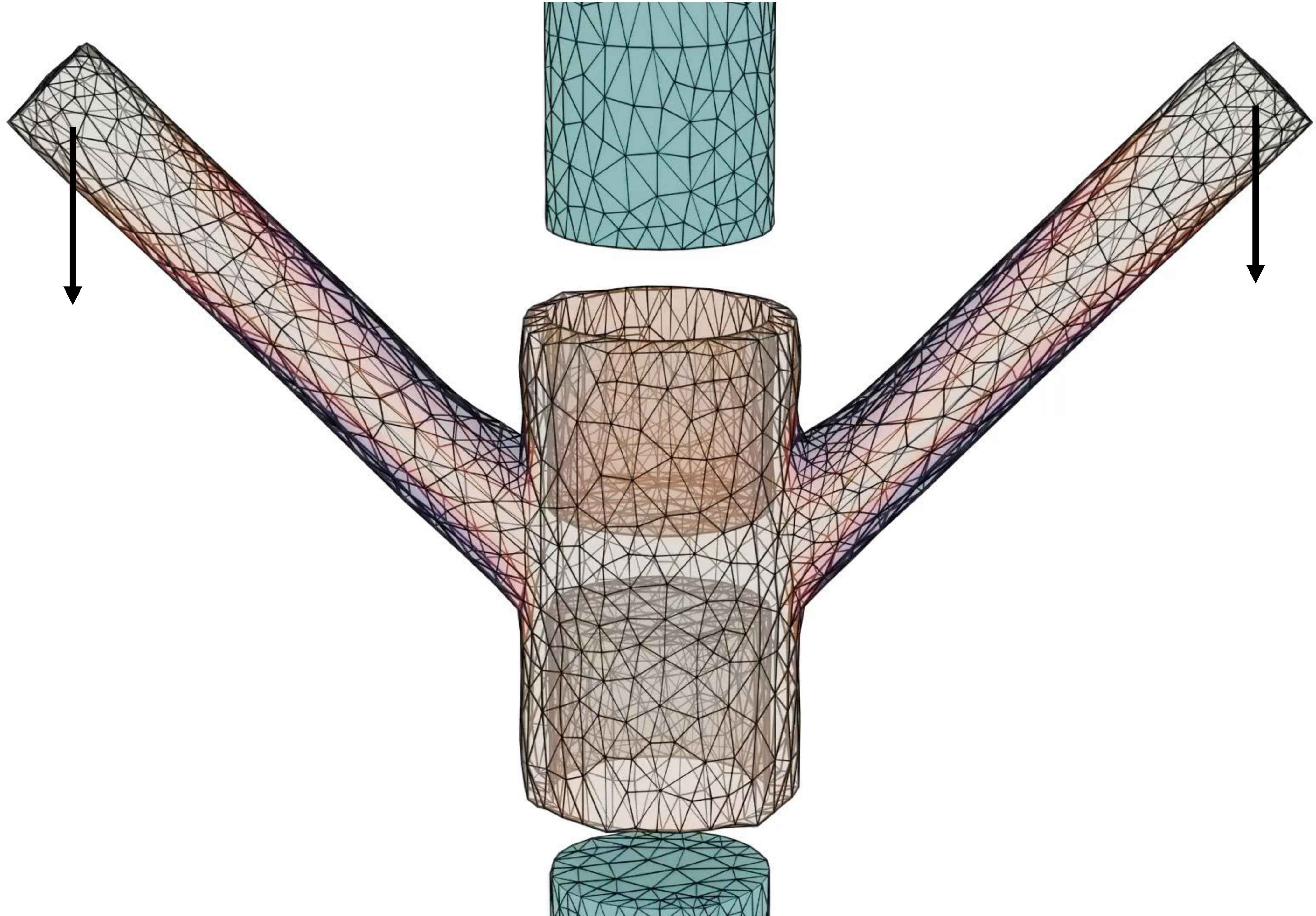
# Deformable-Deformable (RD)



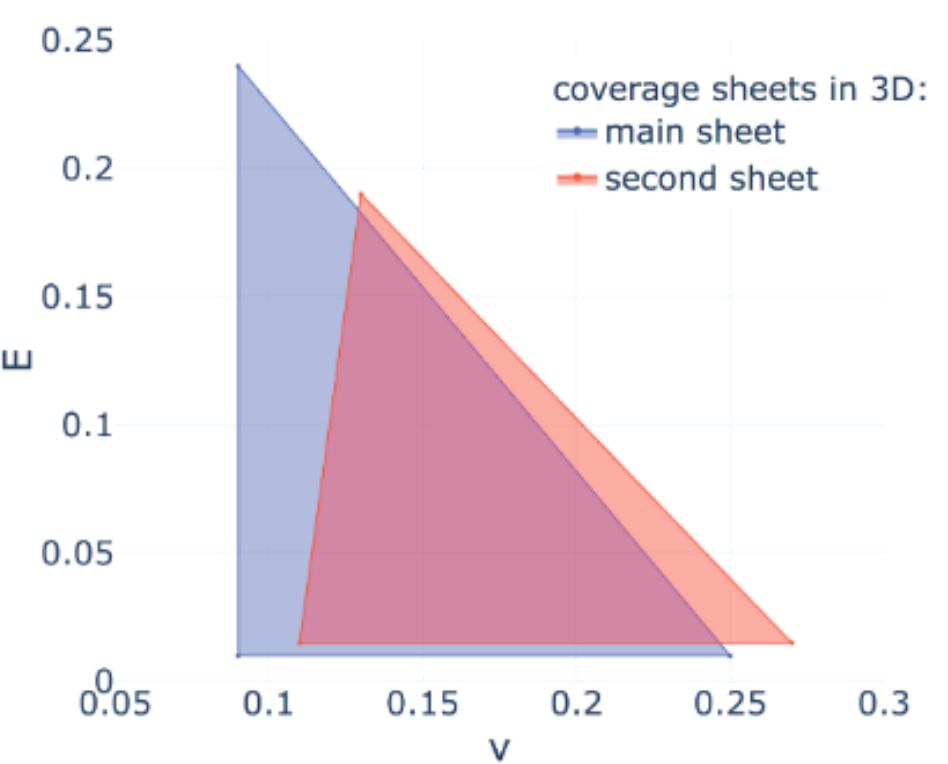
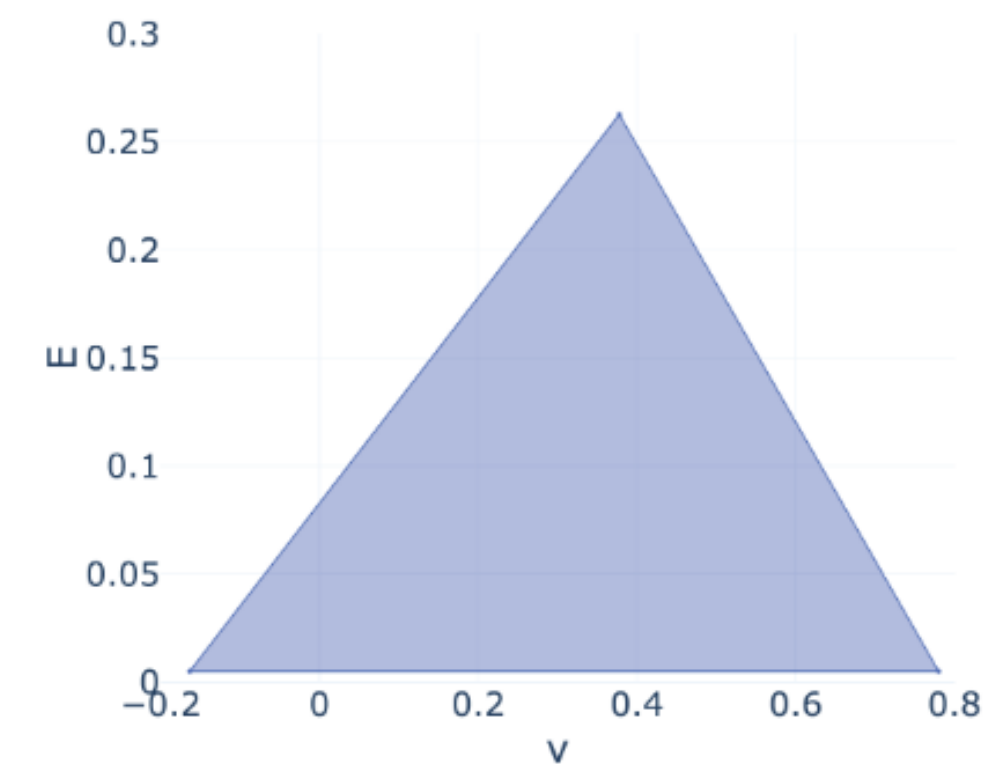
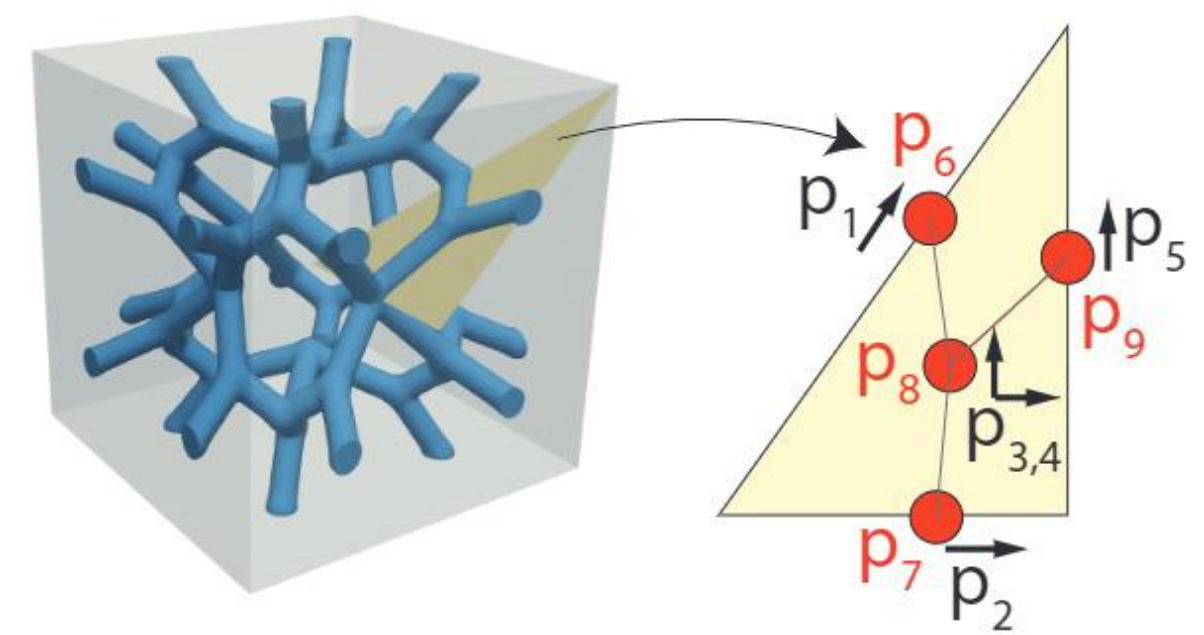
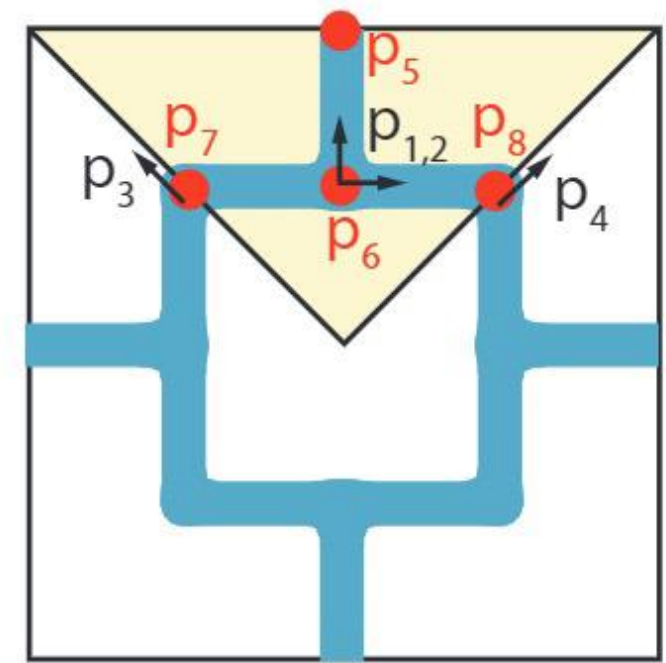
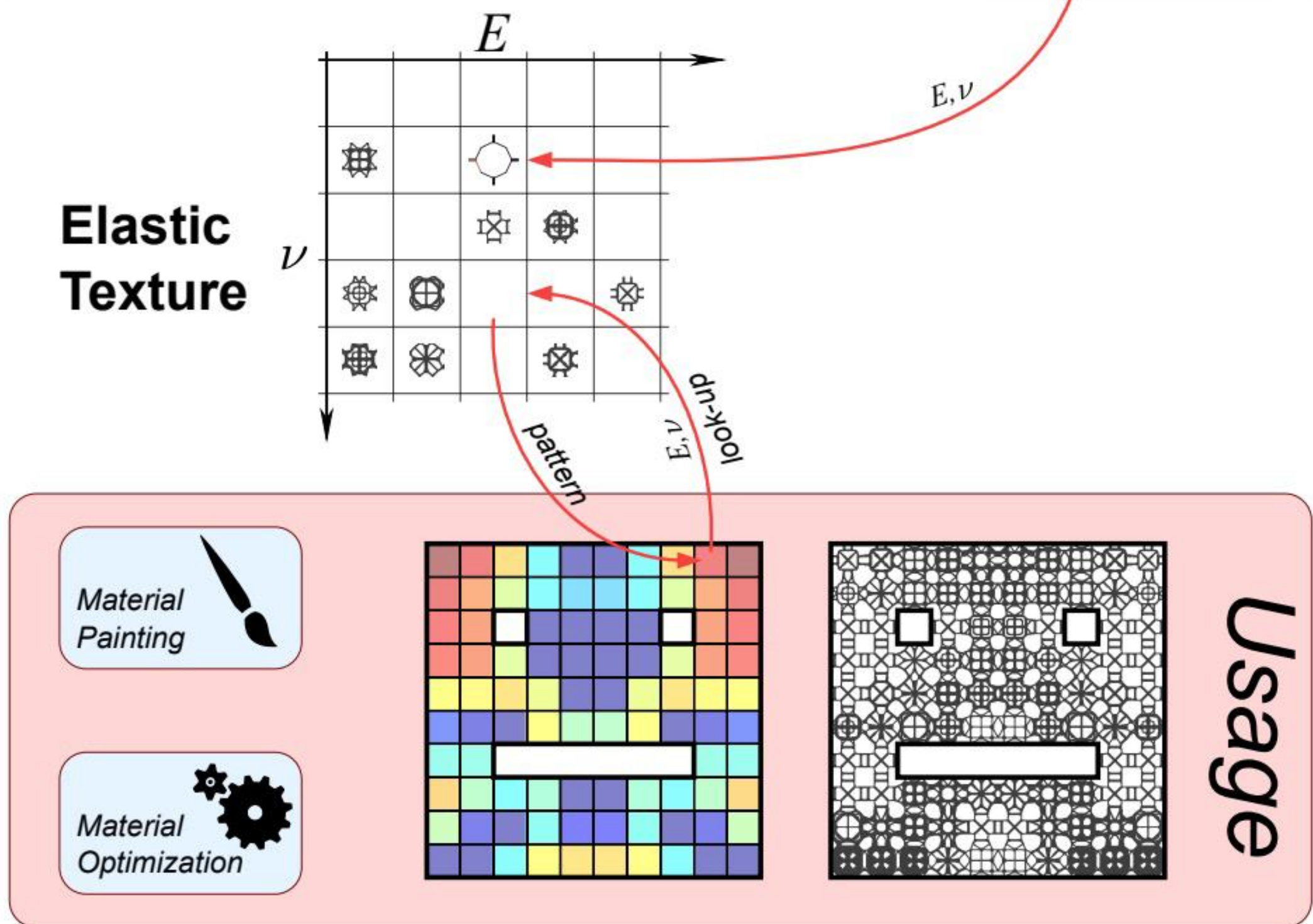
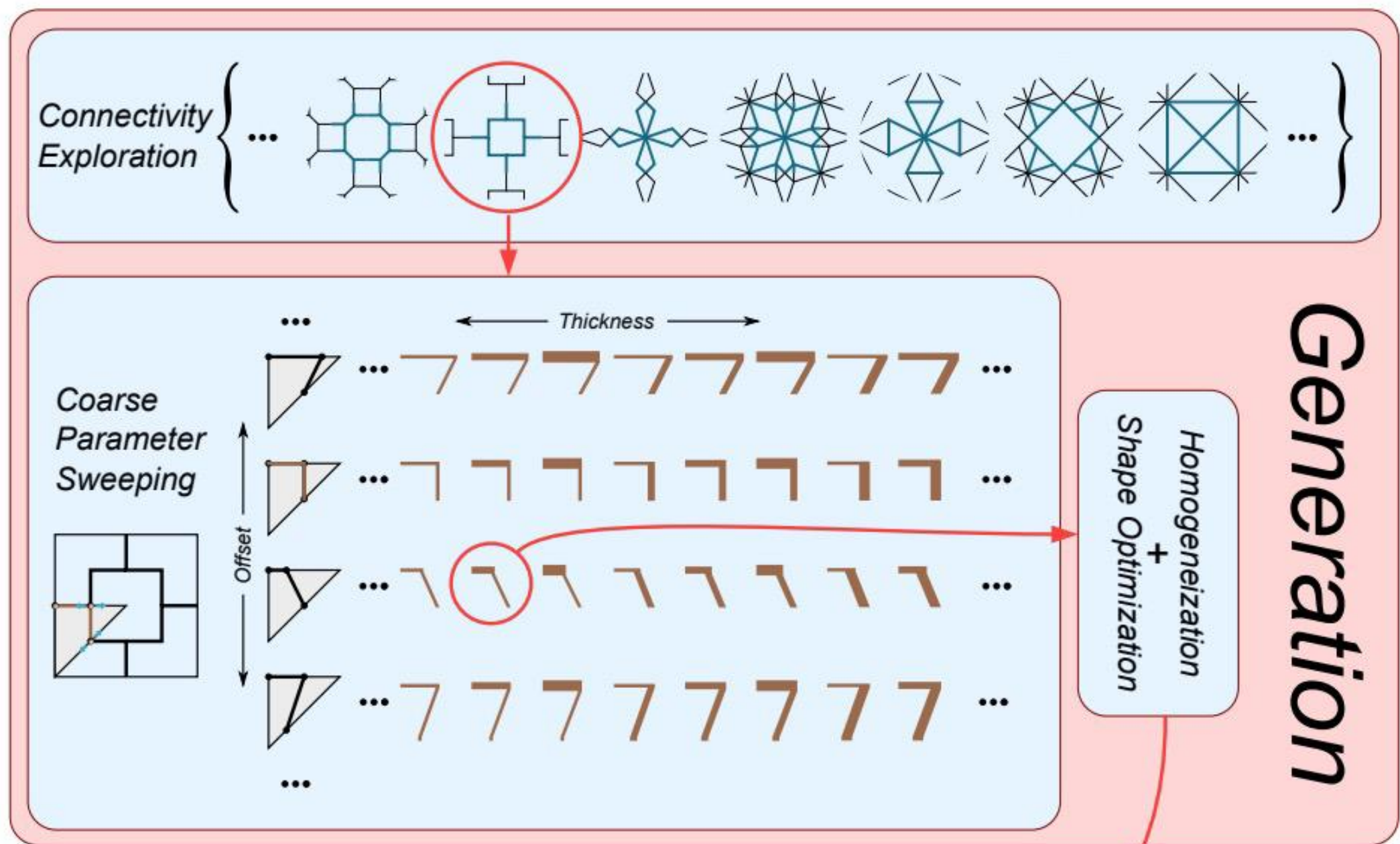




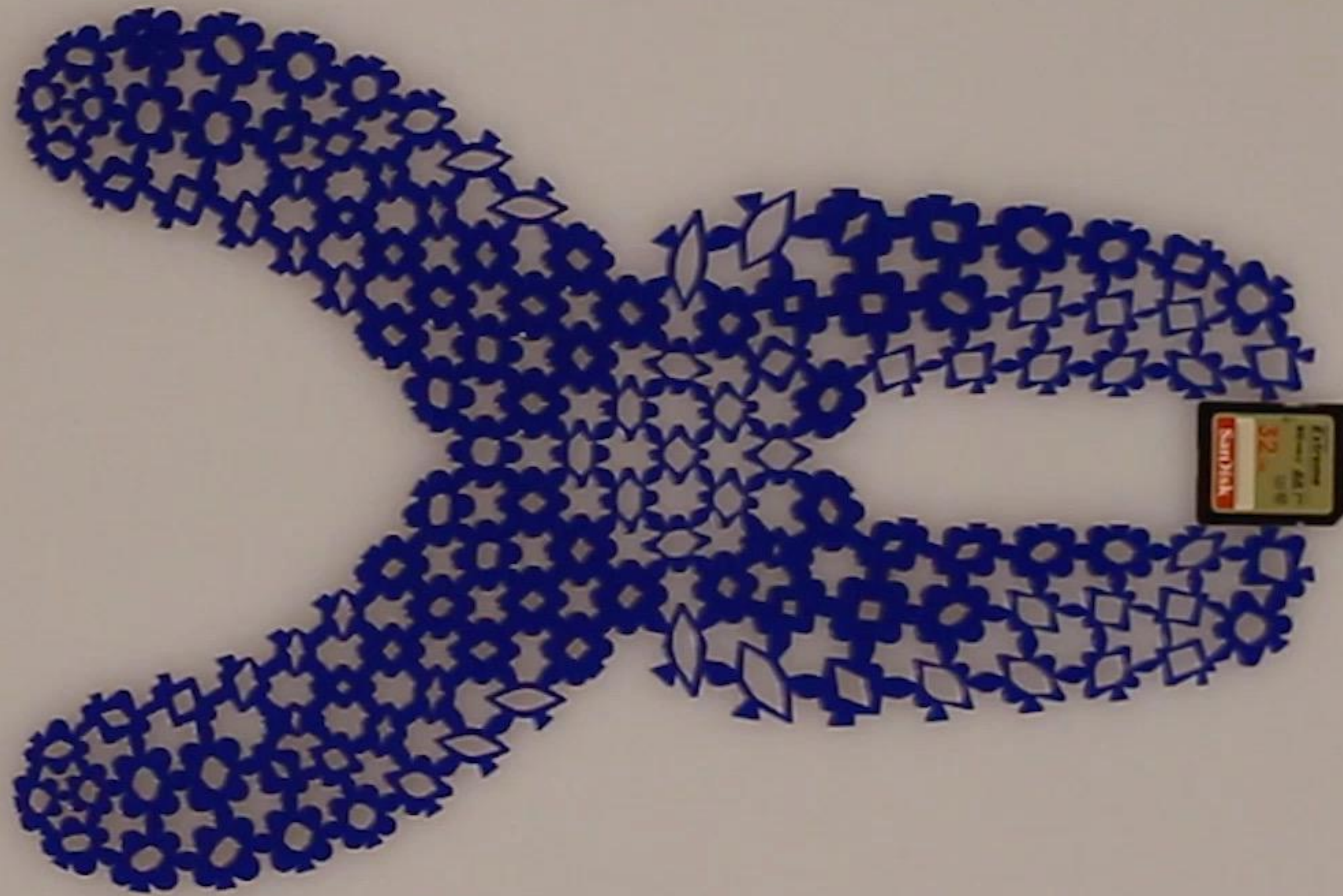










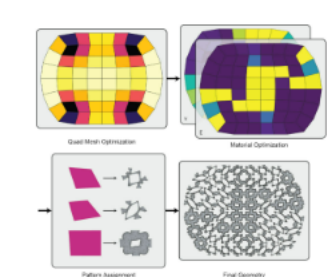


**A Low-Parametric Rhombic Microstructure Family for Irregular Lattices**

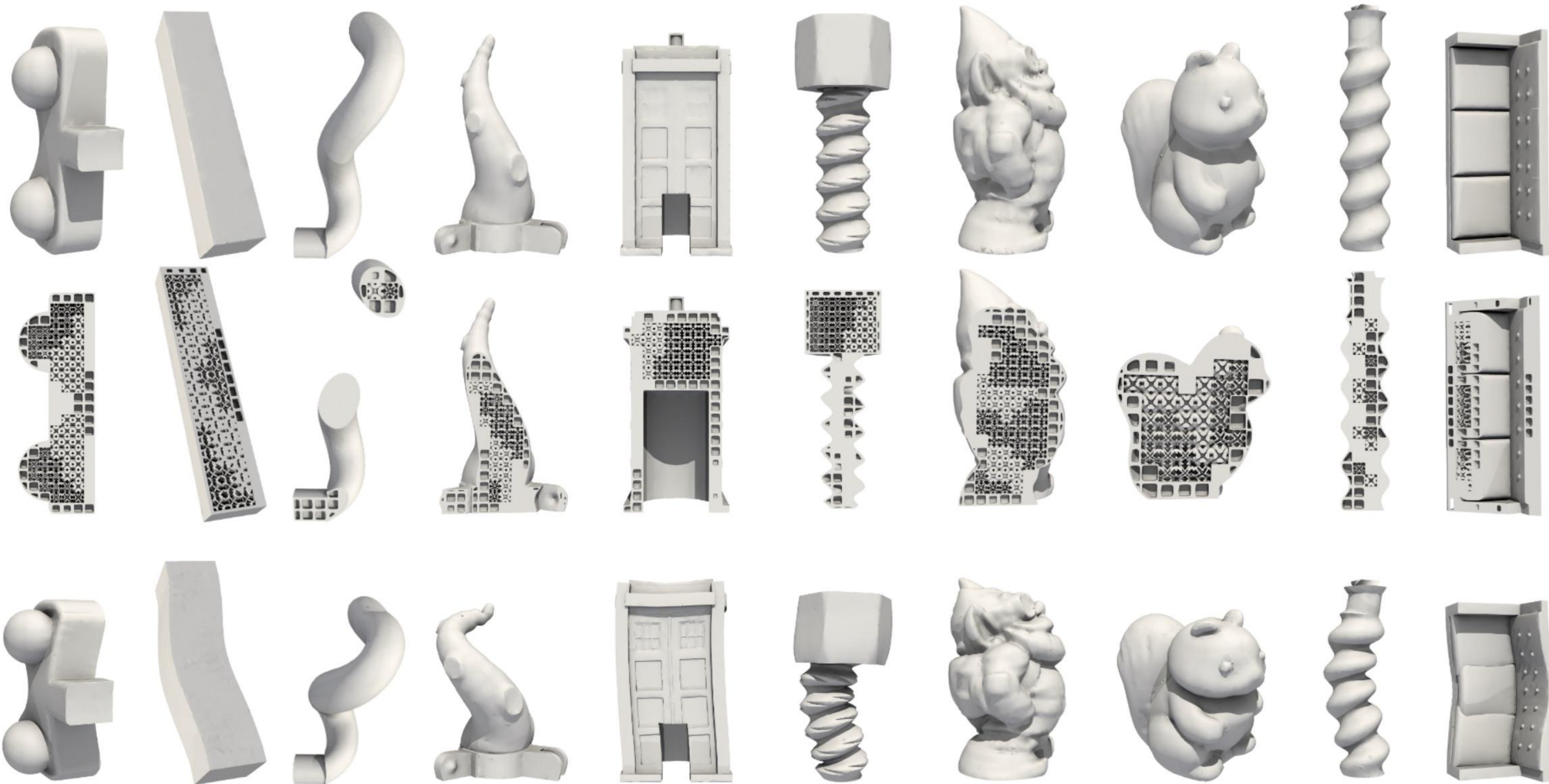
[Davi Colli Tozoni](#), [Jeremie Dumas](#), [Zhongshi Jiang](#), [Julian Panetta](#), [Daniele Panozzo](#), [Denis Zorin](#),

ACM Transaction on Graphics (SIGGRAPH), 2020

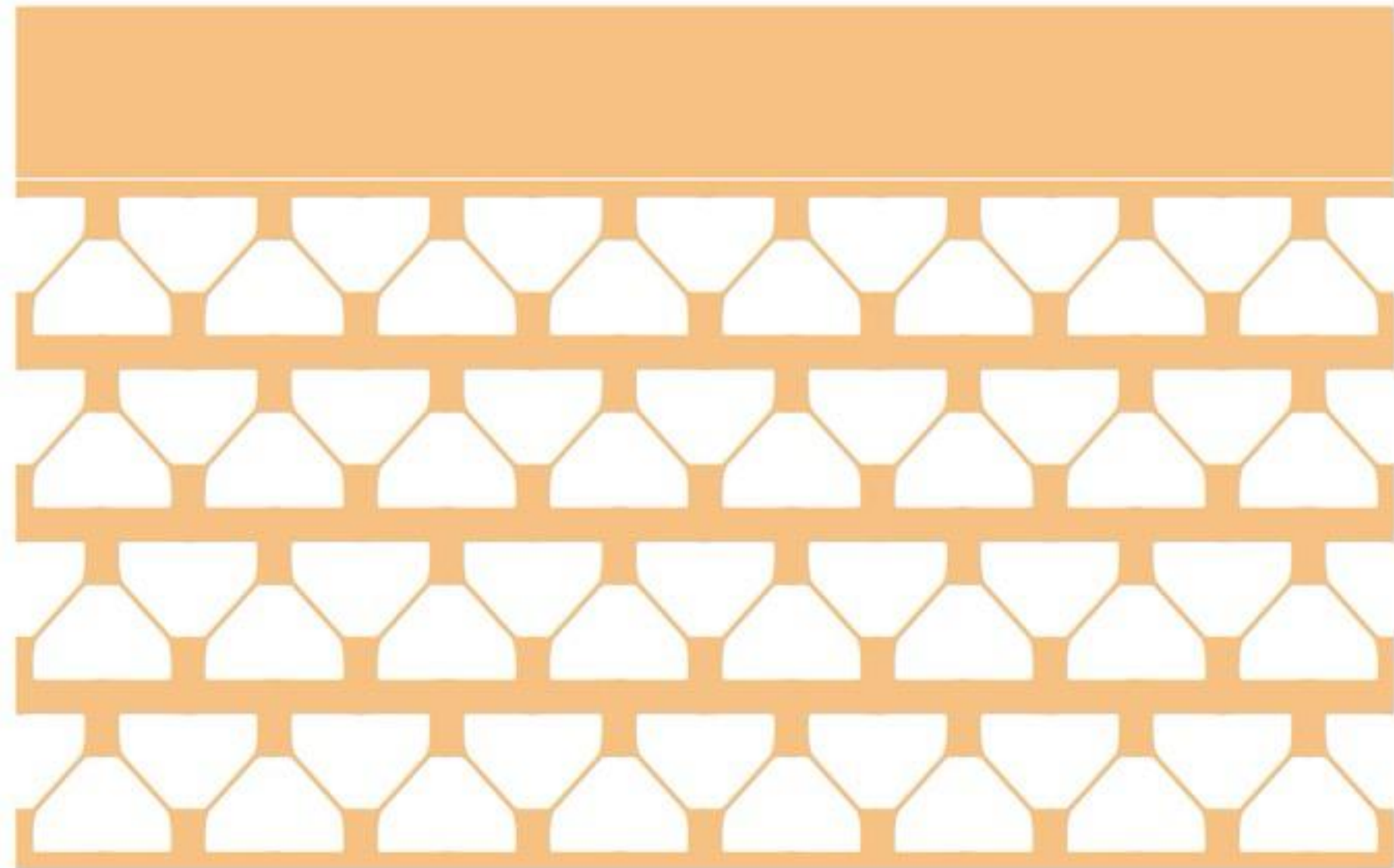
[\[Paper\]](#) [\[Code\]](#) [\[Supplemental\]](#) [\[Talk\]](#)







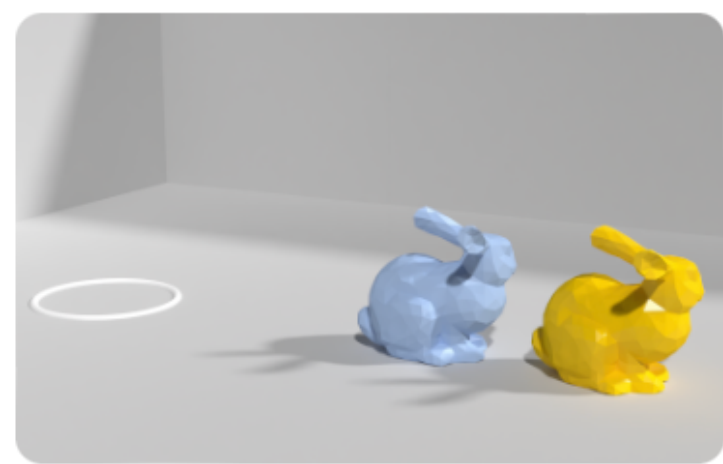
# Elastodynamic and Contact



Initial



# Elastodynamic and Contact



**Differentiable solver for time-dependent deformation problems with contact**

[Zizhou Huang](#), [Davi Colli Tozoni](#), Arvi Gjoka, [Zachary Ferguson](#), [Teseo Schneider](#), [Daniele Panozzo](#), [Denis Zorin](#),

ACM Transaction on Graphics, 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)

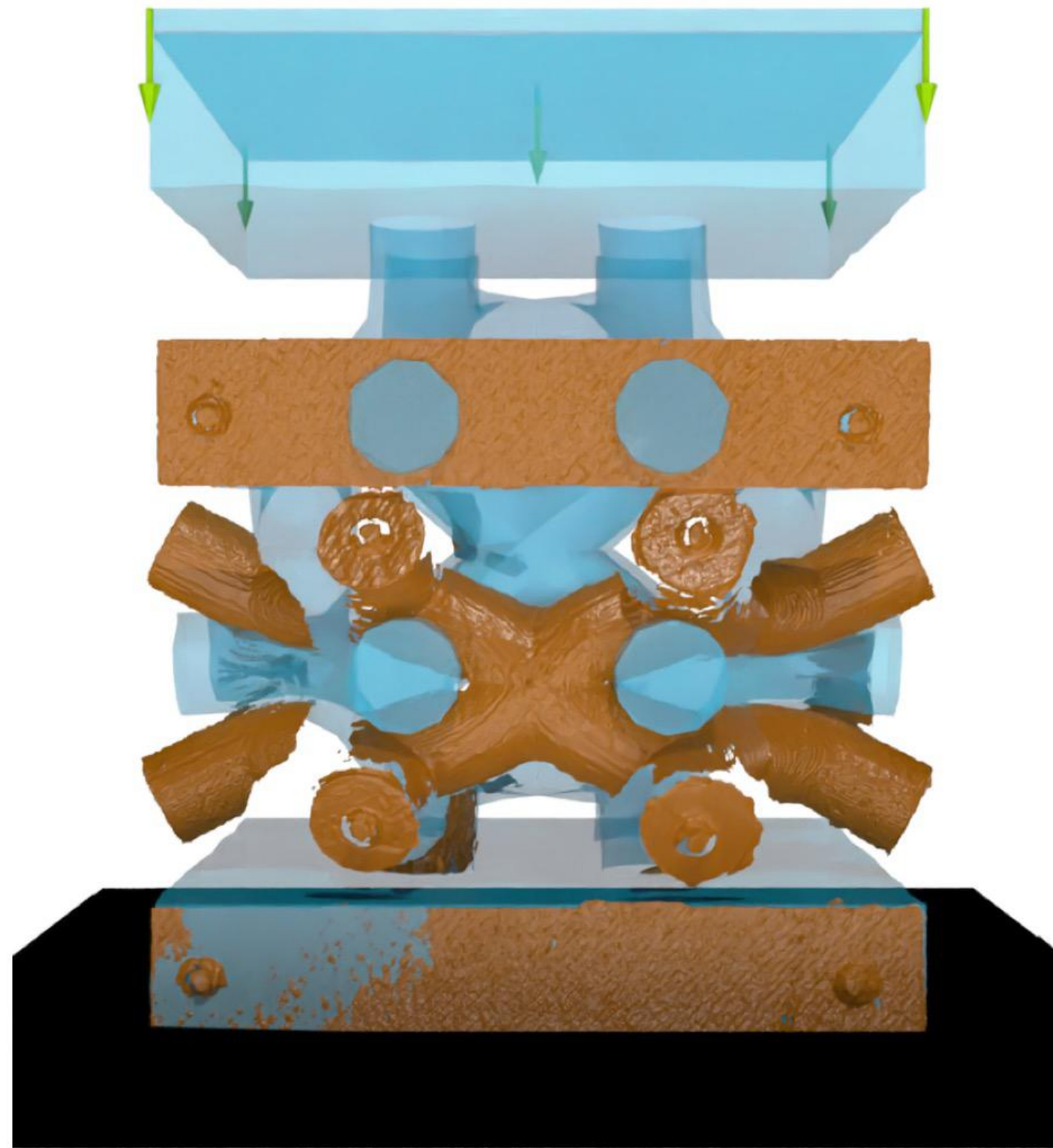


Initial

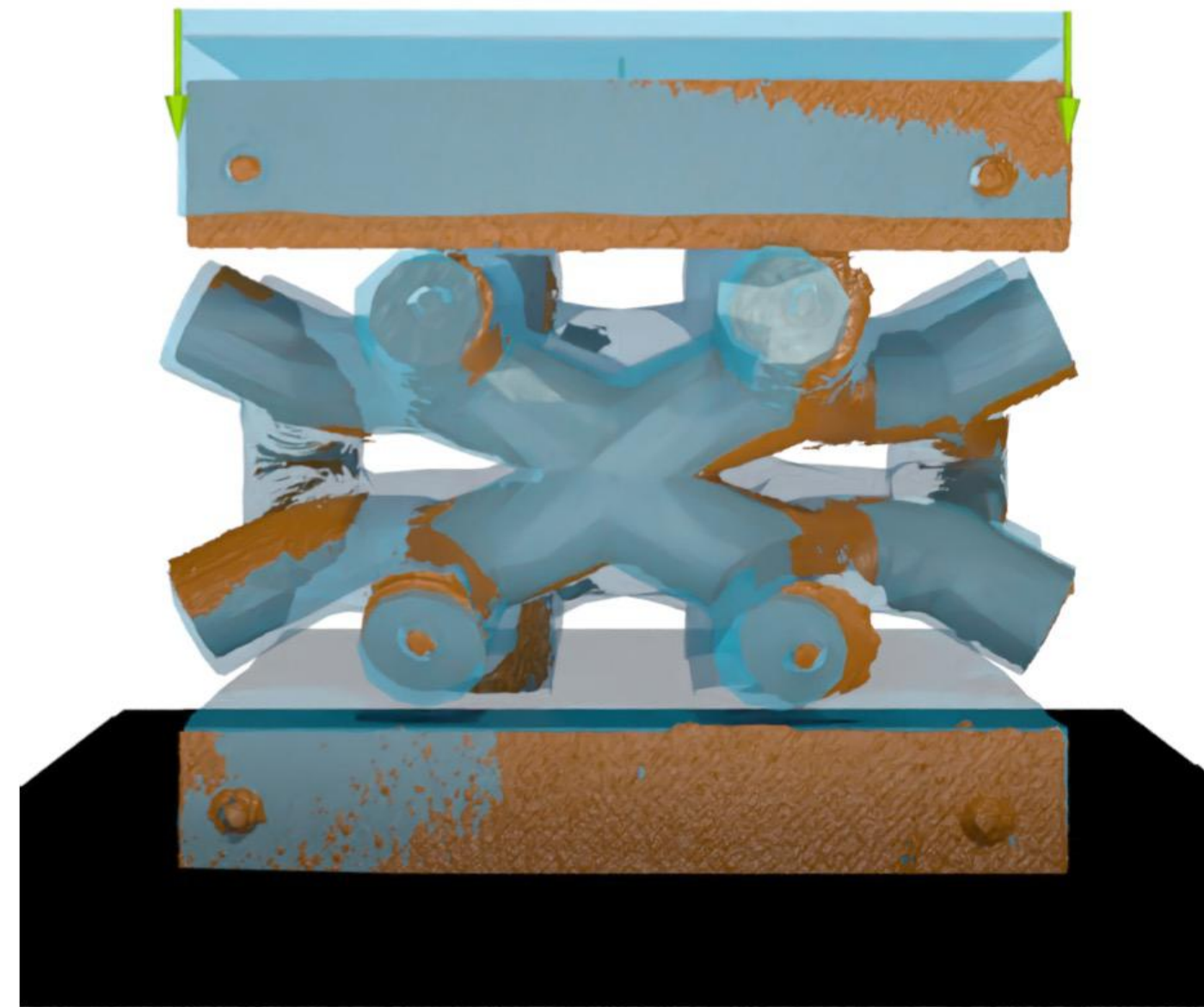
Optimized



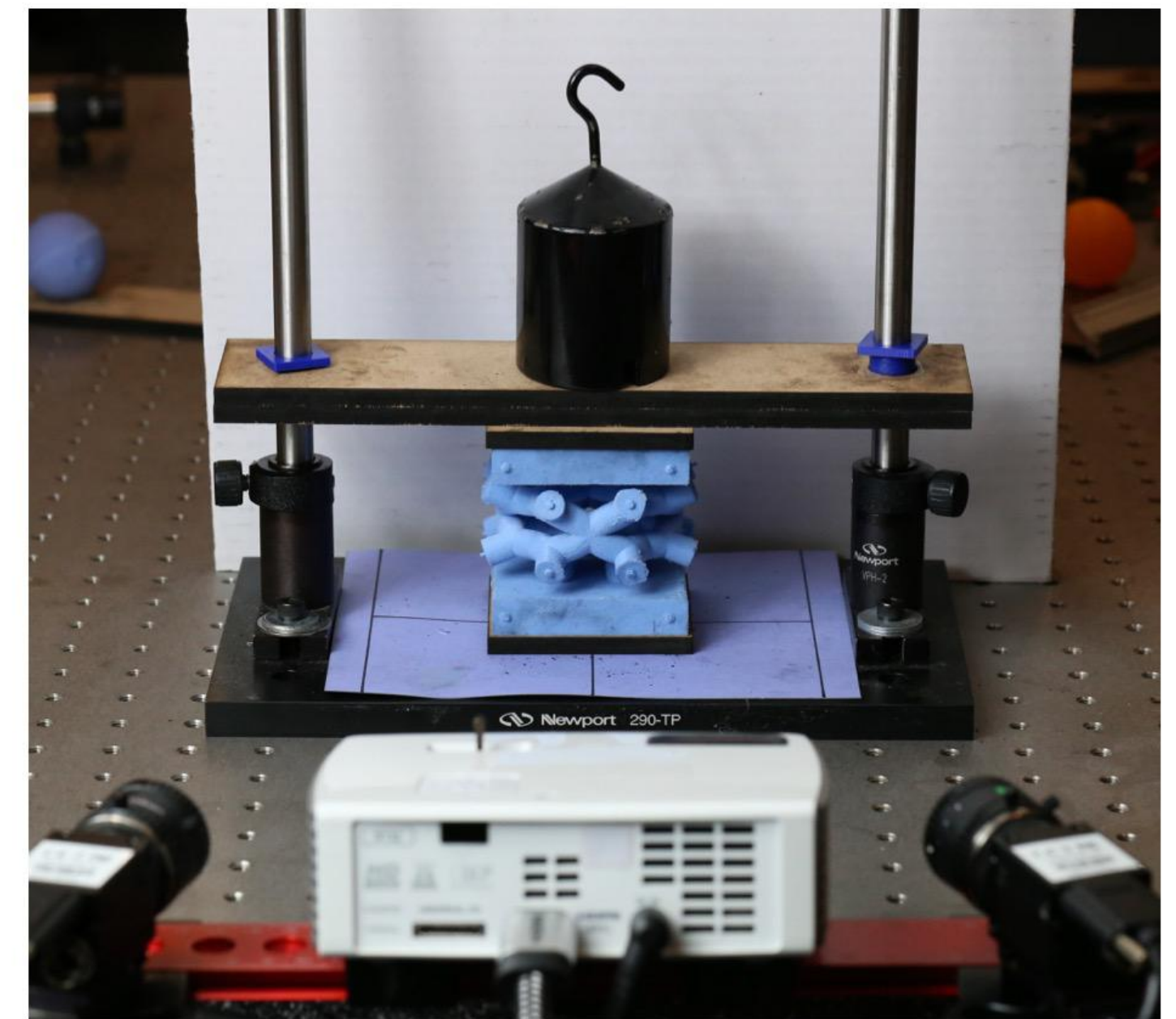
# Indirect Material Testing



Initial.



Optimized.



Physical Experiment Setup.



# Automating Finite Element Analysis

# Forward Simulation





# Forward Simulation

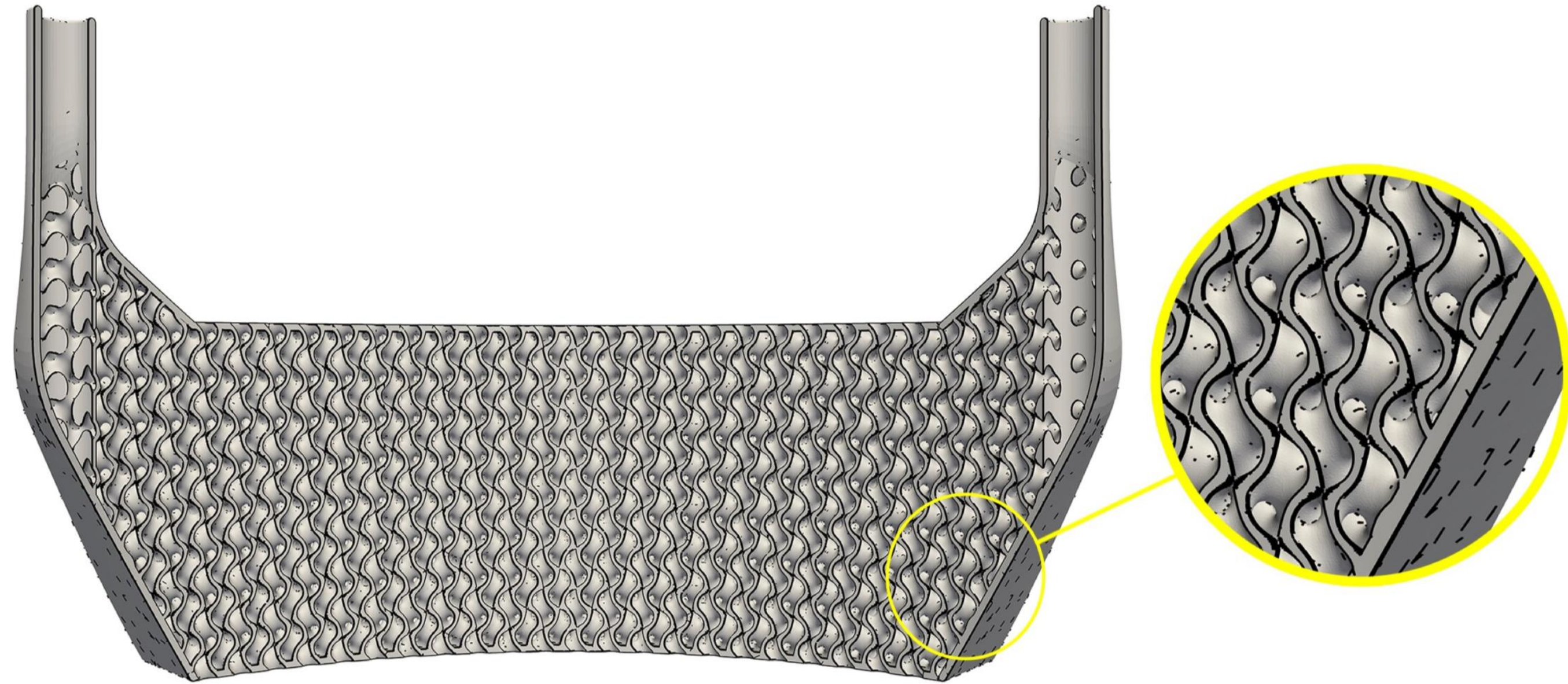




# Complex Geometry



Volume Reduction



Heat Flux Increase

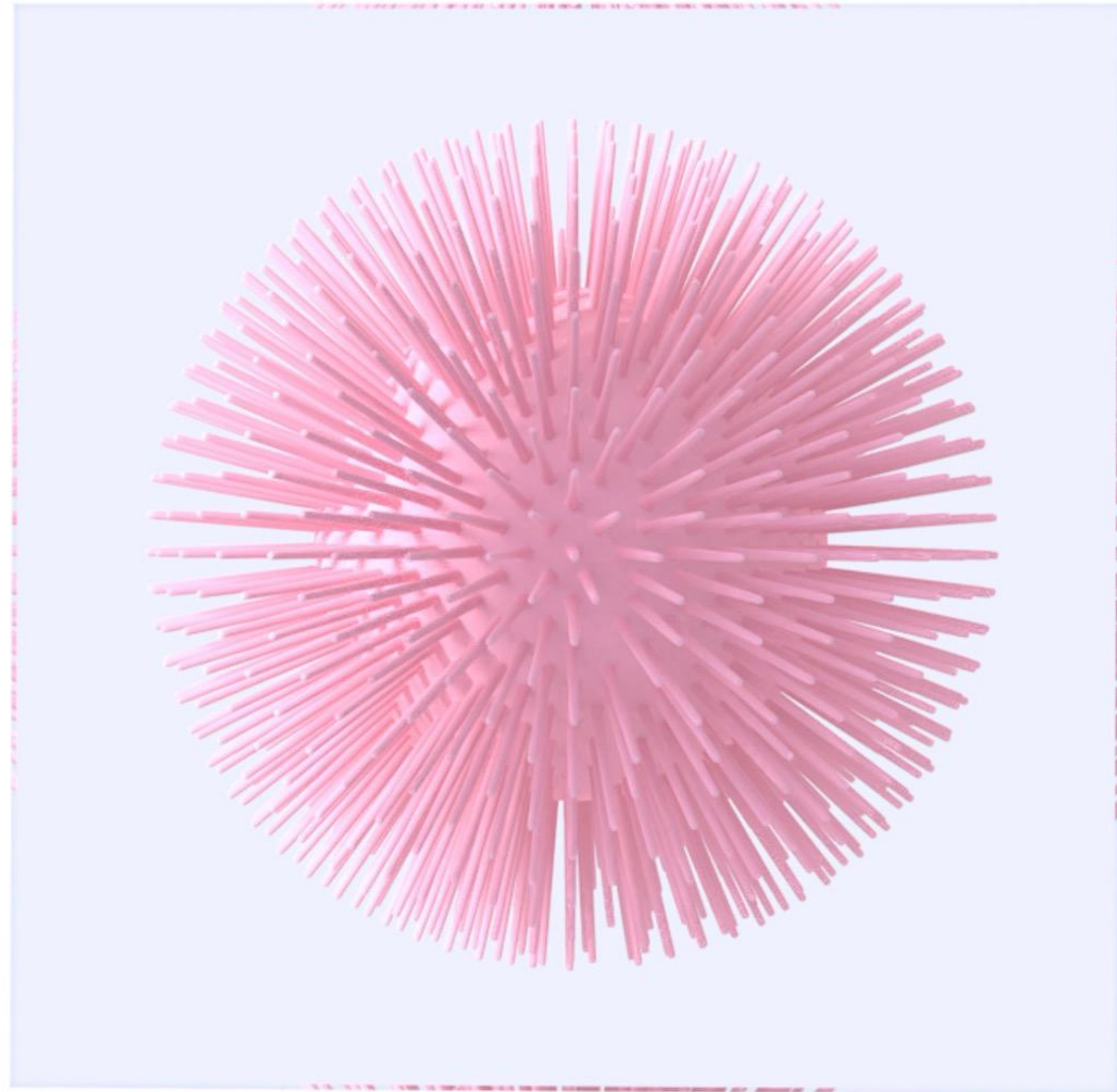


# Complex Contacts

**IPC**



tetrahedra: 2314K  
contacts per step (max): 5.6K  
dt: 0.001  
 $\mu: 0$



# Goals

- Minimal/No requirements on input geometry, which can be implicit (SDF) or explicit (triangle meshes, CAD).
- Fully automatic, parameters should relate to accuracy of the solution or of the physical system (for example Young's modulus for elasticity), not algorithmic details.
- Minimize time, given an error bound.

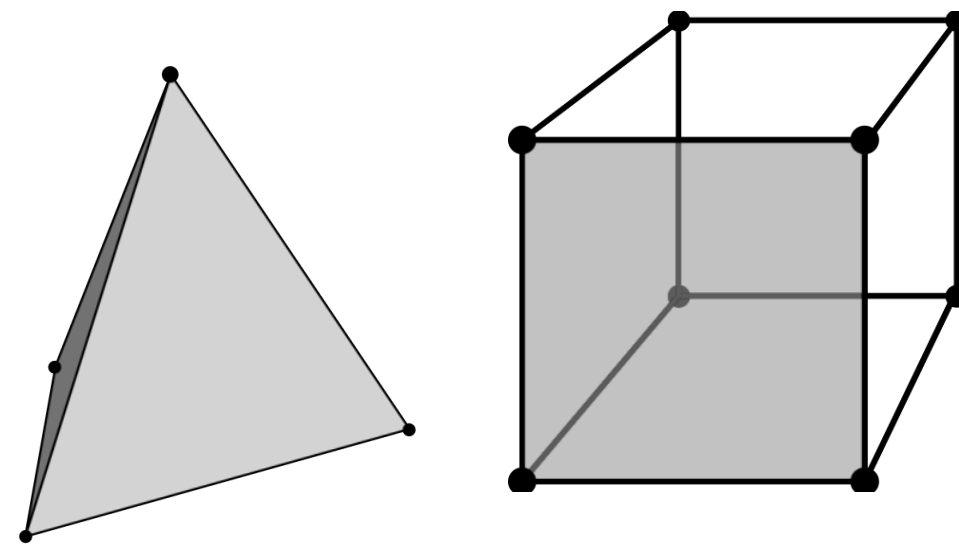


# Observations

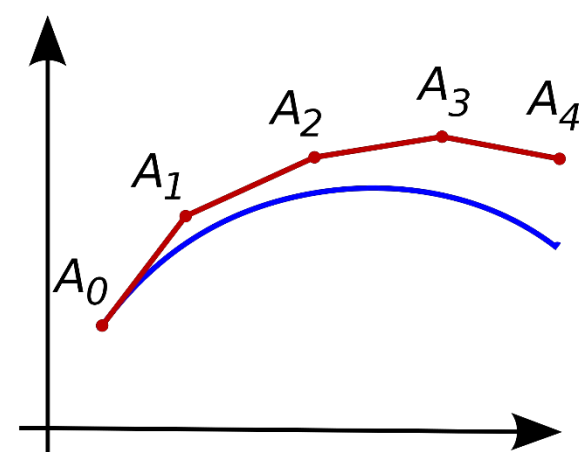
- It is a difficult problem, involving geometry, space discretization, solution of PDEs, contact detection, numerical methods, and physics. Evaluating individual components is insufficient and often misleading.
- I advocate for solving this problem holistically as a single challenge.

# Research Overview

## High Level Decisions

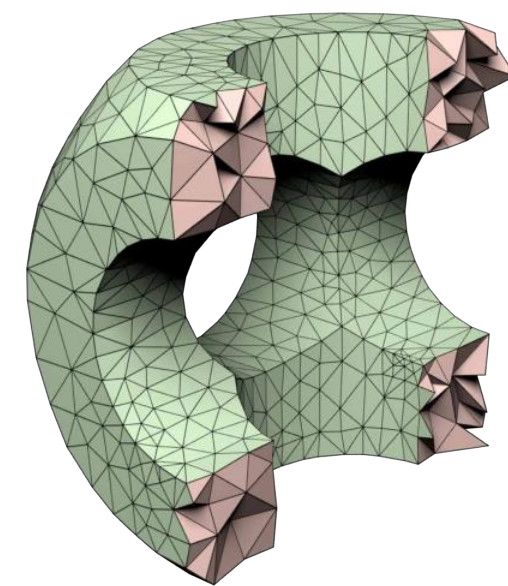


Which space discretization?

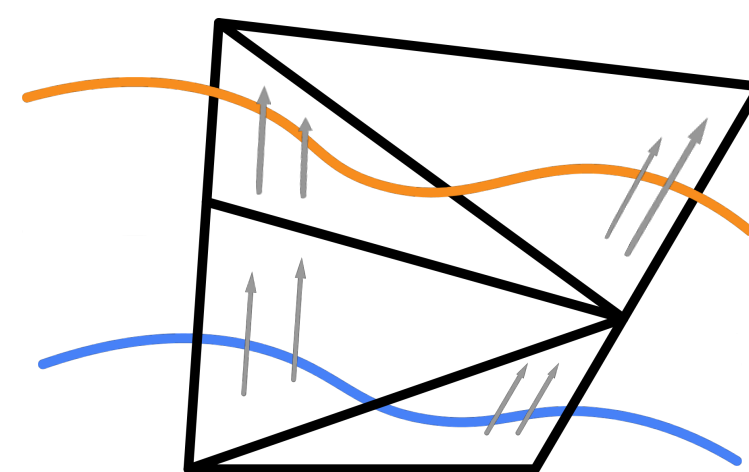


Which time integrator?

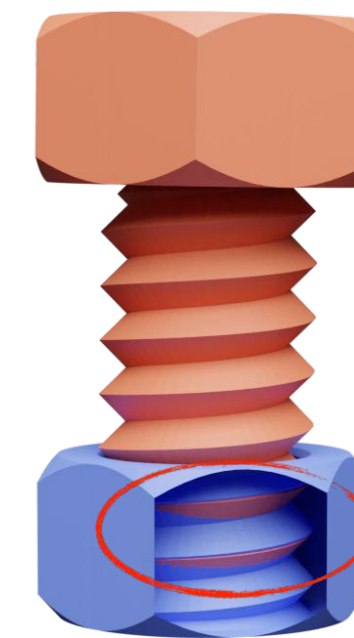
## Realization



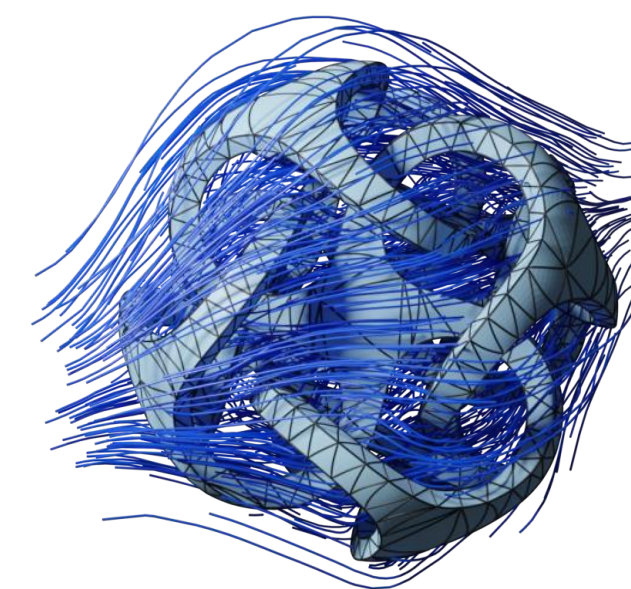
How to discretize space?



How to map input to discretization?

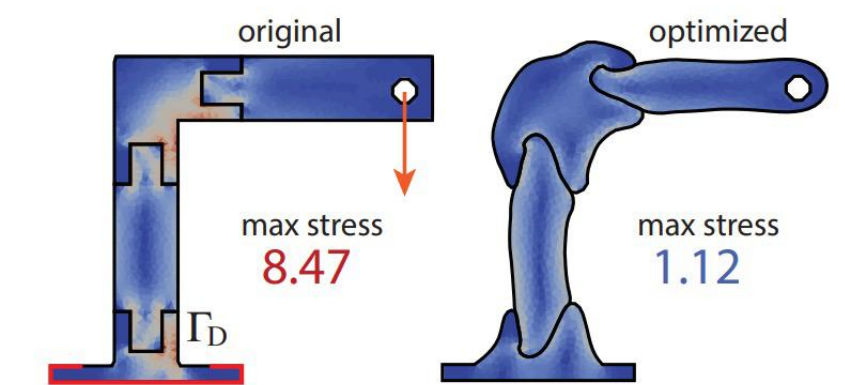


How to handle contacts?

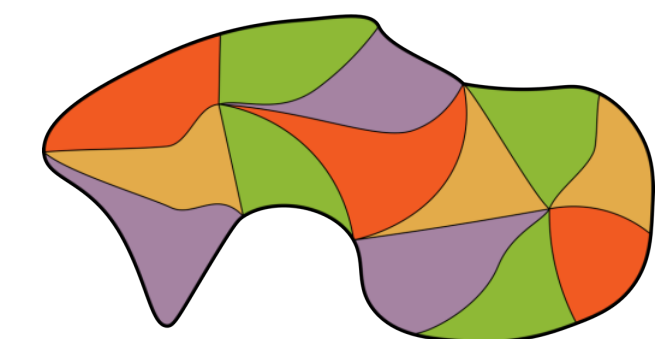


How to solve the PDE?

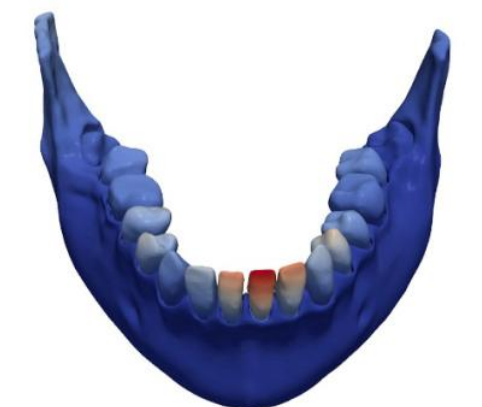
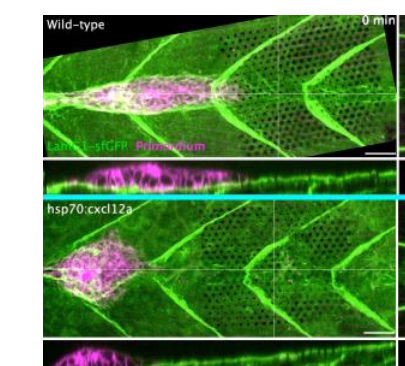
## Applications/Improvement



How to optimize functionals depending on the solution of a PDE?



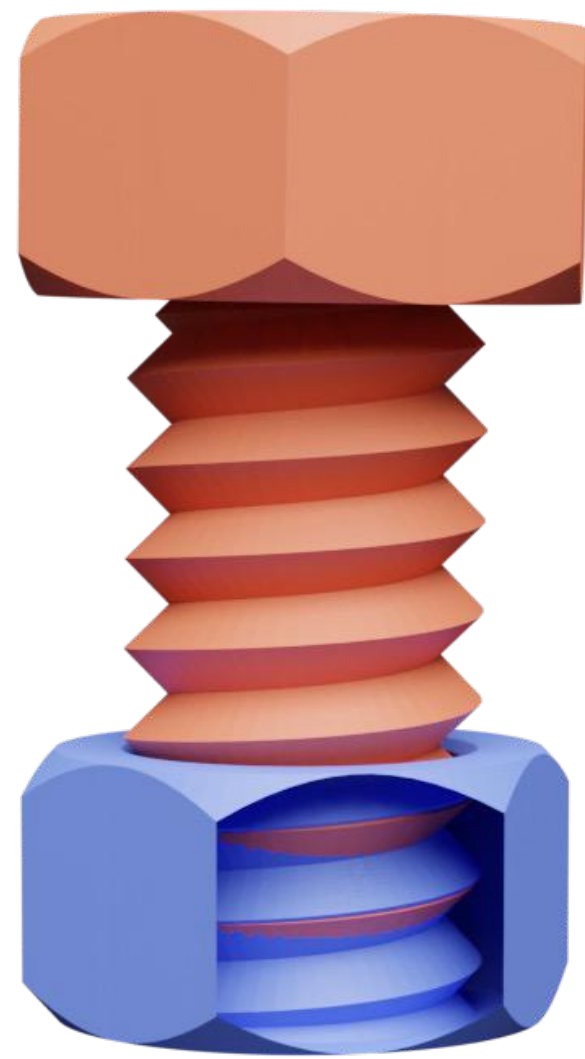
How to accelerate the system to make optimization possible?



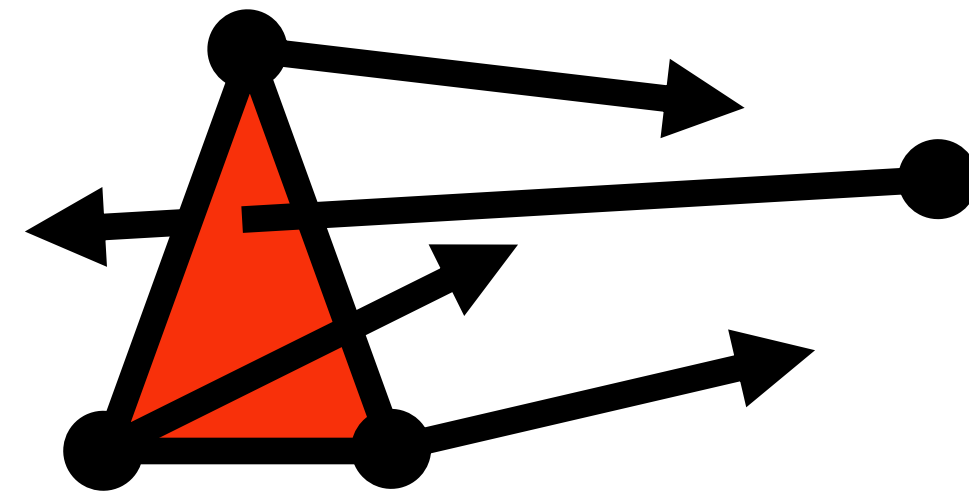
What can we do with it?



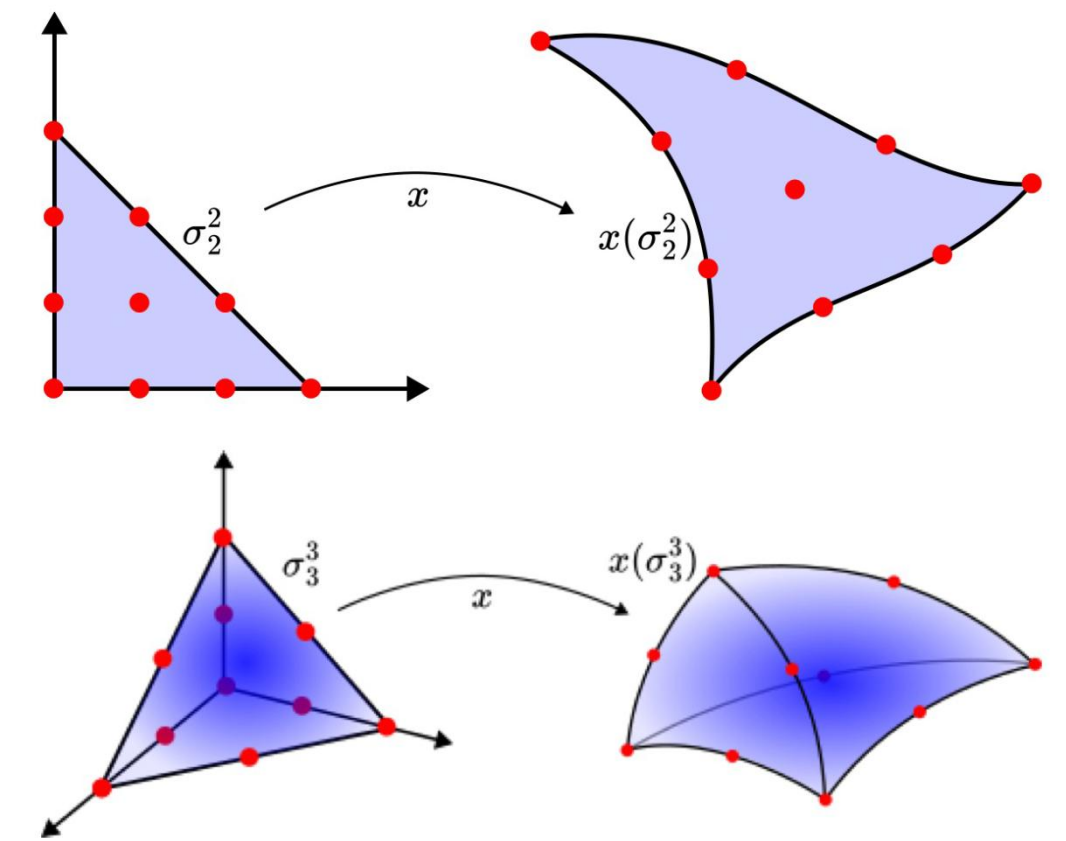
# Talk Overview



Differentiable Contact Elastodynamics  
with an Incremental Potential Formulation

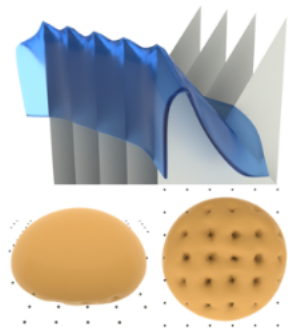


Provably Conservative  
Collision Avoidance



Provably Conservative  
Local Injectivity

# Differentiable Contact Elastodynamics



## Incremental Potential Contact: Intersection- and Inversion-free, Large-Deformation Dynamics

[Minchen Li](#), [Zachary Ferguson](#), [Teseo Schneider](#), [Timothy Langlois](#), [Denis Zorin](#), [Daniele Panozzo](#), [Chenfanfu Jiang](#), [Danny M. Kaufman](#),

ACM Transaction on Graphics (SIGGRAPH), 2020

[\[Paper\]](#) [\[Video\]](#) [\[Supplemental Video\]](#) [\[Code\]](#) [\[S1: Technical\]](#) [\[S2: Comparisons\]](#) [\[S3: Statistics\]](#) [\[Website\]](#)

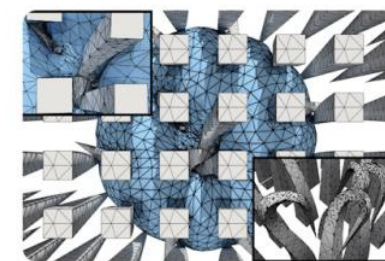


## Intersection-free Rigid Body Dynamics

[Zachary Ferguson](#), [Minchen Li](#), [Teseo Schneider](#), [Francisca Gil Ureta](#), [Timothy Langlois](#), [Chenfanfu Jiang](#), [Denis Zorin](#), [Danny M. Kaufman](#), [Daniele Panozzo](#),

ACM Transaction on Graphics (SIGGRAPH), 2021

[\[Paper\]](#) [\[Video 1\]](#) [\[Video 2\]](#) [\[Website\]](#)

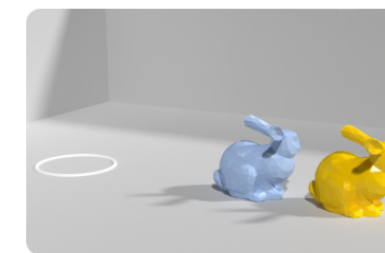


## In-Timestep Remeshing for Contacting Elastodynamics

[Zachary Ferguson](#), [Teseo Schneider](#), [Danny M. Kaufman](#), [Daniele Panozzo](#),

ACM Transaction on Graphics (SIGGRAPH), (DK and DP are co-corresponding authors), 2023

[\[Paper\]](#) [\[Code \(Coming Soon\)\]](#)



## Differentiable solver for time-dependent deformation problems with contact

[Zizhou Huang](#), [Davi Colli Tozoni](#), [Arvi Gjoka](#), [Zachary Ferguson](#), [Teseo Schneider](#), [Daniele Panozzo](#), [Denis Zorin](#),

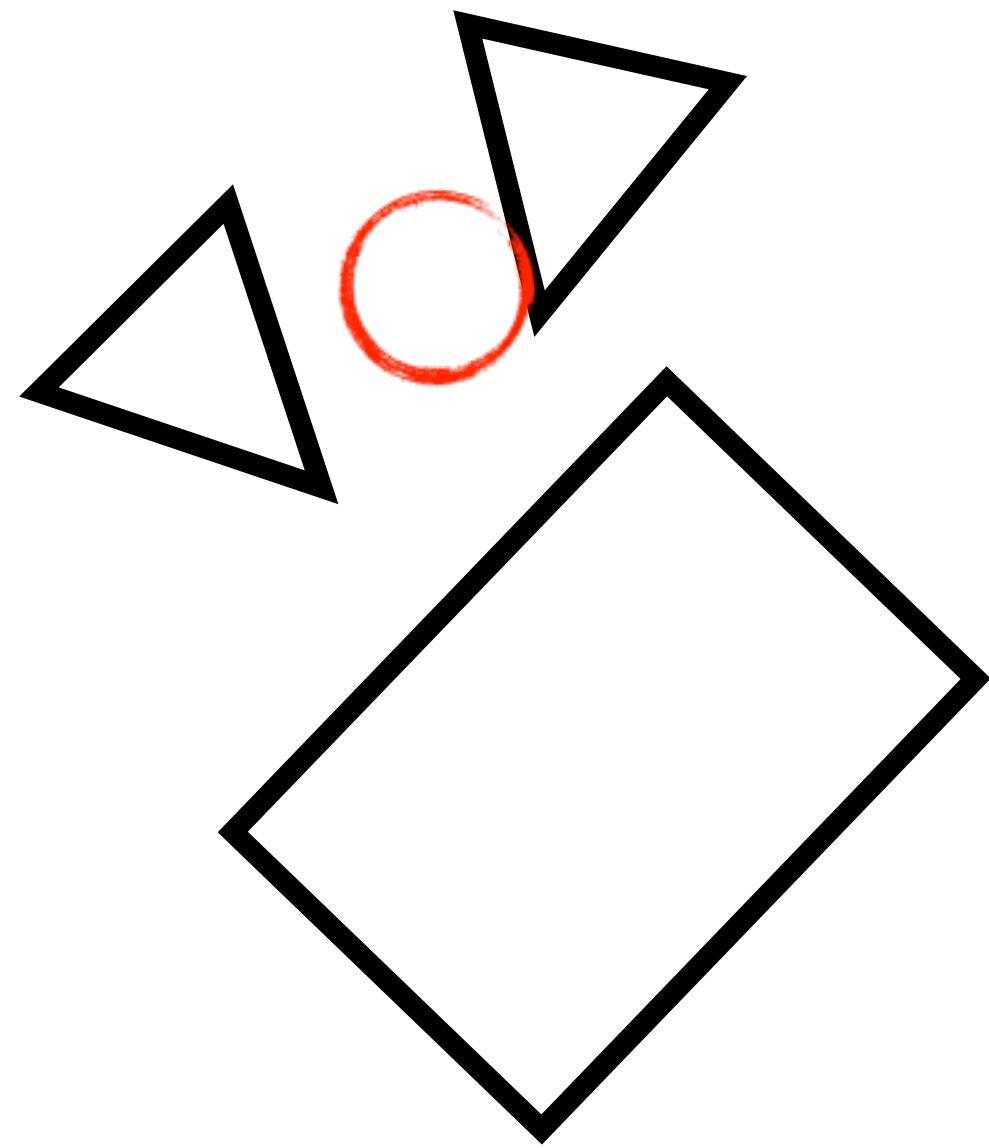
ACM Transaction on Graphics, 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)



# Background

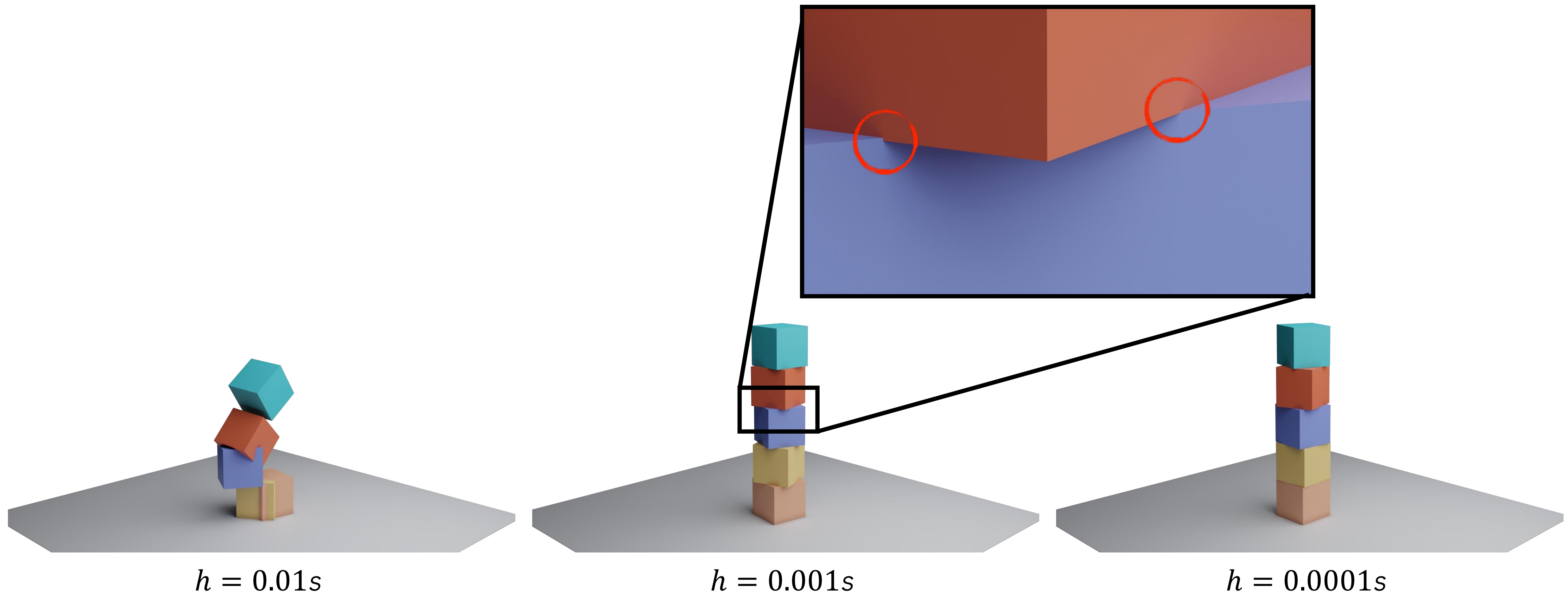
State of the Art



Large time steps

No guarantees!

# A solution might not exist!



Simulated using Chrono [Tasora et al. 2016]



Key:

Finishes successfully

Unable to finish in four hours

Energy or displacement blow up

Intersections

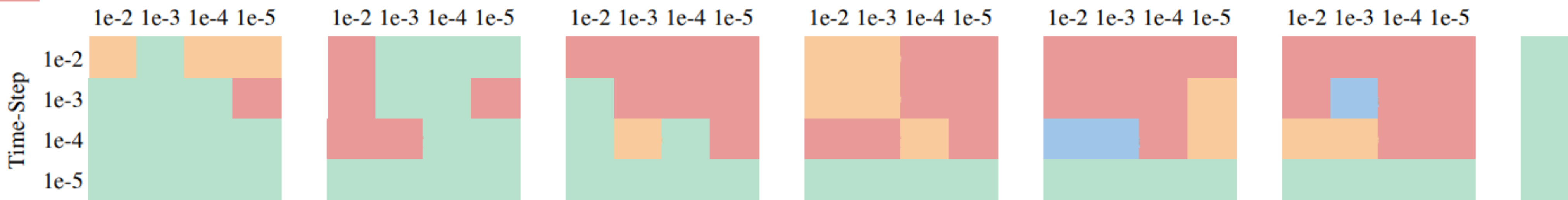
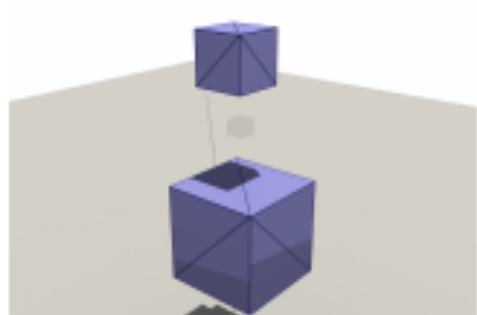


### Sequential Quadratic Programming

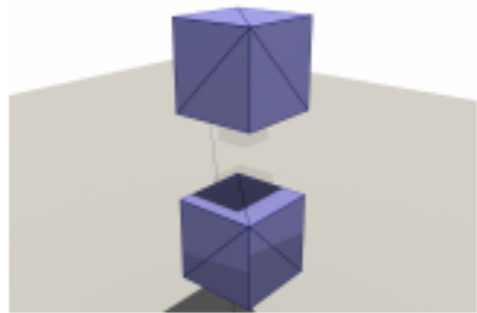
### Quadratic Programming with Nonlinear Constraints

IPC

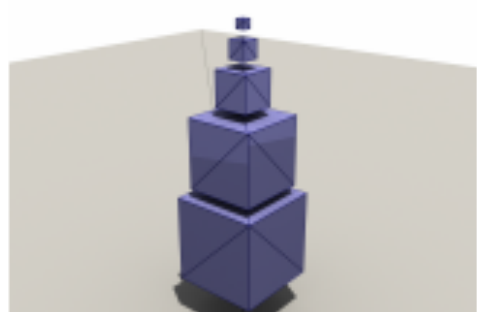
Cube vs Cube



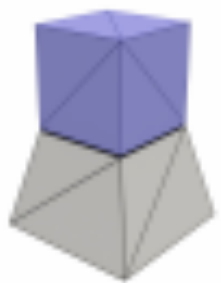
Aligned Cube vs Cube



Five Cube Pyramid

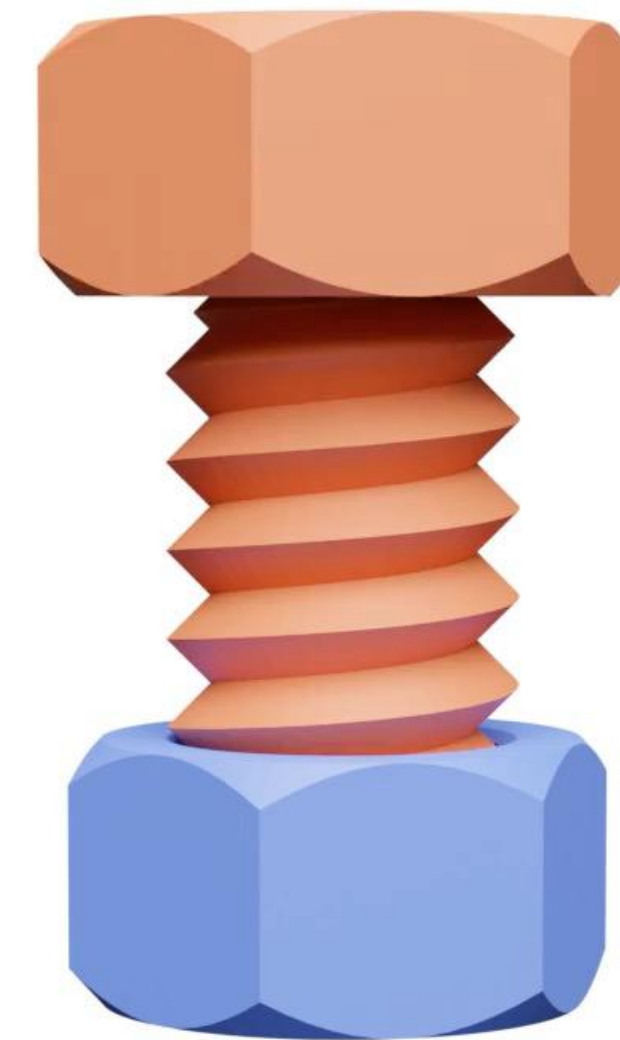
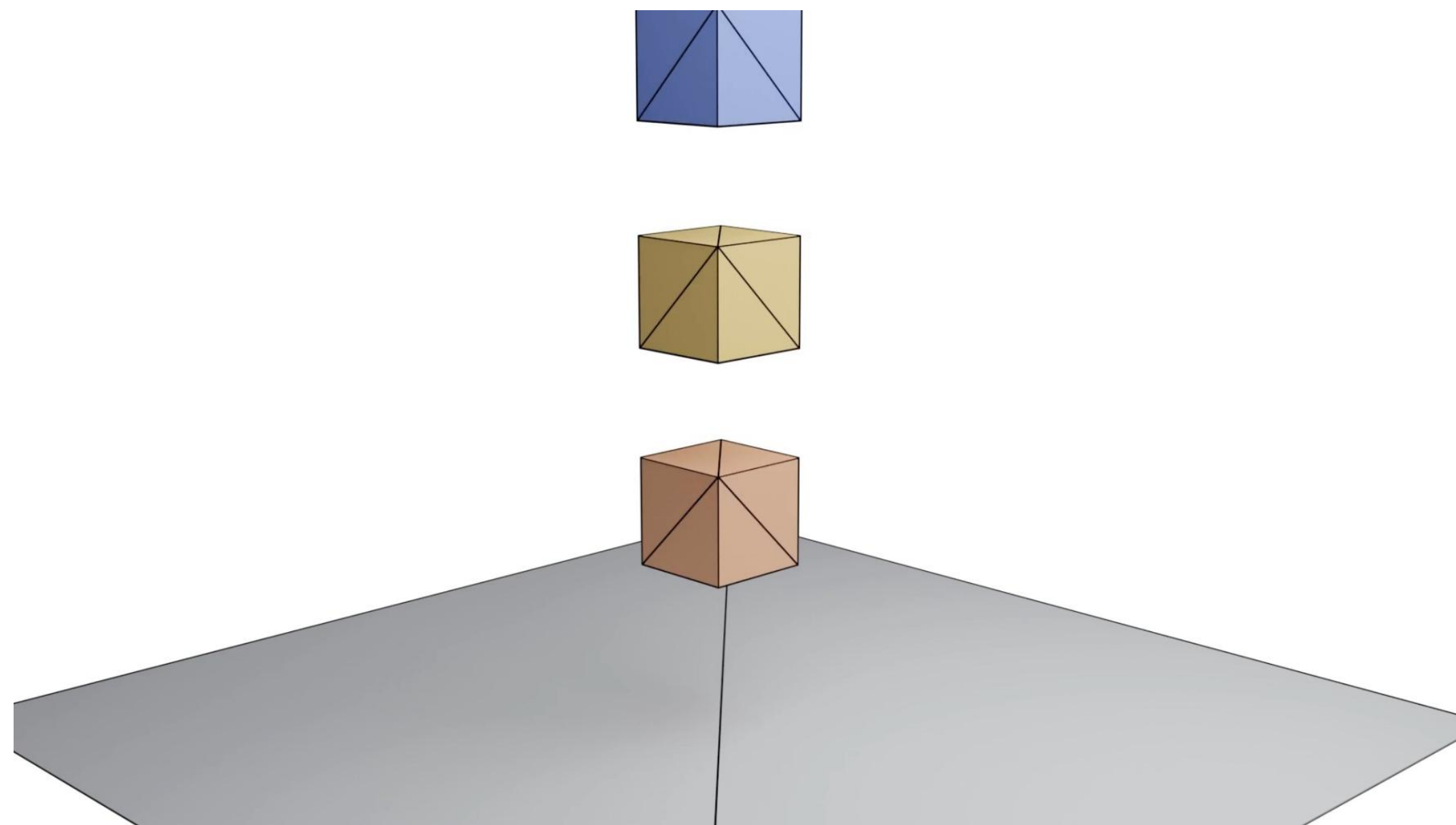


Erleben's Cliff Edges



# Goals

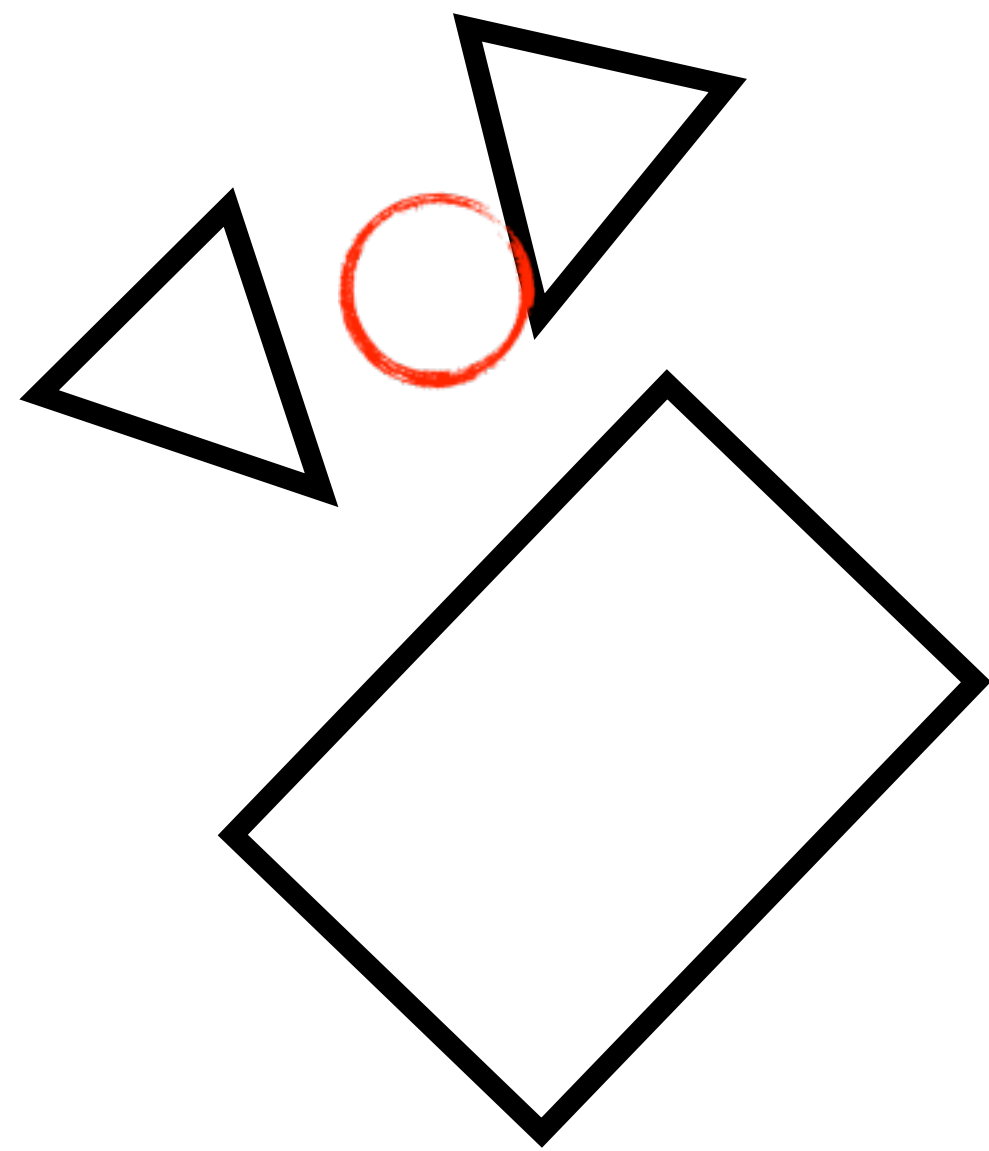
- Invariant 1: No intersection
- Invariant 2: Positive Volume
- Requirement: Differentiable





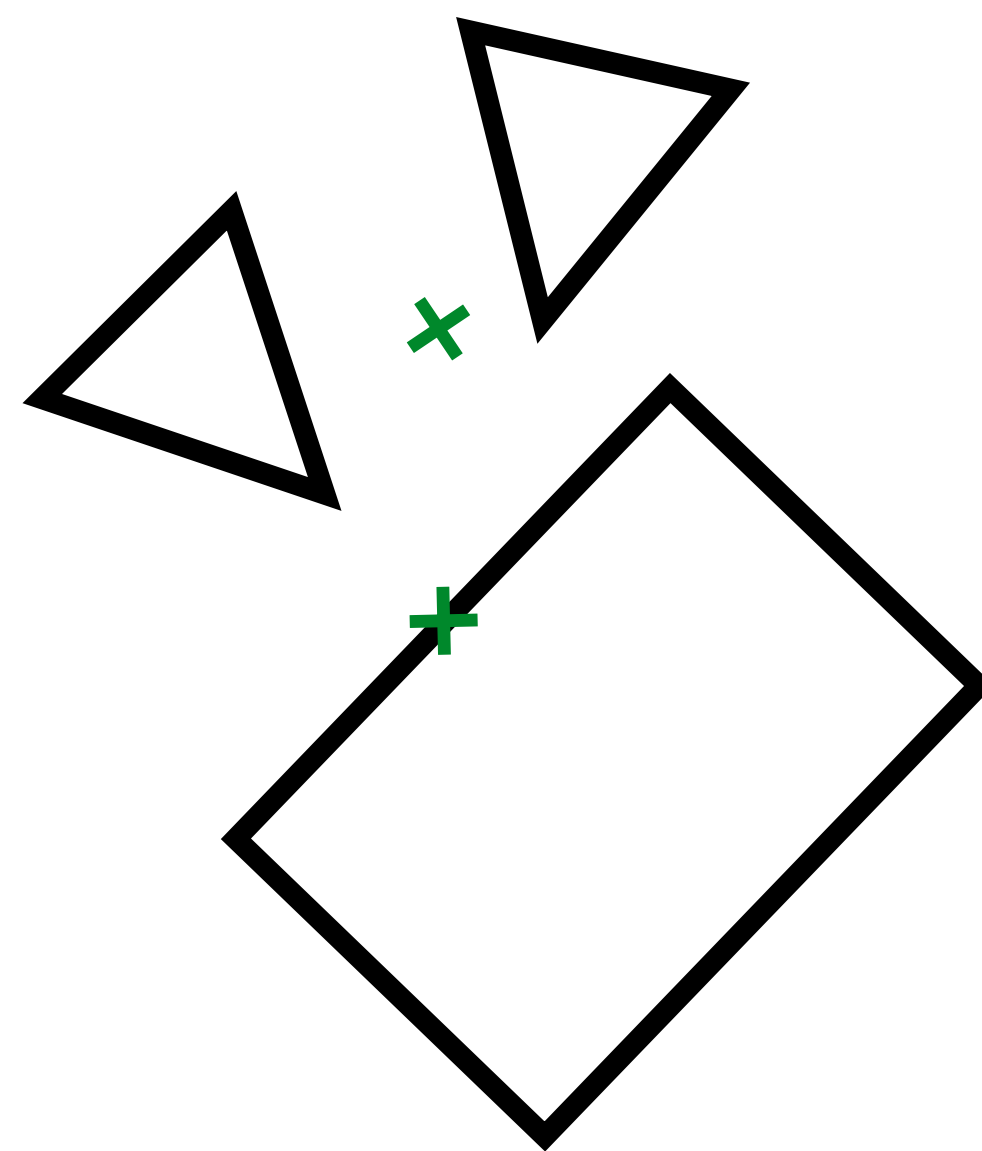
# Solution

State of the Art



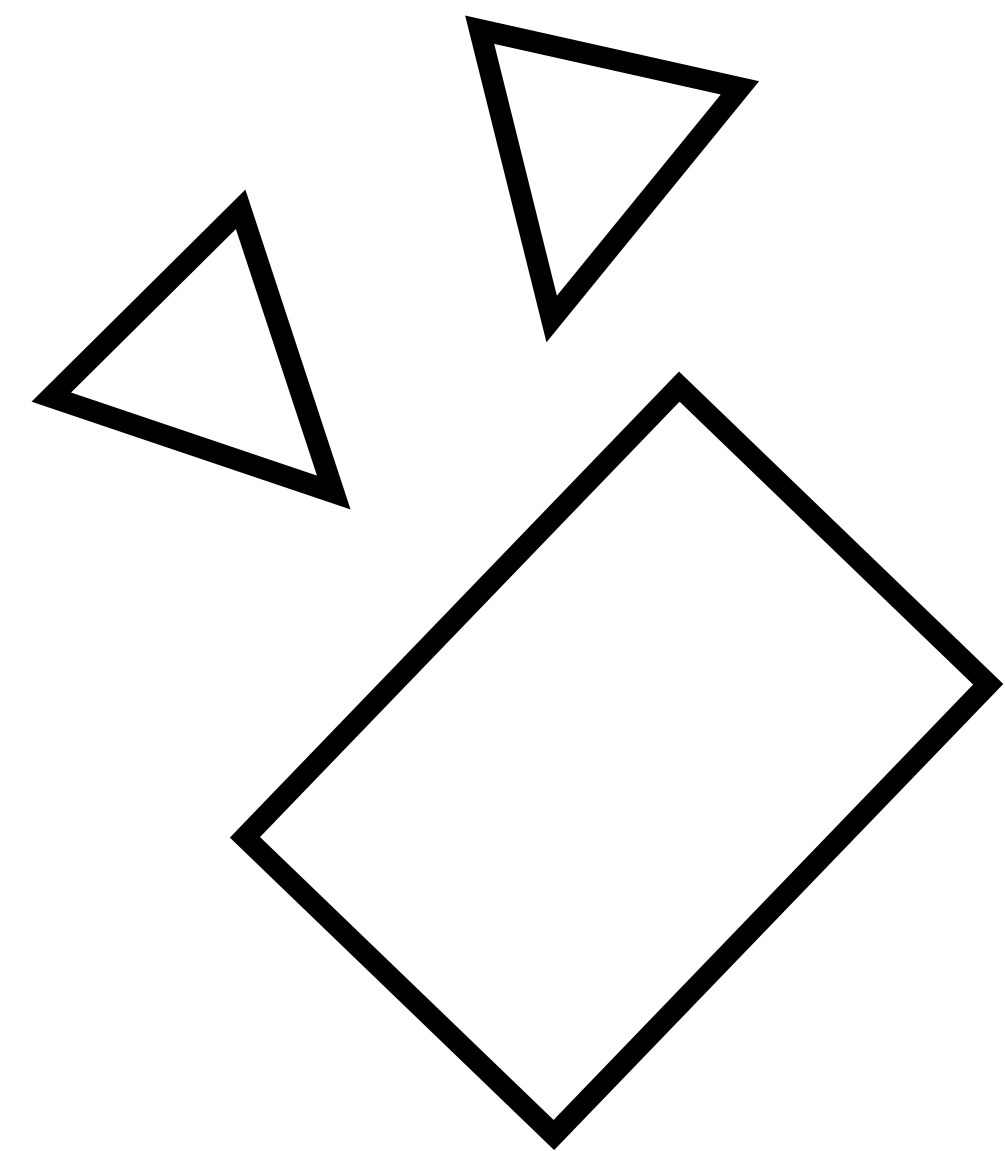
Large time steps  
No guarantees!

Explicit Method



Small time steps!  
Guarantees

Our Approach/IPC



Large time steps  
Guarantees

# Incremental Potential

- Newton's equation of motion ( $x$  = vertex positions):

$$M\ddot{x} = f(x)$$

- Implicit time integration (e.g., implicit Euler):

$$x^{t+1} = x^t + h\dot{x}^t + h^2 M^{-1} f(x^{t+1})$$

- Optimization-based time integration [Kane et al. 2000]:

$$\hat{x} = x^t + h\dot{x}^t$$

$$x^{t+1} = \operatorname{argmin}_x \left( E(x) = \frac{1}{2} (x - \hat{x})^T M (x - \hat{x}) - h^2 x^T f(x) \right)$$

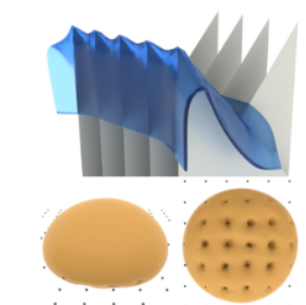
Incremental Potential  
(IP)

Incremental Potential Contact: Intersection- and Inversion-free, Large-Deformation Dynamics

Minchen Li, Zachary Ferguson, Teseo Schneider, Timothy Langlois, Denis Zorin, Daniele Panozzo, Chenfanfu Jiang, Danny M. Kaufman,

ACM Transaction on Graphics (SIGGRAPH), 2020

[Paper] [Video] [Supplemental Video] [Code] [S1: Technical] [S2: Comparisons] [S3: Statistics] [Website]

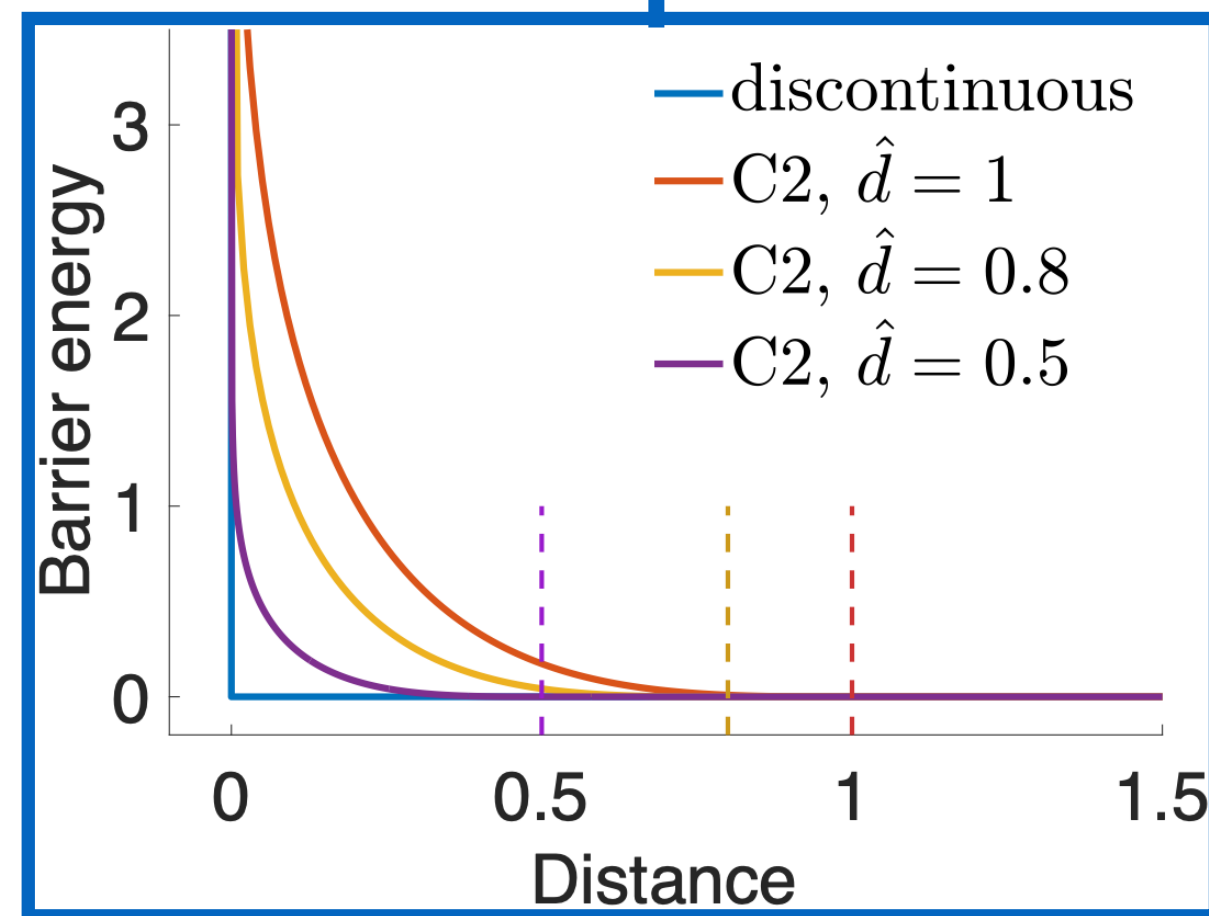




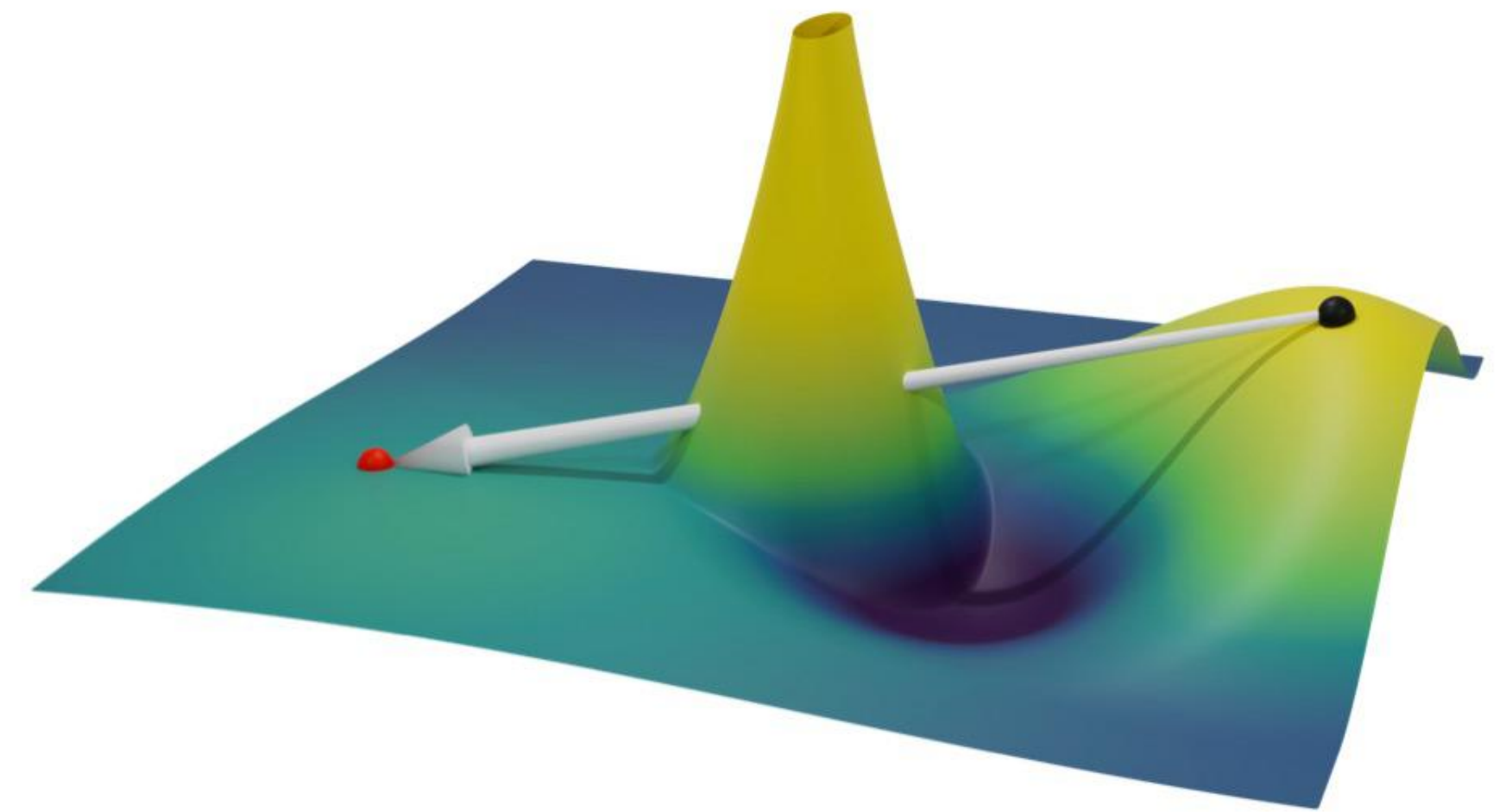
# Contact and Friction\* Potentials

$$x^{t+1} = \underset{x}{\operatorname{argmin}} E(x) + B(x, \hat{d}) + D(x)$$

Inertia &  
Elasticity

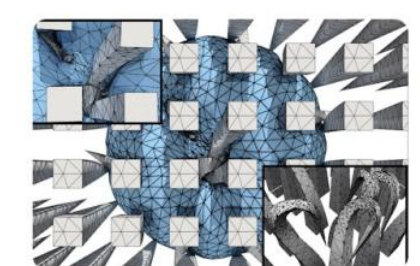
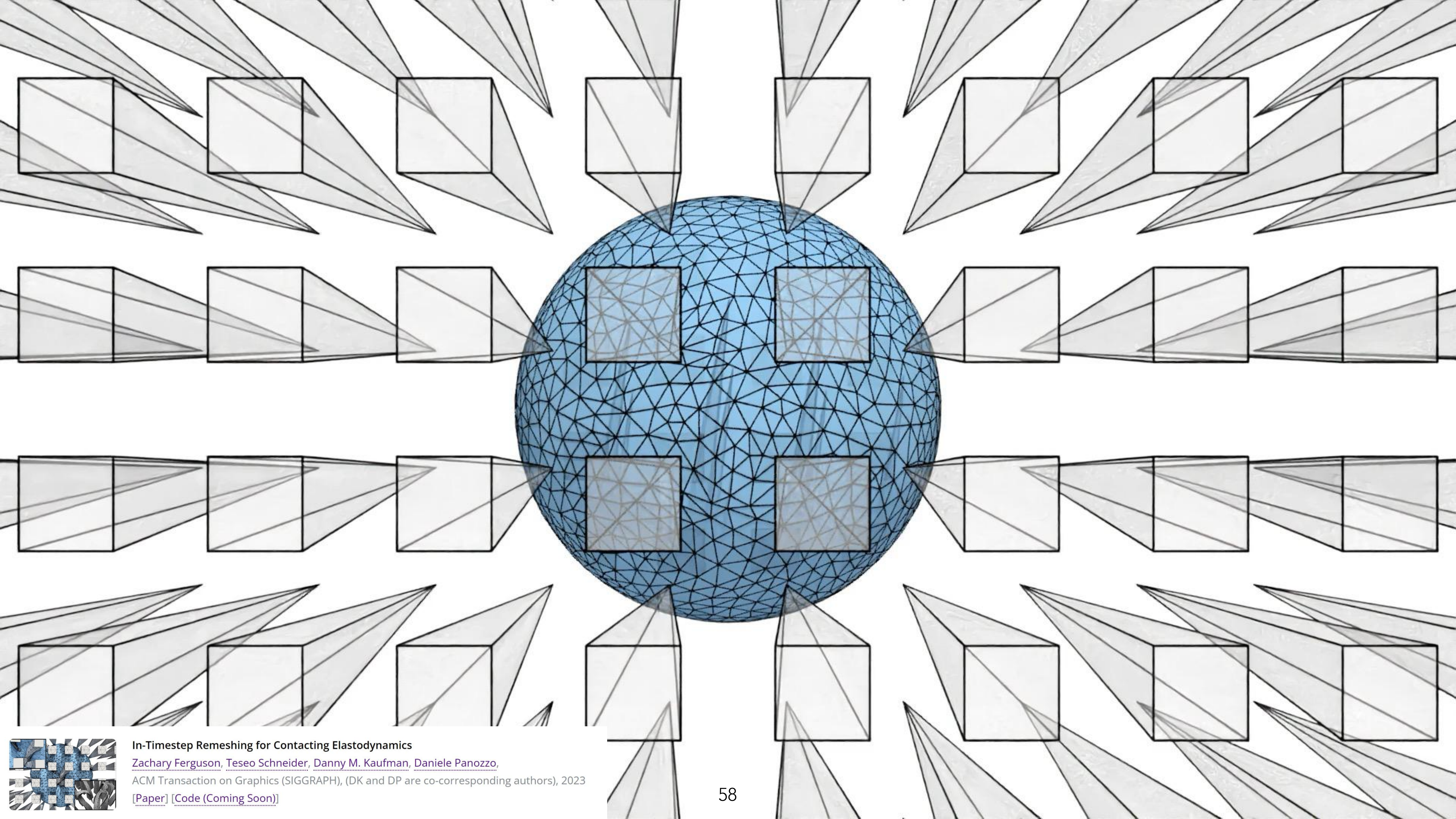


Friction\*



Minimize using Newton's method, with line search to ensure lack of constraints violation





**In-Timestep Remeshing for Contacting Elastodynamics**

[Zachary Ferguson](#), [Teseo Schneider](#), [Danny M. Kaufman](#), [Daniele Panozzo](#),

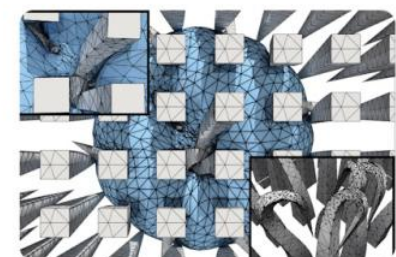
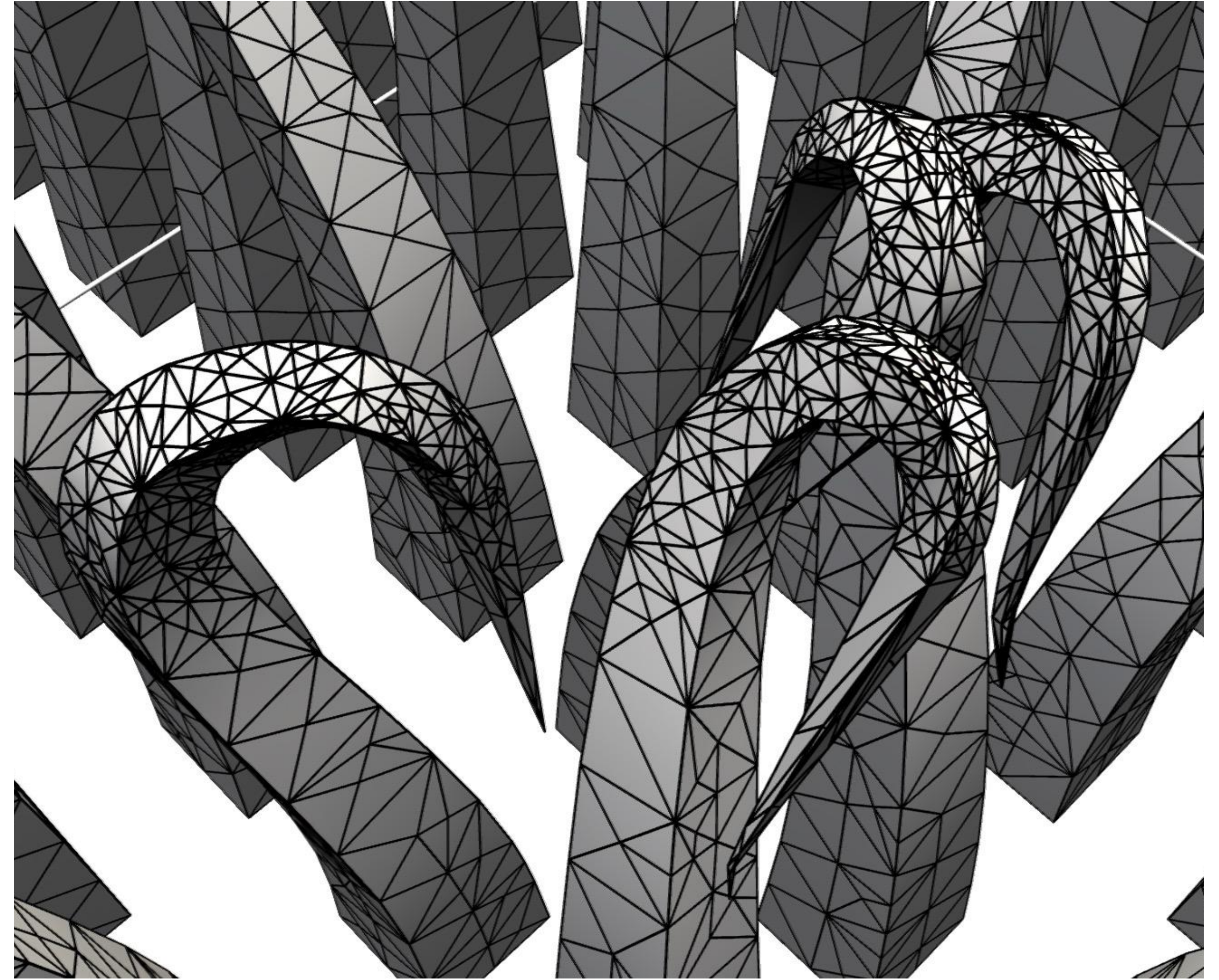
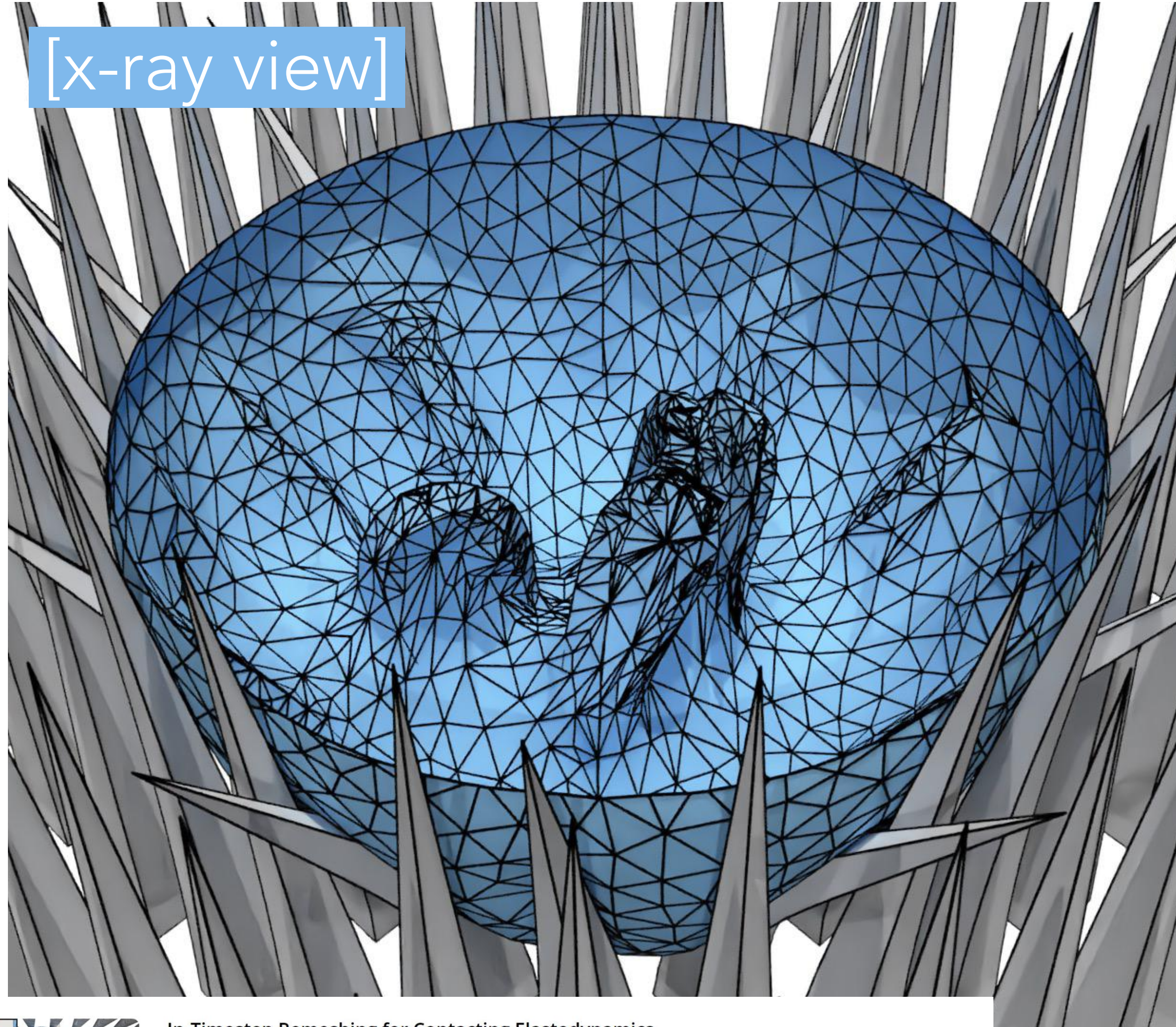
ACM Transaction on Graphics (SIGGRAPH), (DK and DP are co-corresponding authors), 2023

[\[Paper\]](#) [\[Code \(Coming Soon\)\]](#)



# Our Adaptive Meshing

[x-ray view]



**In-Timestep Remeshing for Contacting Elastodynamics**

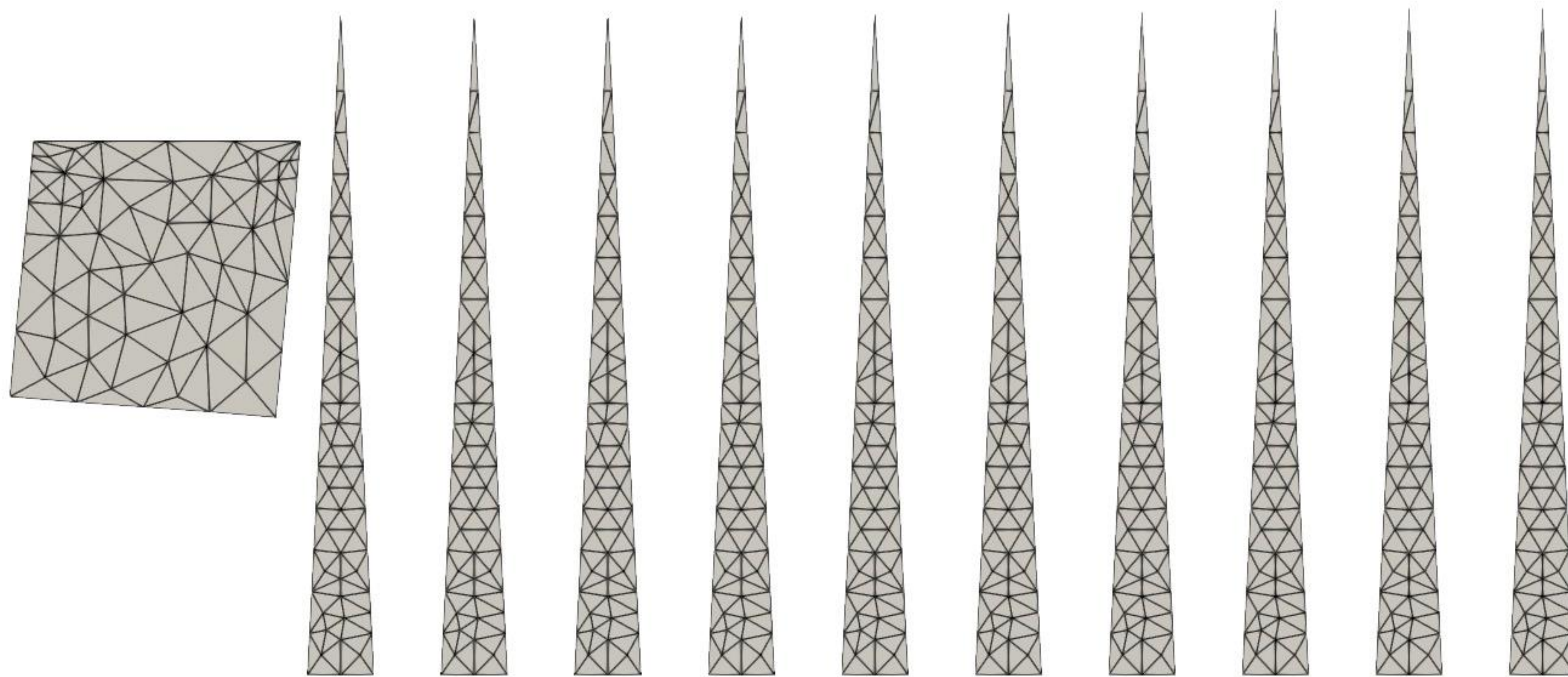
[Zachary Ferguson](#), [Teseo Schneider](#), [Danny M. Kaufman](#), [Daniele Panozzo](#),

ACM Transaction on Graphics (SIGGRAPH), (DK and DP are co-corresponding authors), 2023

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dt: 0.01s

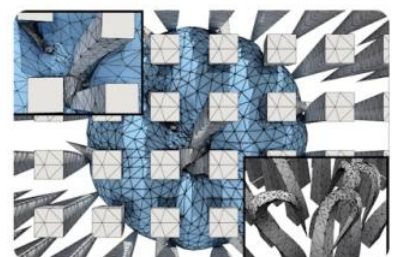


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[\[Paper\]](#) [\[Code \(Coming Soon\)\]](#)

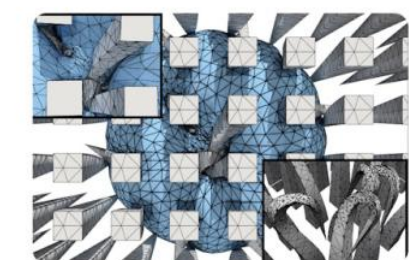
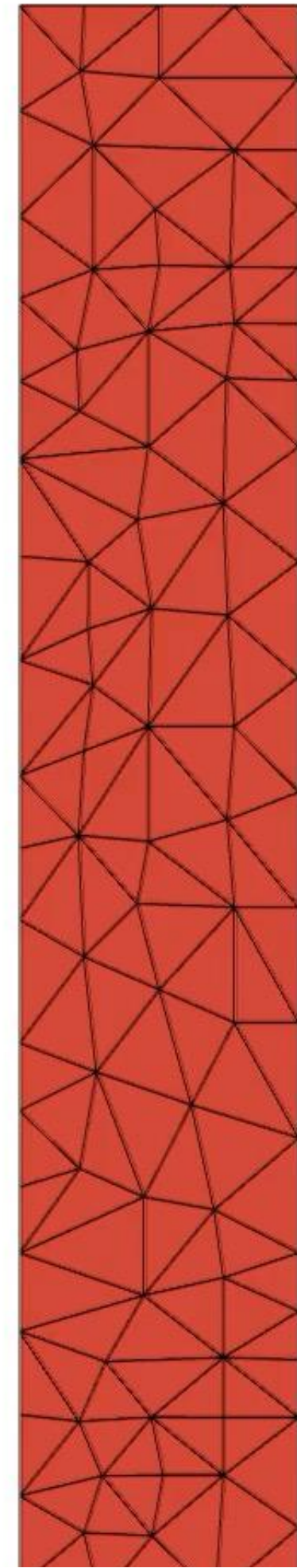




# Bar twisting

No Remeshing

Ours



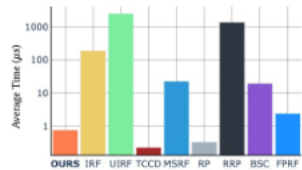
**In-Timestep Remeshing for Contacting Elastodynamics**

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[\[Paper\]](#) [\[Code \(Coming Soon\)\]](#)

# Collision Detection



A Large Scale Benchmark and an Inclusion-Based Algorithm for Continuous Collision Detection

Bolun Wang, [Zachary Ferguson](#), [Teseo Schneider](#), [Xin Jiang](#), [Marco Attene](#), [Daniele Panozzo](#),

ACM Transaction on Graphics, 2021

[\[Paper\]](#) [\[Video\]](#) [\[CCD Code\]](#) [\[Benchmark Code\]](#)

## Orientation-aware Incremental Potential Contact

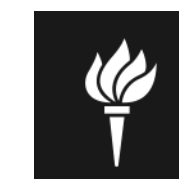
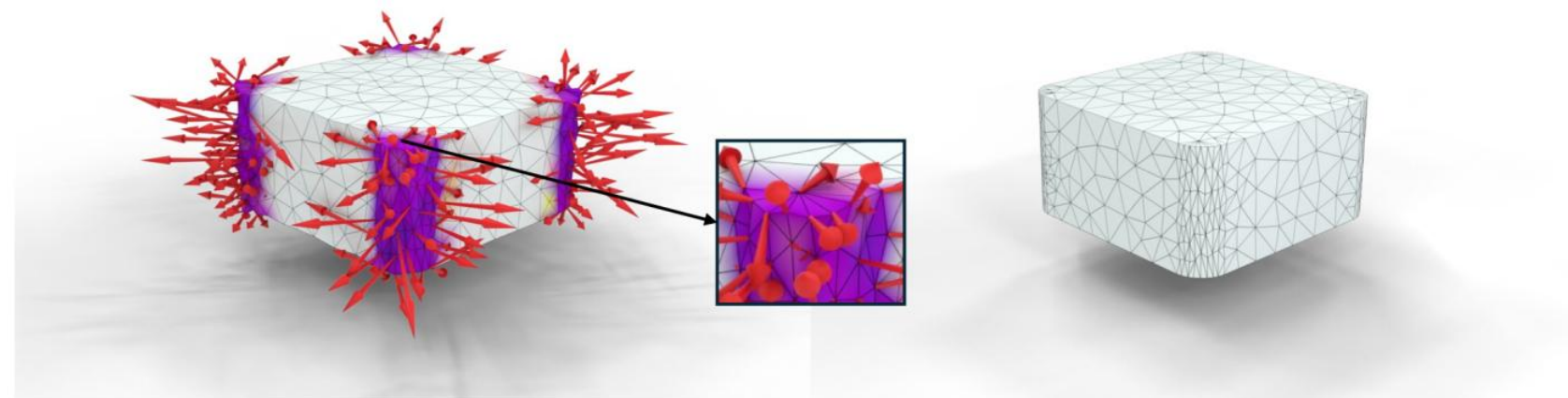
ZIZHOU HUANG, New York University, USA

MAX PAIK, New York University, USA

ZACHARY FERGUSON, Massachusetts Institute of Technology, USA

DANIELE PANOZZO, New York University, USA

DENIS ZORIN, New York University, USA

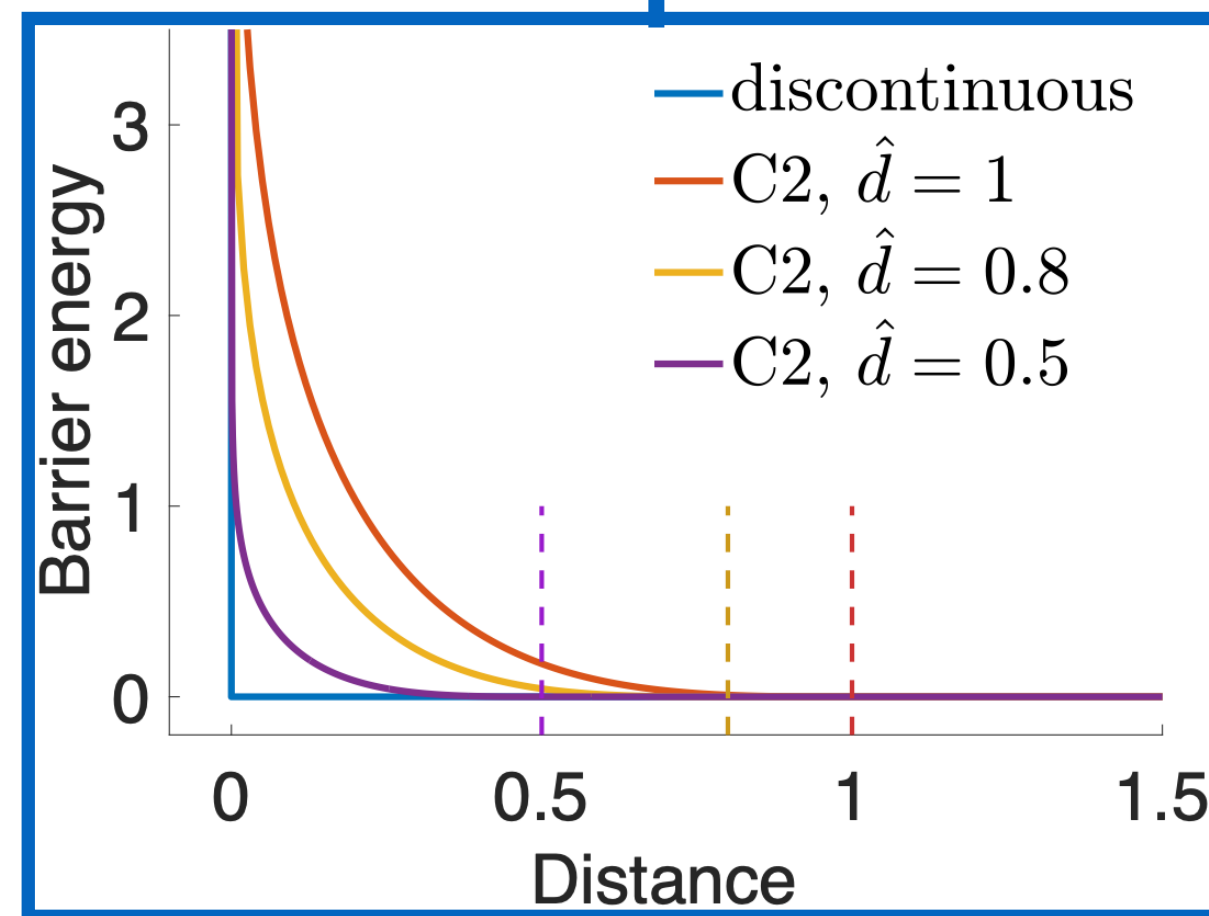




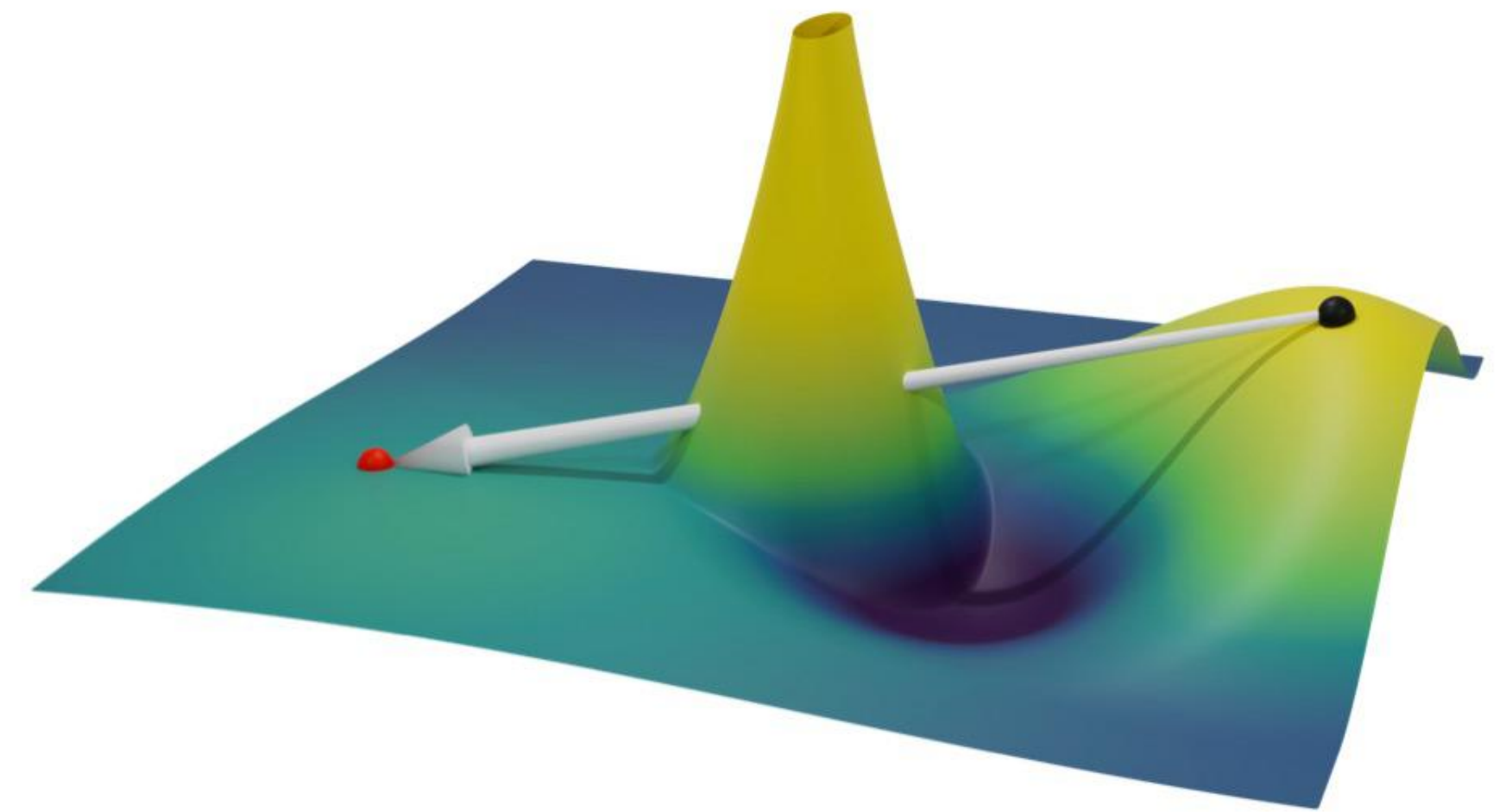
# Contact and Friction\* Potentials

$$x^{t+1} = \underset{x}{\operatorname{argmin}} E(x) + B(x, \hat{d}) + D(x)$$

Inertia &  
Elasticity

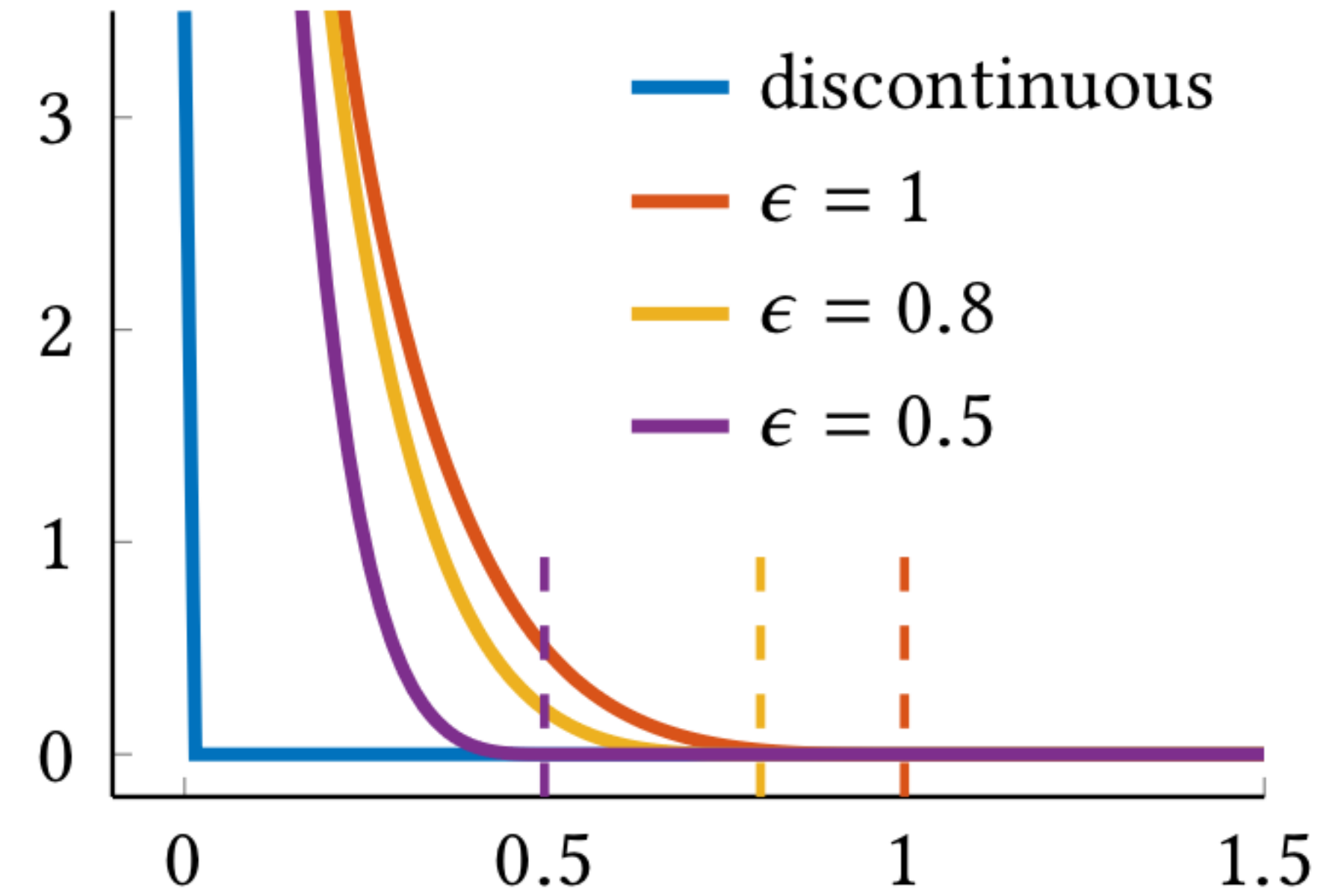
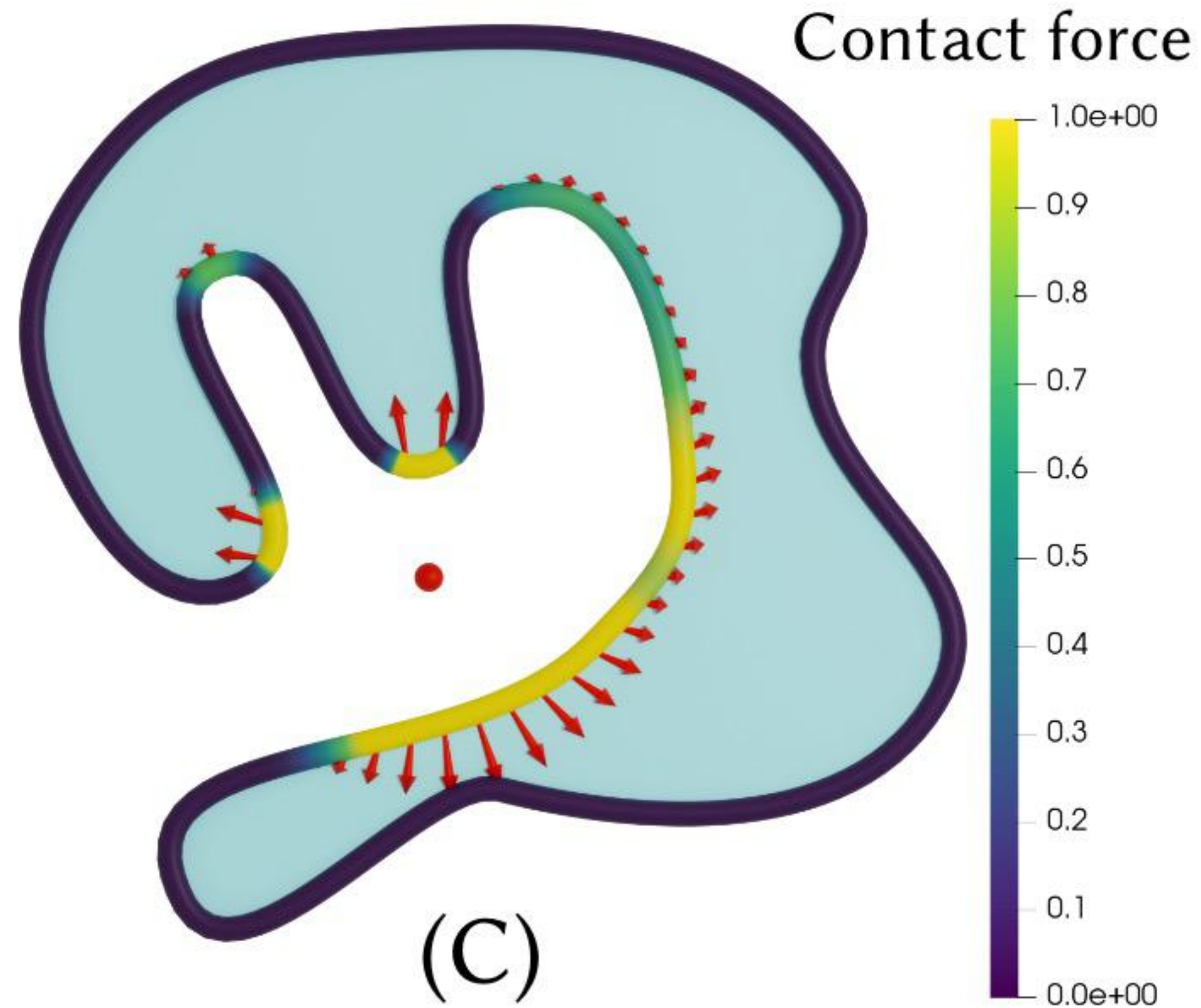


Friction\*



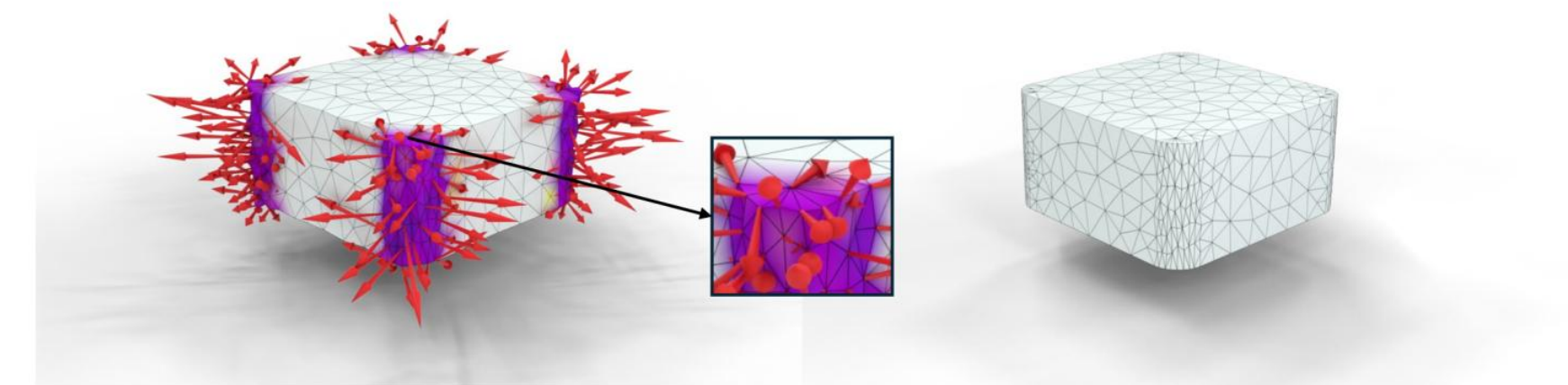
Minimize using Newton's method, with line search to ensure lack of constraints violation

# Contact Potential



## Orientation-aware Incremental Potential Contact

ZIZHOU HUANG, New York University, USA  
MAX PAIK, New York University, USA  
ZACHARY FERGUSON, Massachusetts Institute of Technology, USA  
DANIELE PANOZZO, New York University, USA  
DENIS ZORIN, New York University, USA

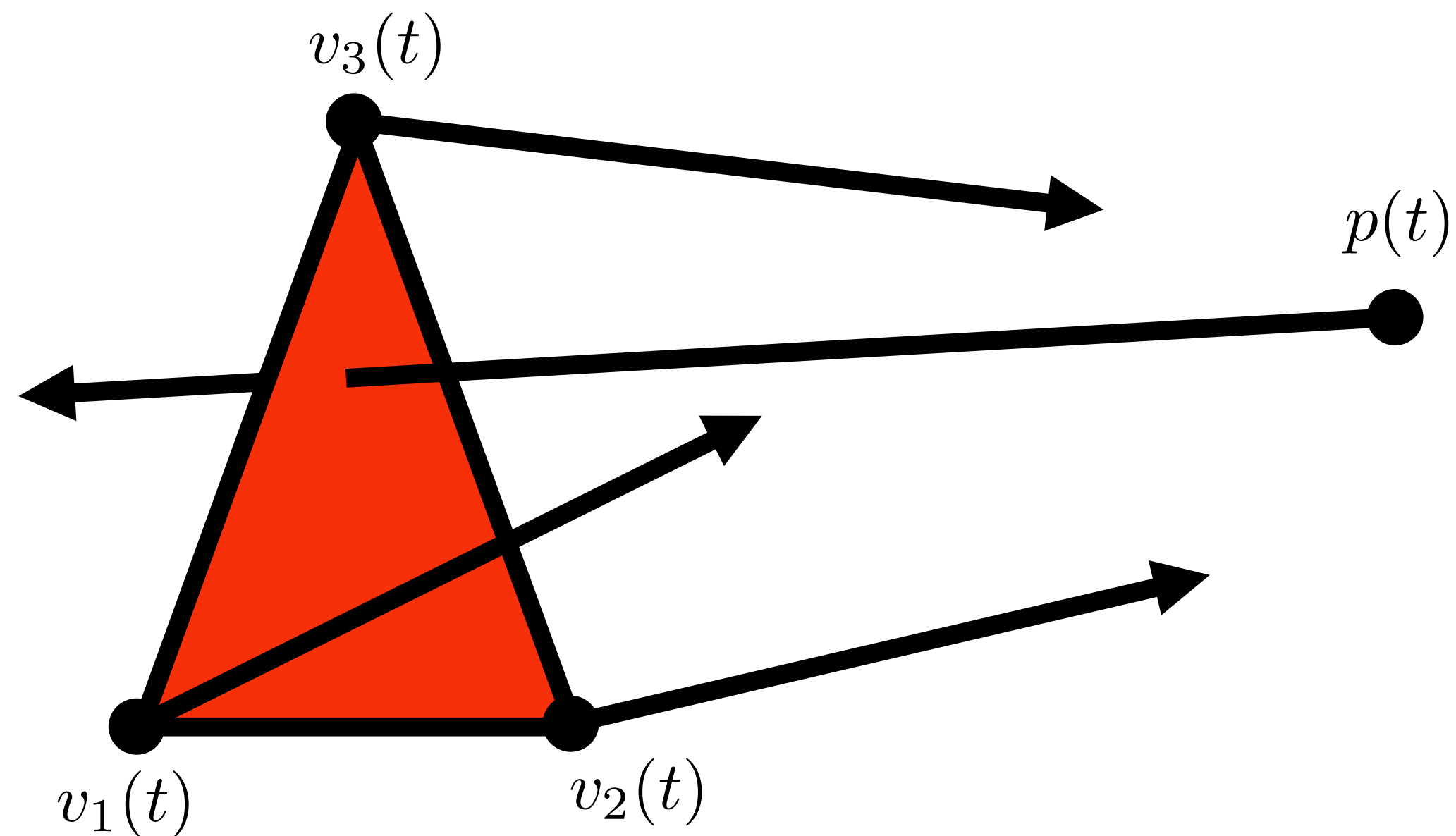




# Point to Triangle

Multivariate  
(Snyder 1992)

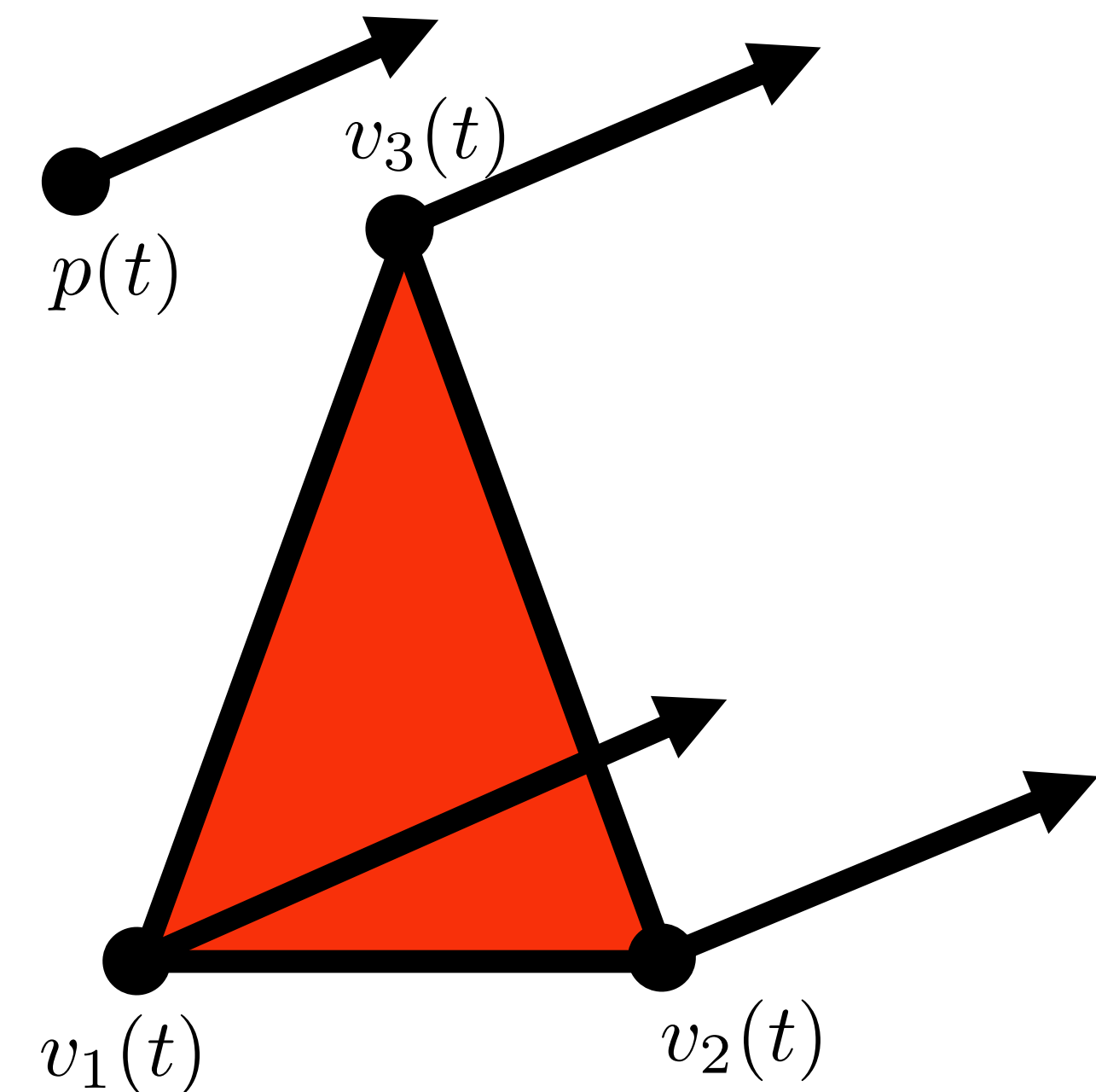
$$F_{\text{vf}}(t, u, v) = p(t) - ((1 - u - v)v_1(t) + uv_2(t) + vv_3(t)),$$



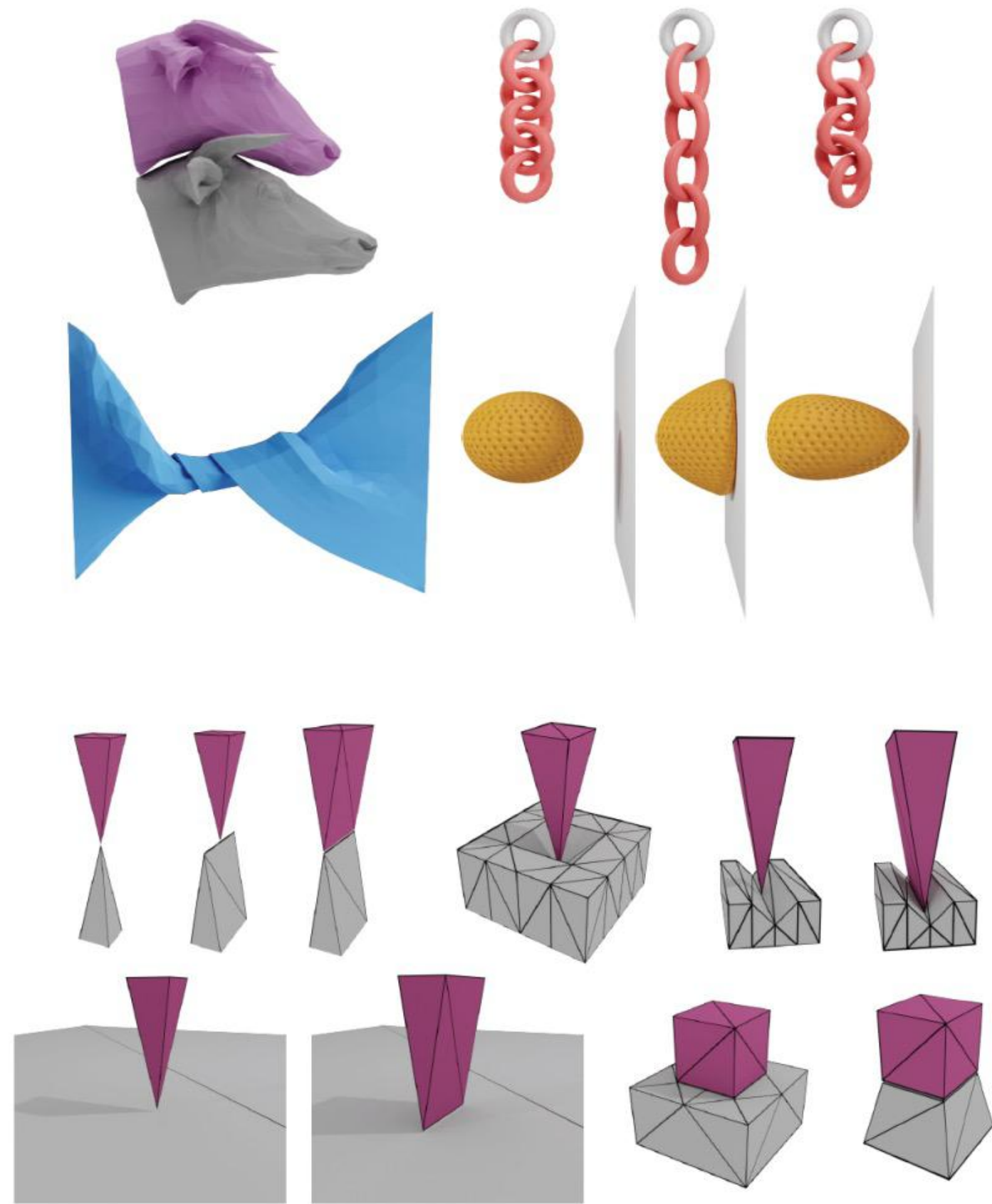
Univariate  
(modern approach)

$$f(t) = \langle n(t), q(t) \rangle = 0,$$

$$n(t) = (v_2(t) - v_1(t)) \times (v_3(t) - v_1(t)) \text{ and } q(t) = p(t) - v_1(t)$$



# Continuous Collision Detection

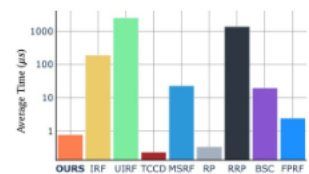


A Large Scale Benchmark and an Inclusion-Based Algorithm for Continuous Collision Detection

Bolun Wang, [Zachary Ferguson](#), [Teseo Schneider](#), [Xin Jiang](#), [Marco Attene](#), [Daniele Panozzo](#),

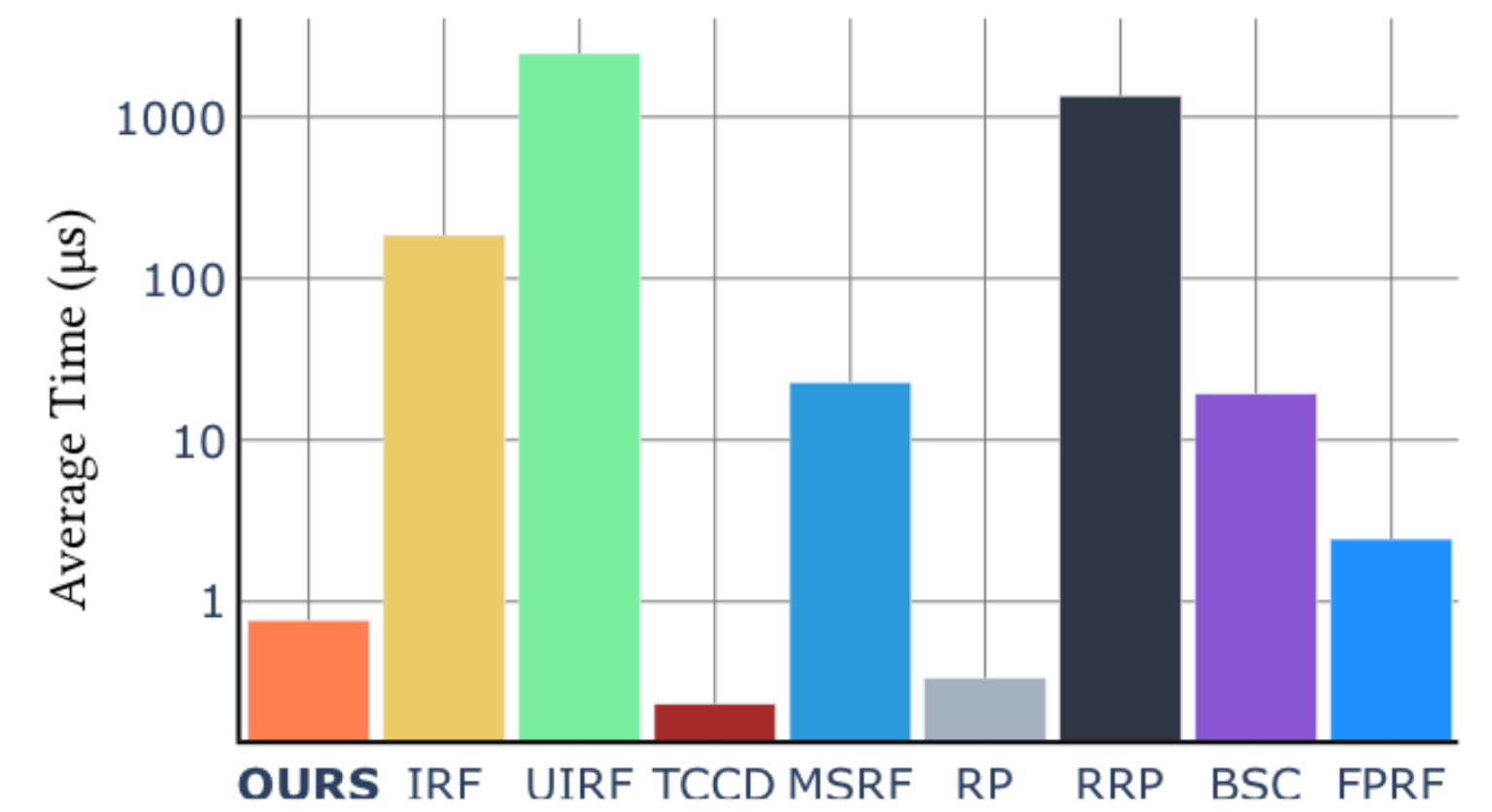
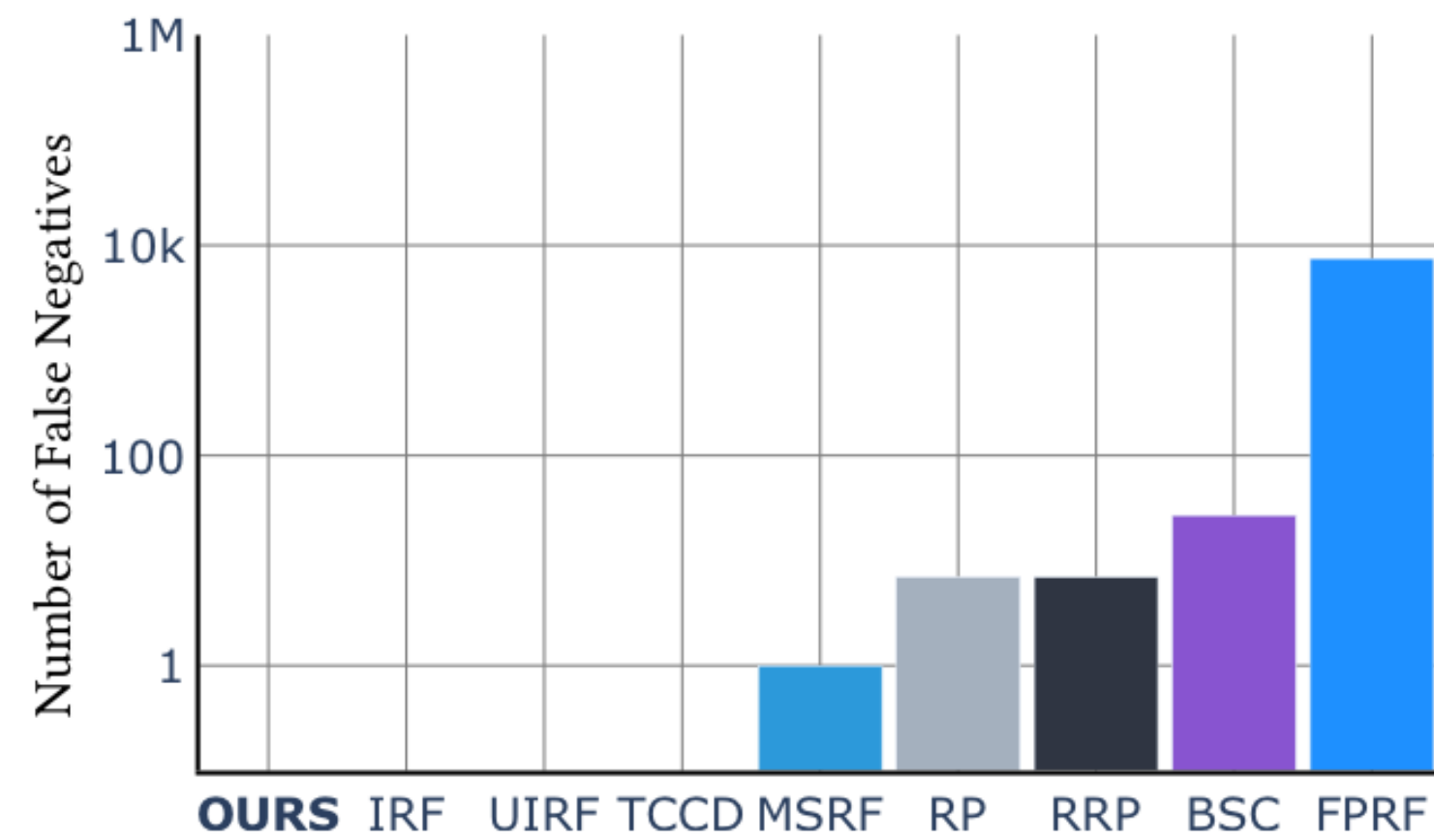
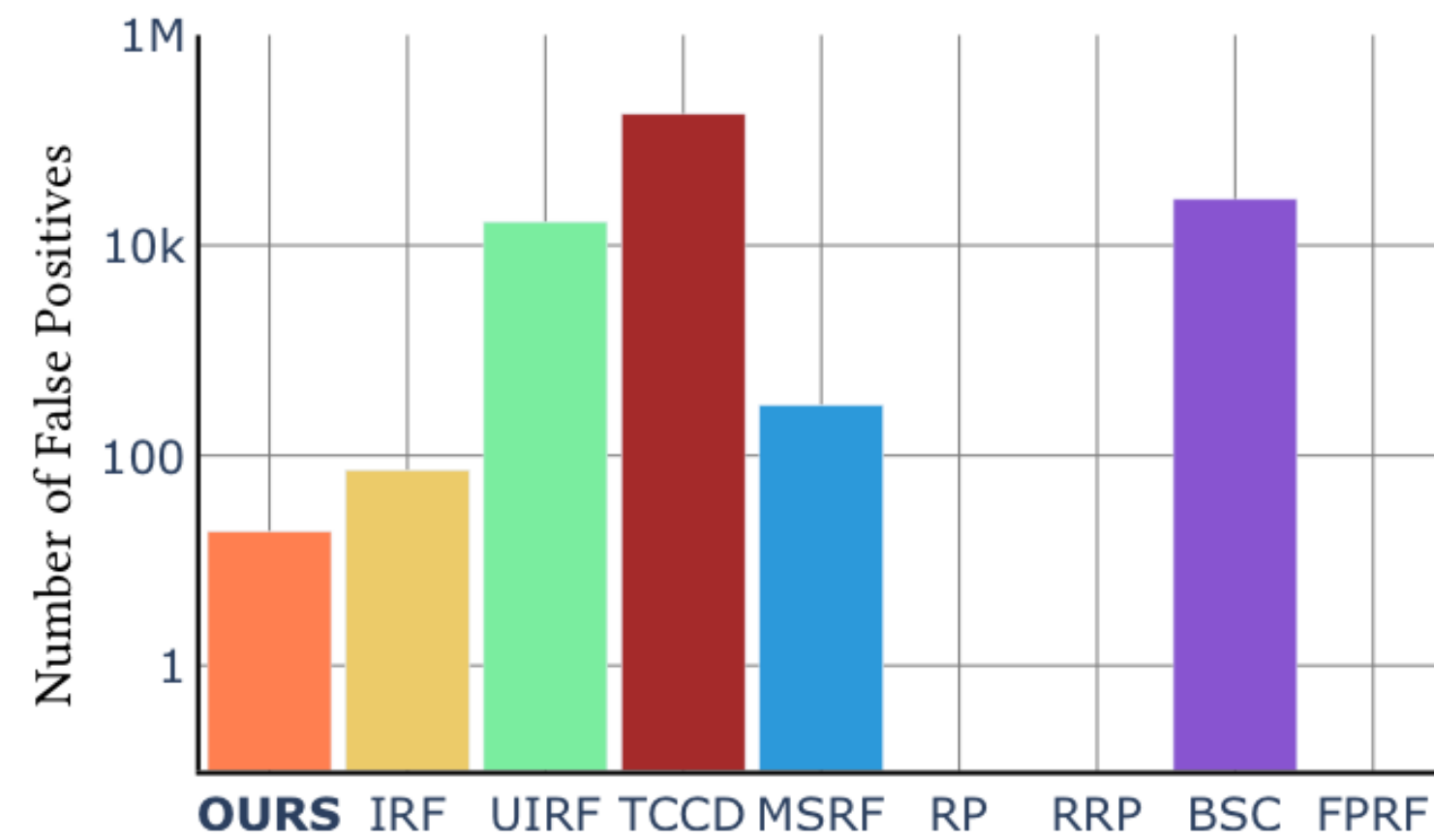
ACM Transaction on Graphics, 2021

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# Benchmark

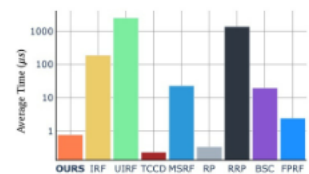


A Large Scale Benchmark and an Inclusion-Based Algorithm for Continuous Collision Detection

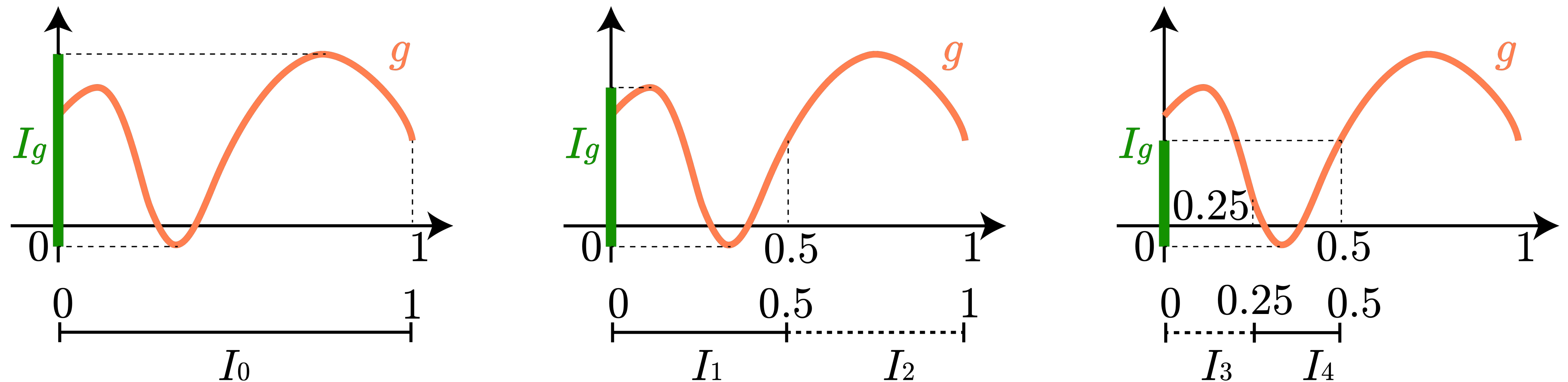
Bolun Wang, Zachary Ferguson, Teseo Schneider, Xin Jiang, Marco Attene, Daniele Panizzo,

ACM Transaction on Graphics, 2021

[\[Paper\]](#) [\[Video\]](#) [\[CCD Code\]](#) [\[Benchmark Code\]](#)



# Bisection Root Finding



The inclusion function  $I_g$  is:

1. A conservative estimation (Inclusion) of the co-domain.
2. Convergent ( $size(I_g) \rightarrow 0$  when  $I_i \rightarrow 0$ ).



# Algorithm

---

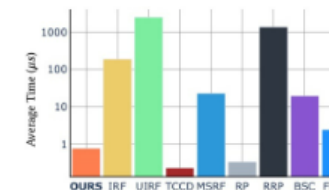
**Algorithm 1** Inclusion-based root-finder

---

```
1: function SOLVE( $I_0, g, \delta$ )
2:    $res \leftarrow \emptyset$ 
3:    $S \leftarrow \{I_0\}$ 
4:    $\ell \leftarrow 0$ 
5:   while  $L \neq \emptyset$  do
6:      $I \leftarrow \text{POP}(L)$ 
7:      $I_g \leftarrow \square g(I)$        $\triangleright$  Compute the inclusion function
8:     if  $0 \in I_g$  then
9:       if  $w(I) < \delta$  then       $\triangleright I$  is small enough
10:         $res \leftarrow R \cup \{I\}$ 
11:       else
12:          $I_1, I_2 \leftarrow \text{SPLIT}(I)$ 
13:          $S \leftarrow S \cup \{I_1, I_2\}$ 
14:    $\ell \leftarrow \ell + 1$ 
return  $res$ 
```

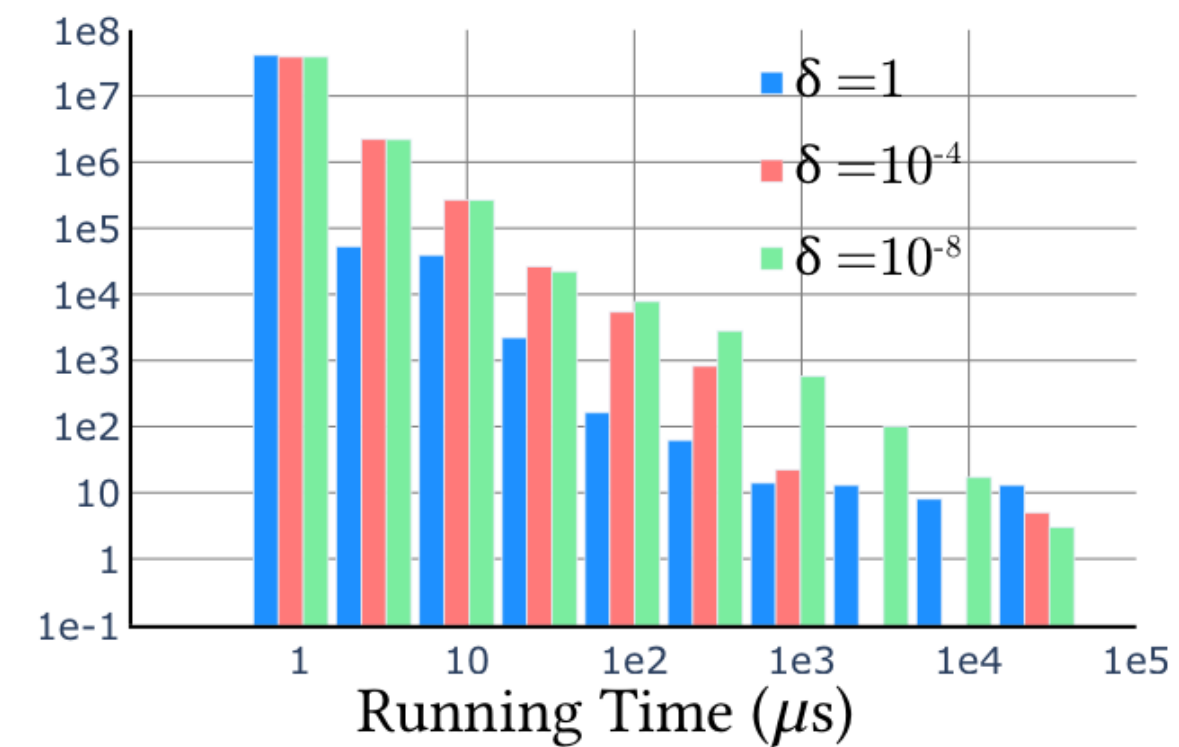
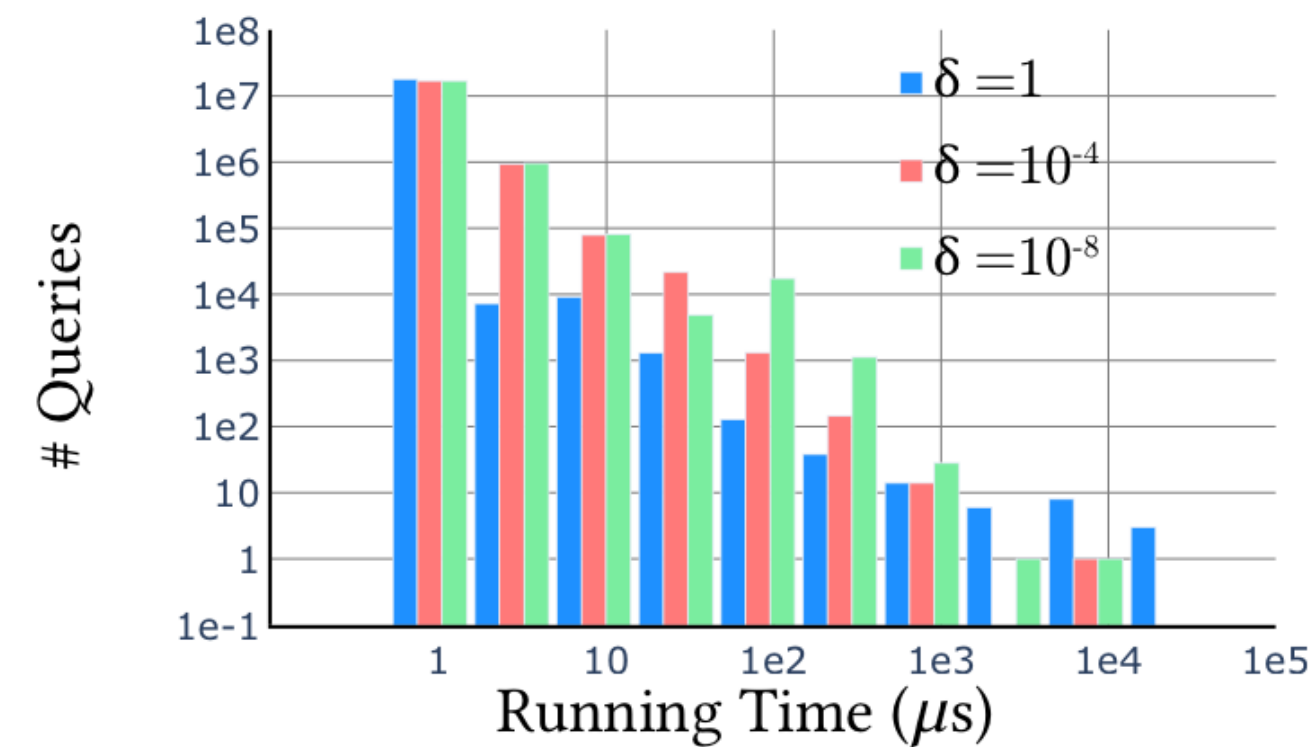
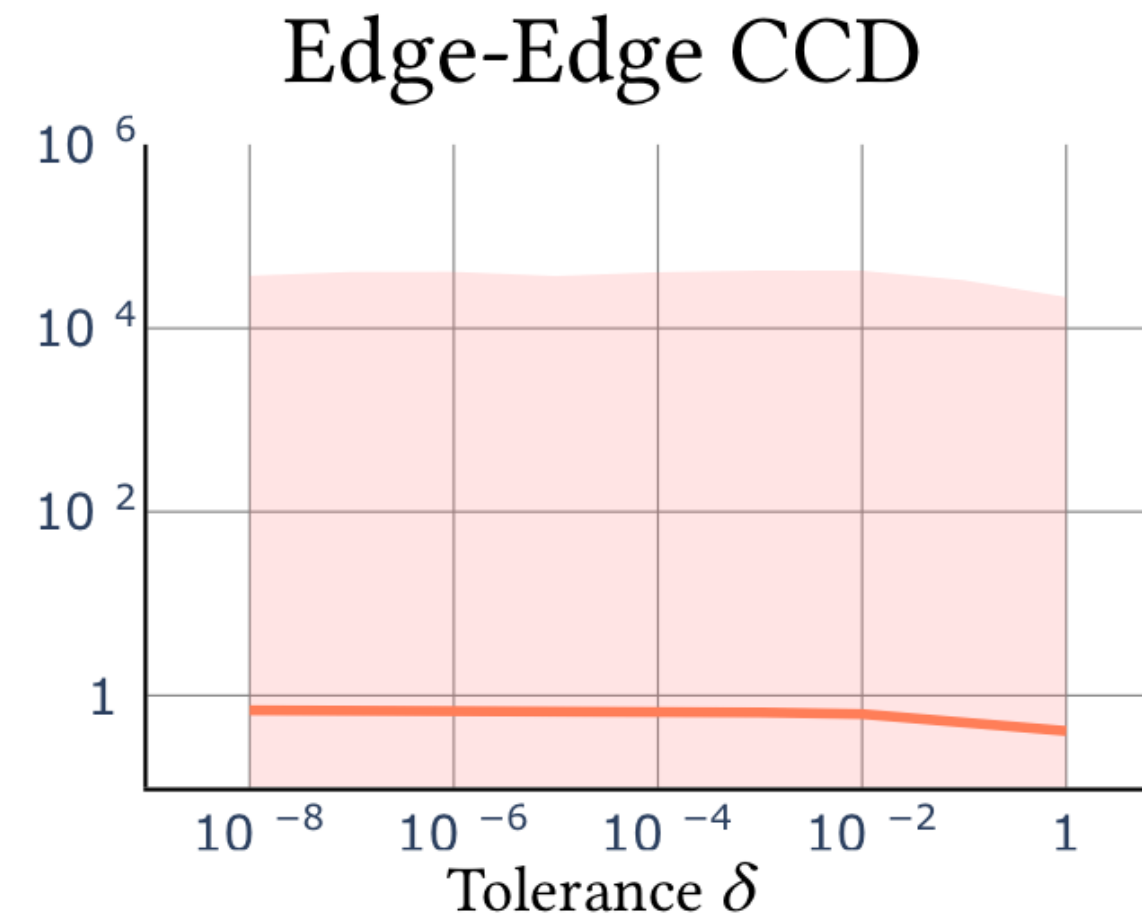
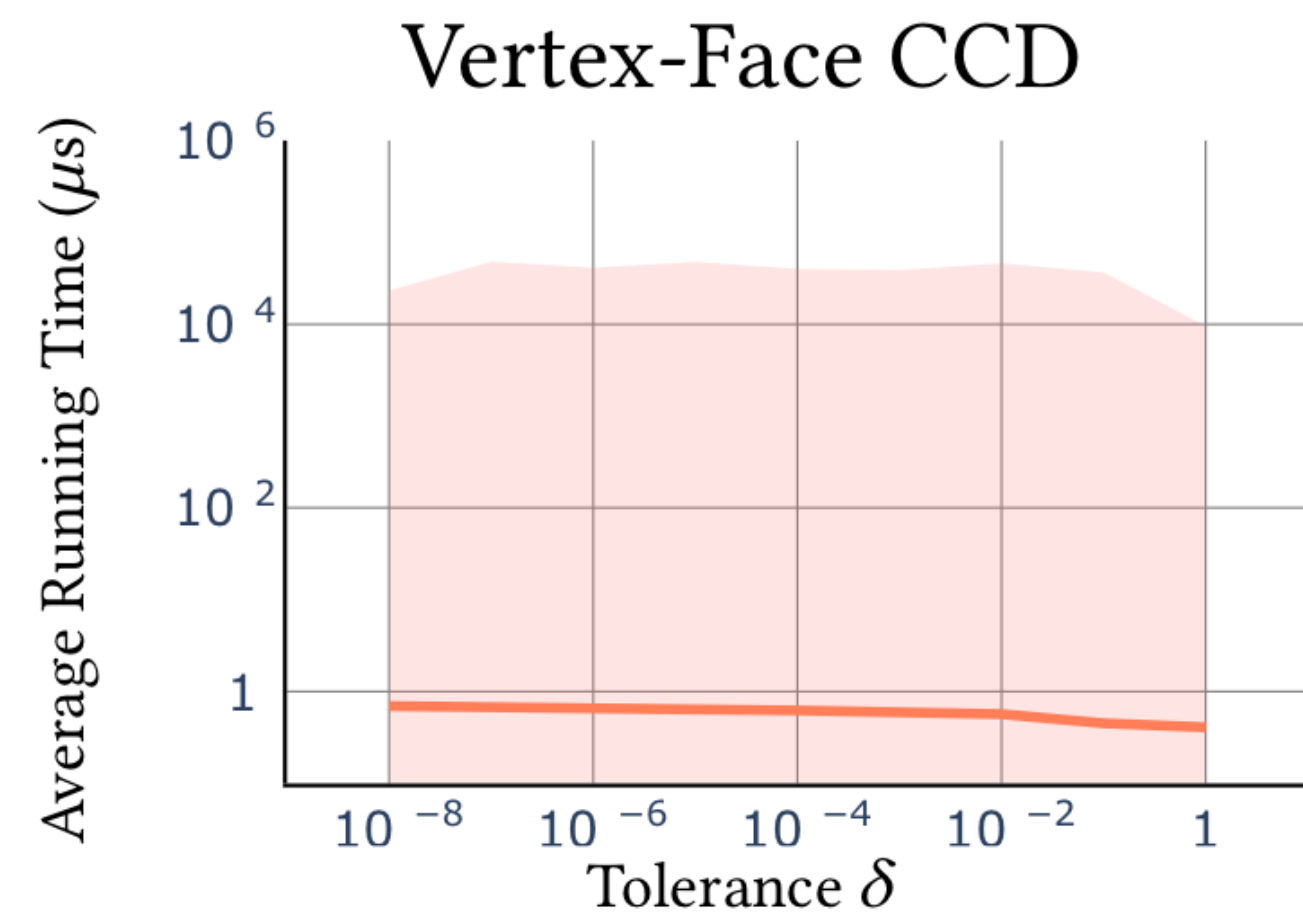
---

- How to define the inclusion function?
- Implementation with intervals is slow
- We discovered (see the paper for the derivation) that a pure floating point inclusion function can be devised for this specific polynomial!



A Large Scale Benchmark and an Inclusion-Based Algorithm for Continuous Collision Detection  
Bolun Wang, [Zachary Ferguson](#), [Teseo Schneider](#), Xin Jiang, [Marco Attene](#), [Daniele Panozzo](#),  
ACM Transaction on Graphics, 2021  
[\[Paper\]](#) [\[Video\]](#) [\[CCD Code\]](#) [\[Benchmark Code\]](#)

# Performance



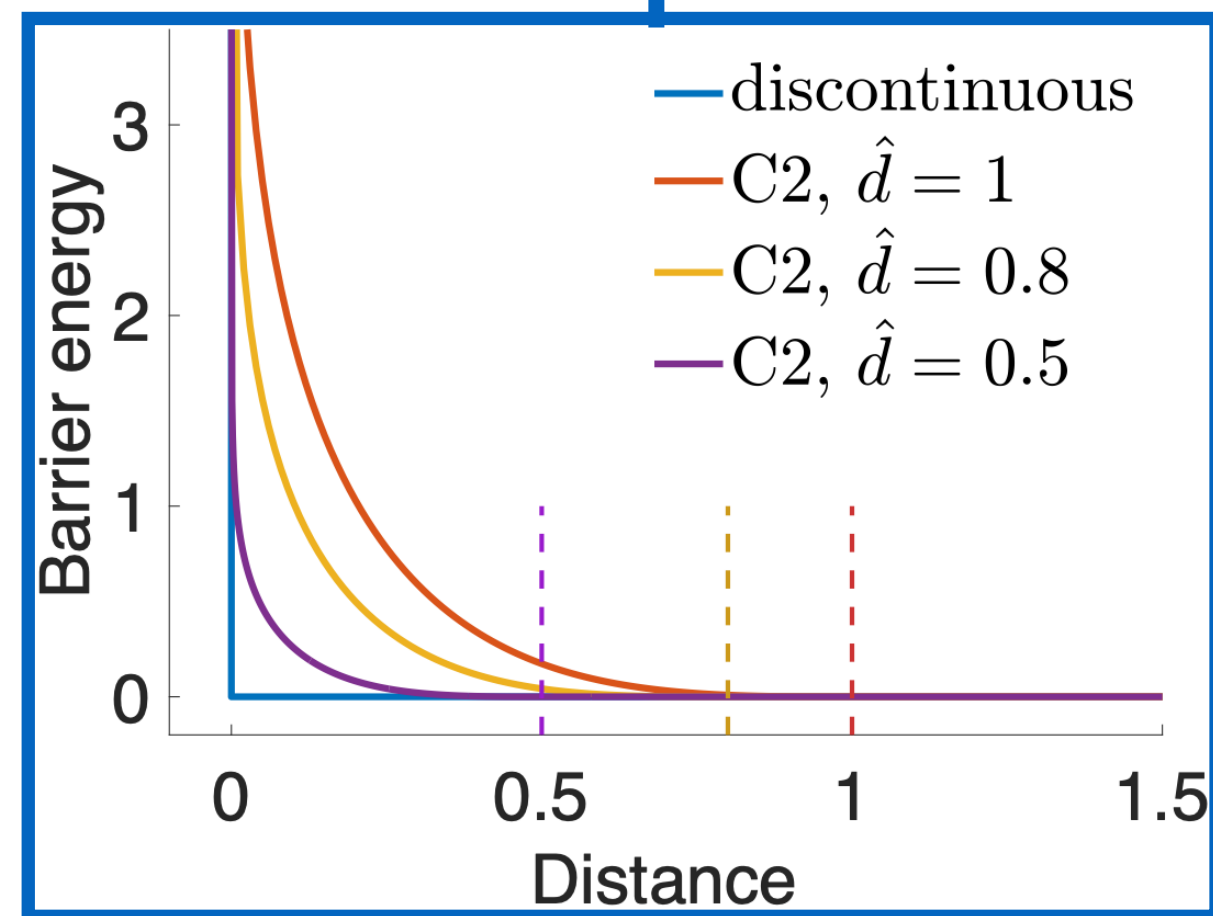
- Running time are suitable for large scale scenes, and fully parallelizable too!
- Additionally, since we only care about the first intersection, we can heavily prune.
- Implementation is challenging as the order of operations matters here!



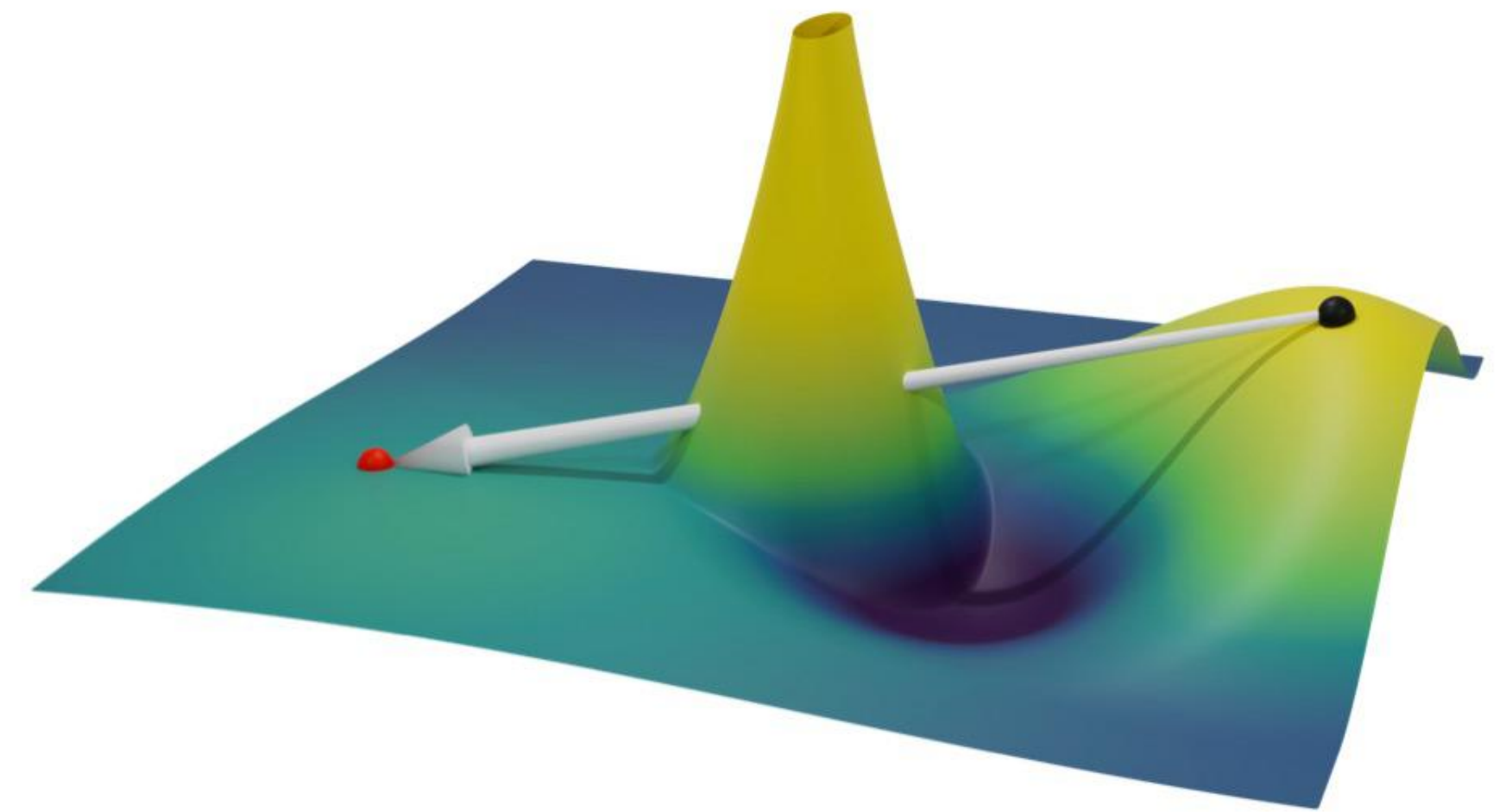
# Contact and Friction\* Potentials

$$x^{t+1} = \underset{x}{\operatorname{argmin}} E(x) + B(x, \hat{d}) + D(x)$$

Inertia &  
Elasticity

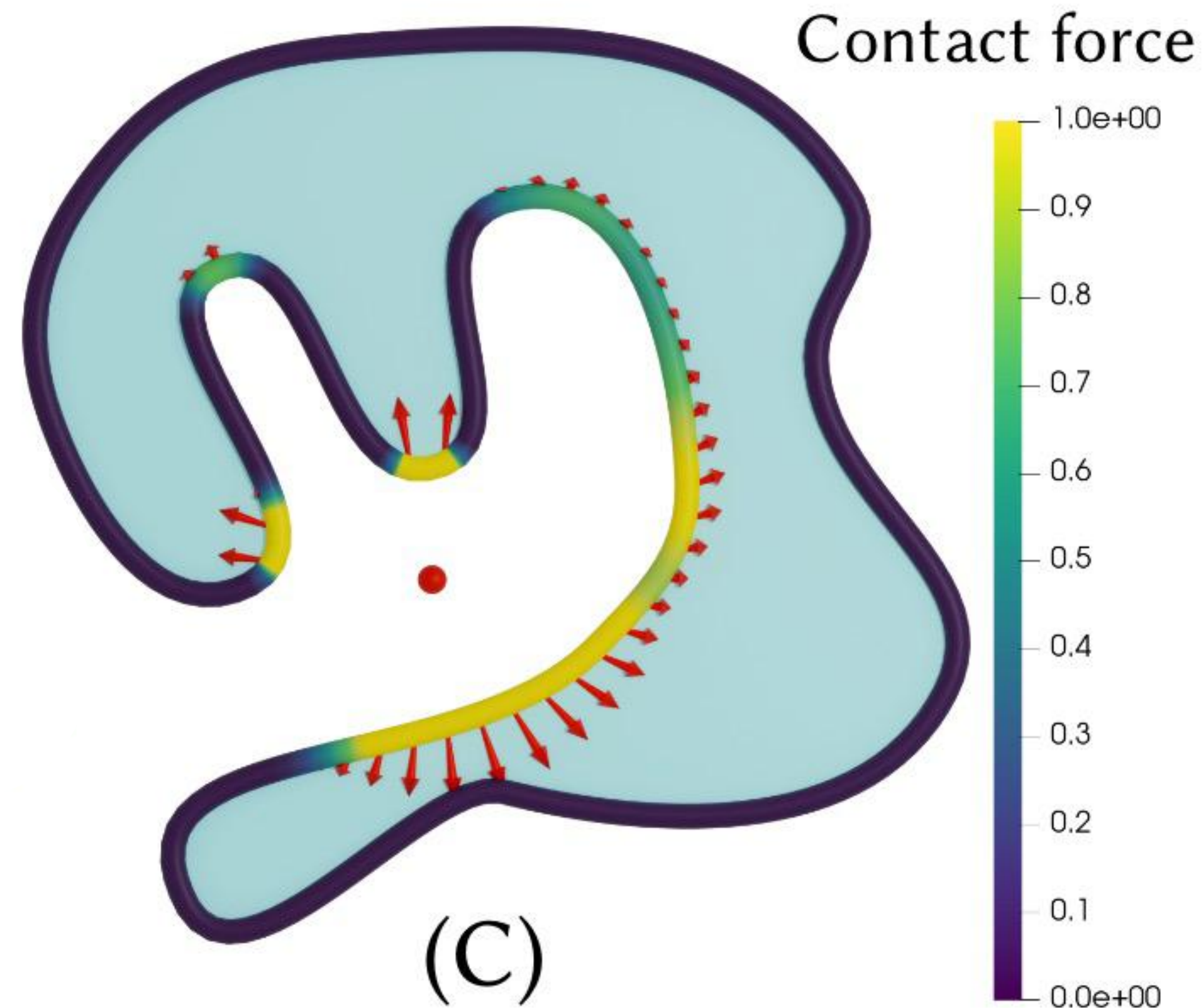


Friction\*



Minimize using Newton's method, with line search to ensure lack of constraints violation

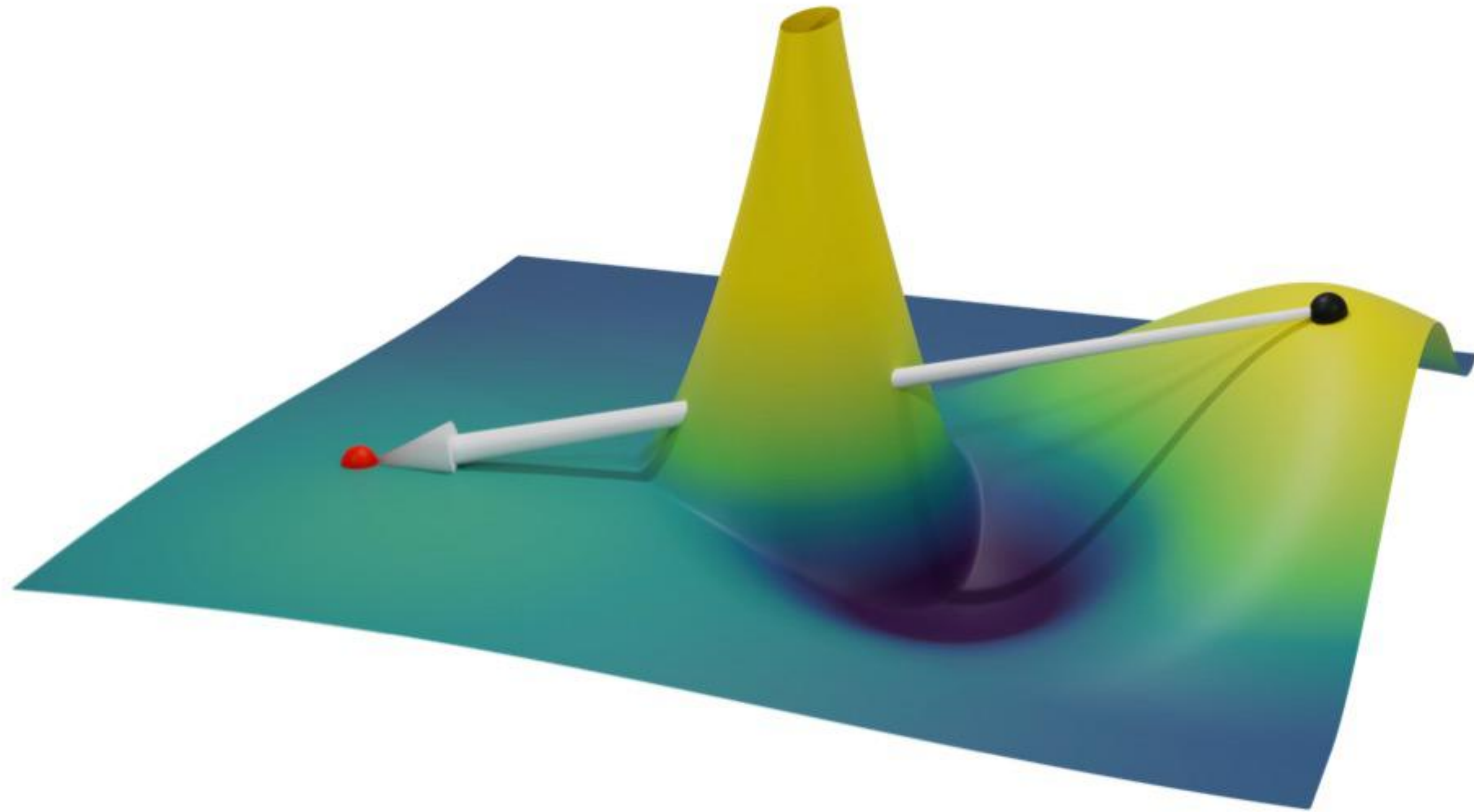
# Quadrature for Contact Potential



- The contact potential must be integrated with custom quadrature rules, so that the discrete quadrature diverges when the potential is infinite.
- We add a quadrature point on the closest contact point.
- This is crucial for convergence, otherwise the line search would stop convergence.



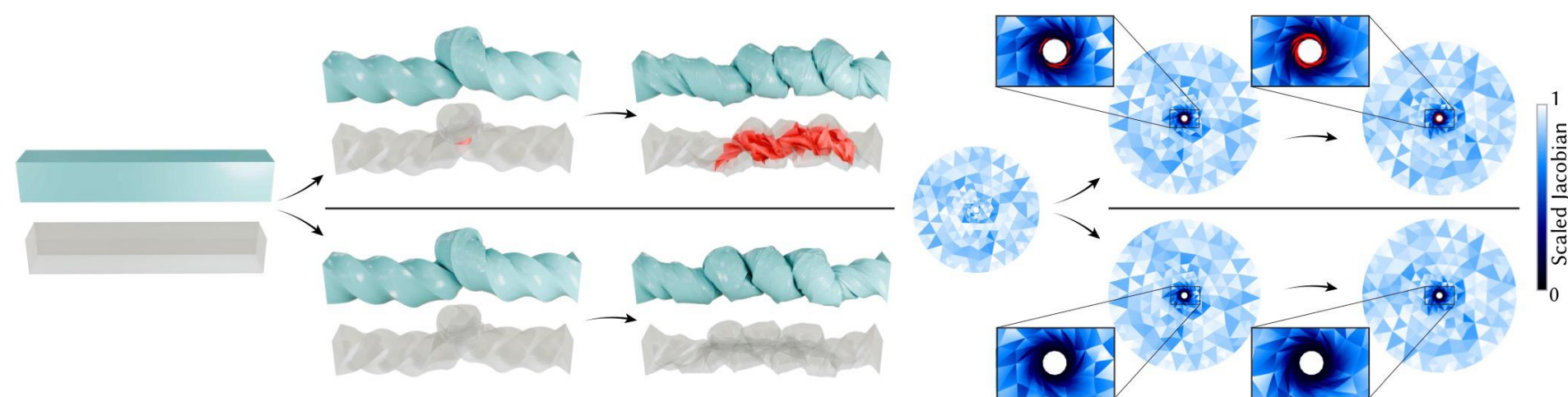
# Quadrature for Contact Potential



# High-Order Continuous Geometrical Validity

## High-Order Continuous Geometrical Validity

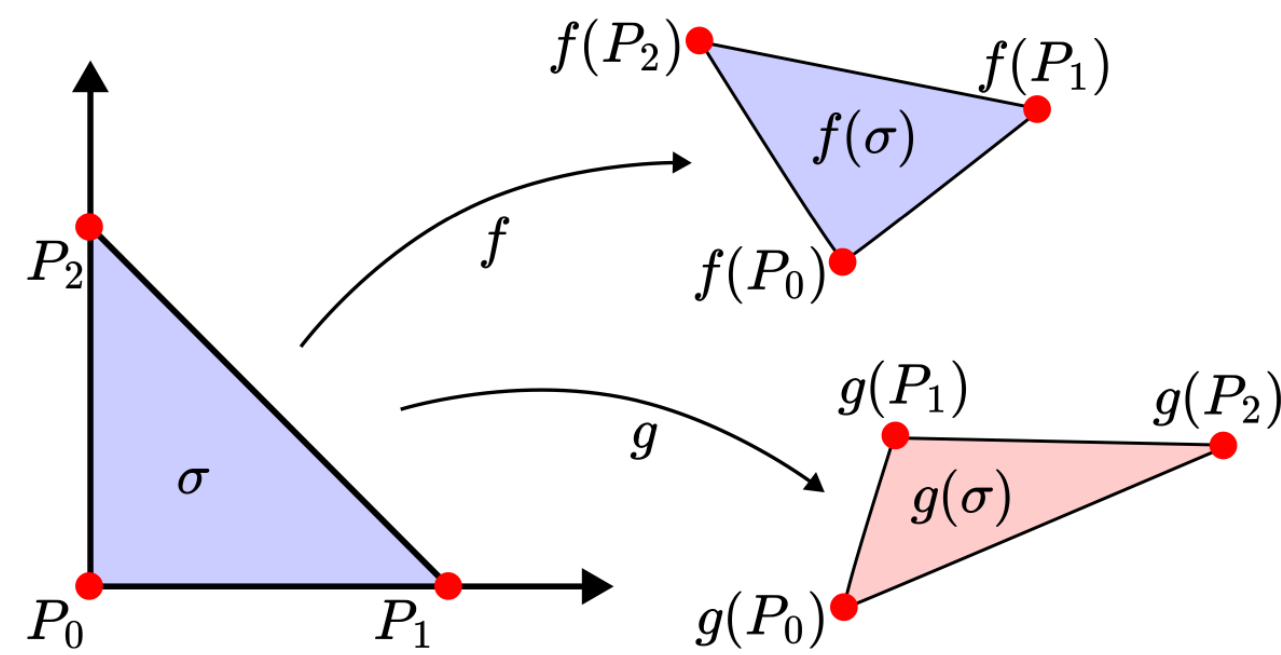
FEDERICO SICHETTI, University of Genoa, Italy  
ZIZHOU HUANG, New York University, United States  
MARCO ATTENE, CNR-IMATI: GENOVA, Italy  
DENIS ZORIN, New York University, United States  
ENRICO PUPPO, University of Genoa, Italy  
DANIELE PANOZZO, New York University, United States



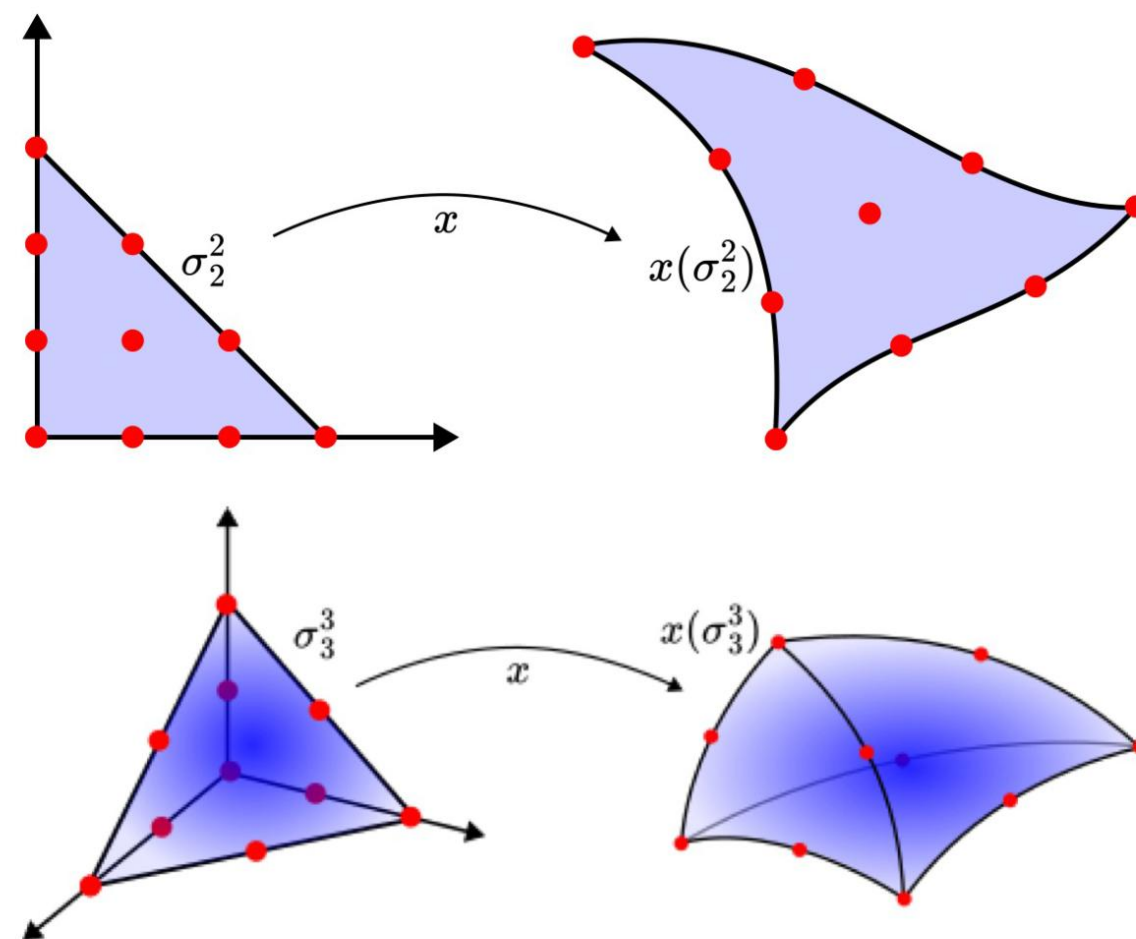


# Geometric Map and Basis

## Geometric Map



## FEM Basis

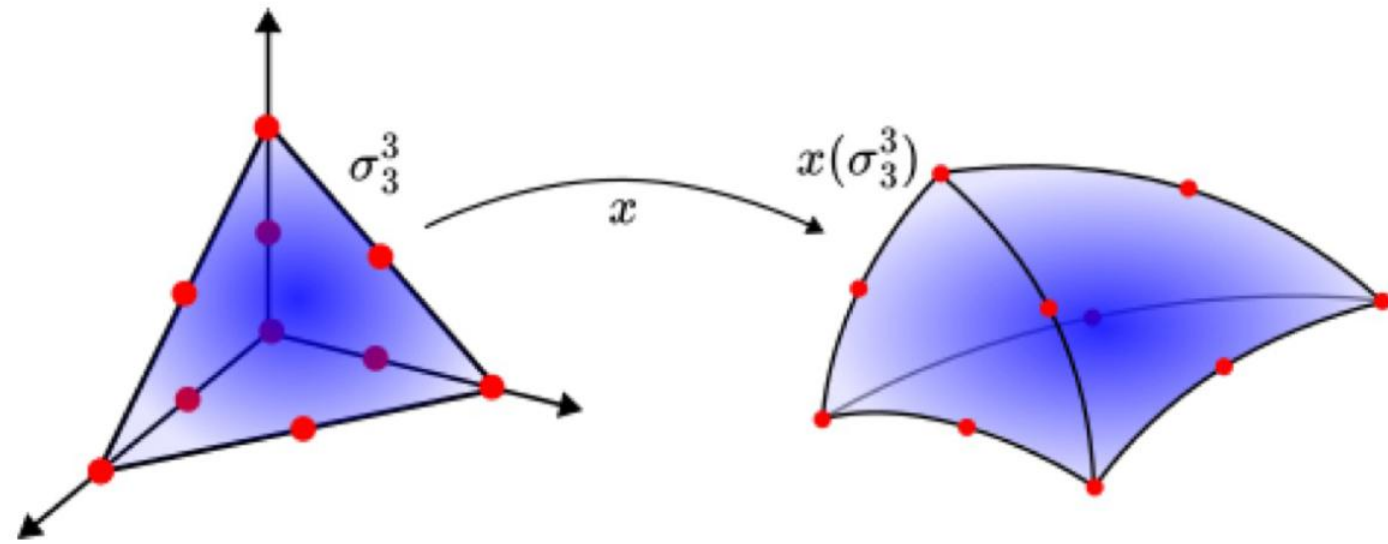


- Both can be either linear or higher order.
- The geometric map must be always injective (usually an issue for meshing).
- If we solve for displacement, then the composition of geometric map and displacement must be injective.

# Positivity of Jacobian

- Let's call it geometric map for simplicity:

$$x : \sigma_s^n \longrightarrow \mathbb{R}^n$$



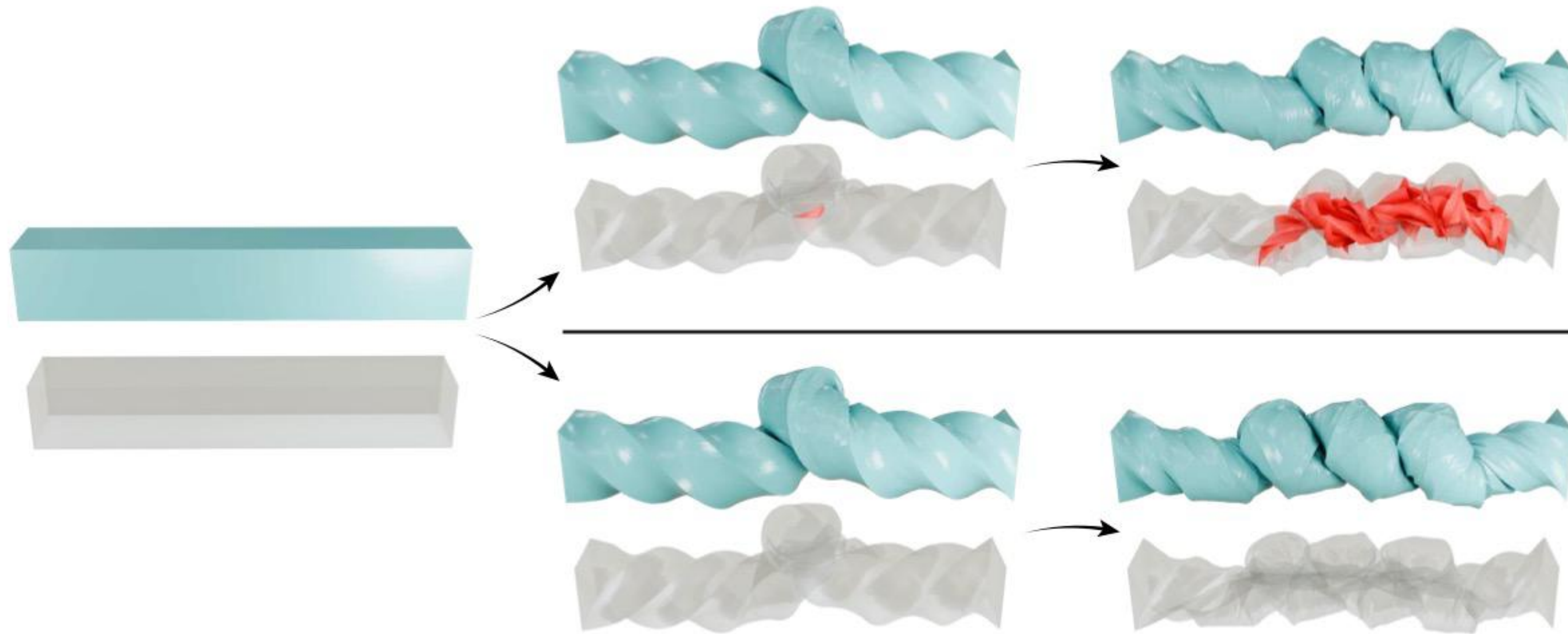
- The injectivity condition is usually expressed as:

$$|J_x| > 0$$

- And tested on quadrature points.

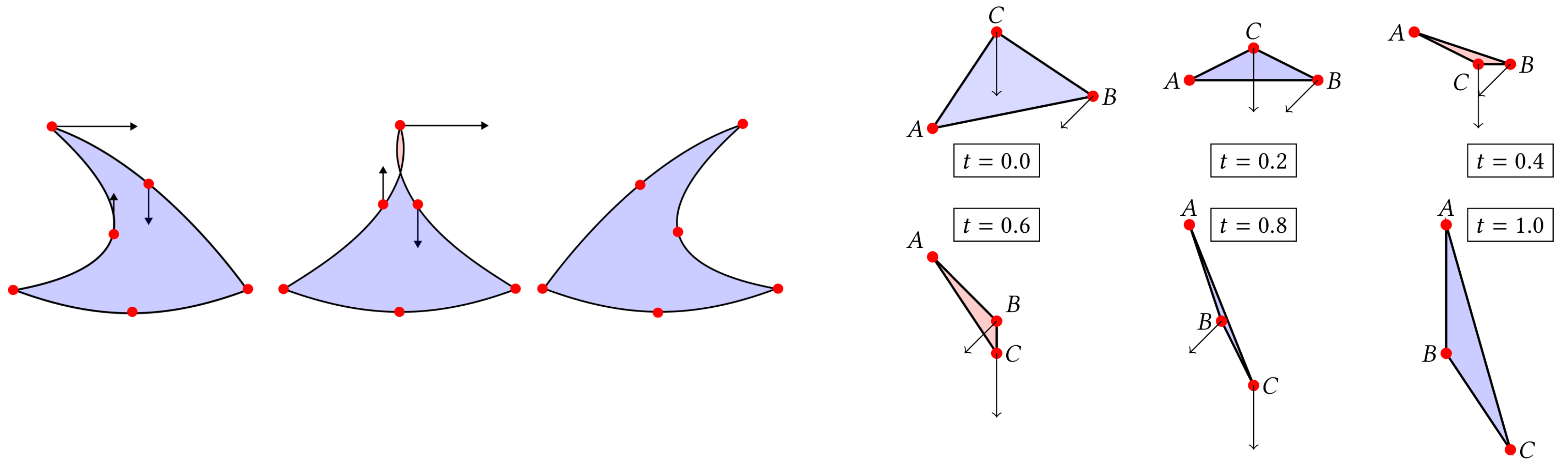


# This is insufficient!



- Testing it on quadrature points misses violations very often: we discovered that all PolyFEM high-order displacement simulations suffer from this issue... [Johnen et al 2014], GMSH has a solution, not robust to rounding but way better than sampling.

# This is still insufficient!



- It ignores what is happening over the time dimension. Testing at discrete time step is insufficient.



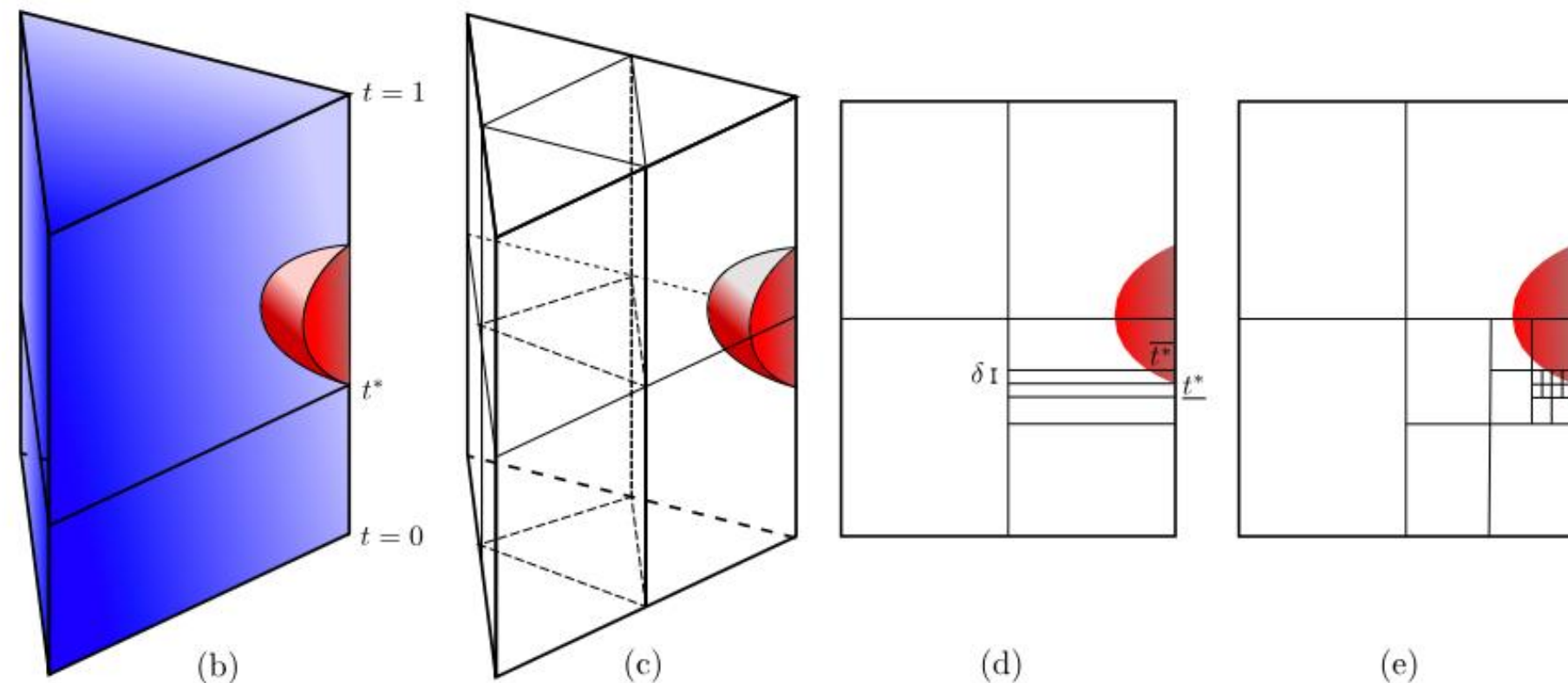
# Time-Dependent Positivity Check

DEFINITION 3 (DYNAMIC REFERENCE ELEMENT). Let  $\sigma_s^n$  be a reference element. The dynamic element  $\bar{\sigma}_s^n$  of  $\sigma_s^n$  is the  $(n+1)$ -dimensional reference element  $\sigma_s^{n+1} = \sigma_s^n \times [0, 1]$ .

Assuming linear trajectories, the *dynamic geometric map*  $\bar{x} : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$  of order  $p$  for  $\bar{\sigma}_s^n$  is expressed by linear interpolation of the  $n$ -dimensional geometric maps  $x^0(\xi)$  and  $x^1(\xi)$  of the static element at the two consecutive time steps:

$$\bar{x}(\xi, t) = x^0(\xi) + t(x^1(\xi) - x^0(\xi)). \quad (2)$$

This map is of order  $p$  in the  $\xi$  variables and linear in  $t$ .



- The new condition to check is:

$$|J_{\bar{x}}| > 0$$

- Similarly to before, we use bisection, but it is more nuanced here

# Algorithm

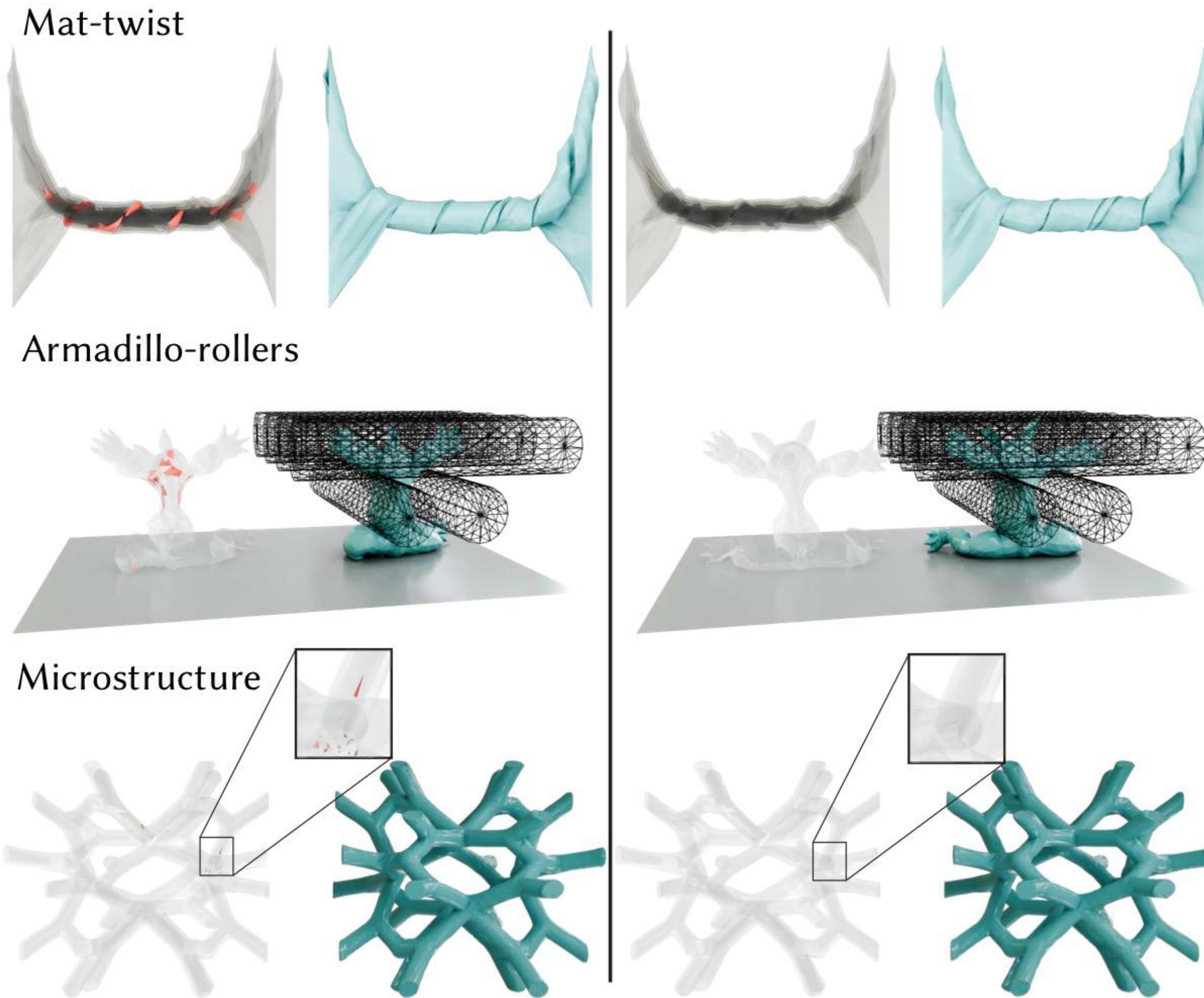
**Algorithm 1** Maximum valid time step with inclusion functions

```
1: function MAXVALIDSTEP( $J, \delta, l_{\max}$ )
2:    $P \leftarrow$  PRIORITYQUEUE( $<$ )  $\triangleright$  priority queue for subdomains
3:    $\overline{t^*} \leftarrow 1$   $\triangleright$  initialize upper bound of  $t^*$ 
4:    $\underline{t^*} \leftarrow 0$   $\triangleright$  initialize lower bound of  $t^*$ 
5:   PUSH( $P, \sigma$ )
6:    $F \leftarrow$  FALSE  $\triangleright$  flag of whether an invalidity has been found
7:    $l \leftarrow 0$   $\triangleright$  maximum subdivision depth reached so far
8:   while TRUE do
9:     if  $F \wedge (\overline{t^*} - \underline{t^*} \leq \delta) \wedge (\underline{t^*} > 0)$  then  $\triangleright$  reached accuracy
10:    | return  $\underline{t^*}$   $\triangleright$  conservative estimate of  $t^*$ 
11:    if ISEMPY( $P$ ) then
12:    | return 1
13:     $S \leftarrow$  POP( $P$ )  $\triangleright$  get the next subdomain from  $P$ 
14:     $l \leftarrow \max\{l, \text{DEPTH}(S)\}$   $\triangleright$  update maximum depth
15:    if  $l > l_{\max}$  then  $\triangleright$  maximum level reached: give up
16:    | return  $\underline{t^*}$   $\triangleright$  conservative estimate of  $t^*$ 
17:     $\underline{t^*} \leftarrow$  STARTTIME( $S$ )  $\triangleright$  everything before this time is valid
18:     $I \leftarrow \square_{\min} J(S)$   $\triangleright$  check minimum inclusion
19:    if HIGH( $I$ )  $\leq 0$  then  $\triangleright$  there is an invalidity in  $S$ 
20:    | if ENDTIME( $S$ )  $< \overline{t^*}$  then
21:    | |  $F \leftarrow$  TRUE
22:    | |  $\overline{t^*} \leftarrow$  ENDTIME( $S$ )
23:    | | PUSH( $P, \psi^-(S)$ )  $\triangleright$  bisect on the  $t$  axis only
24:    | | PUSH( $P, \psi^+(S)$ )  $\triangleright$  bisect on the  $t$  axis only
25:    | else if  $\neg(\text{LOW}(I) > 0)$  then
26:    | | for  $q \in \{1, \dots, Q\}$  do
27:    | | | PUSH( $P, \psi^q(S)$ )  $\triangleright$  subdivide on  $\xi$  and bisect on  $t$ 
28: function  $<(S_0, S_1)$   $\triangleright$  priority function
29: | if STARTTIME( $S_0$ )  $\neq$  STARTTIME( $S_1$ ) then  $\triangleright$  lower time first
30: | | return STARTTIME( $S_0$ )  $<$  STARTTIME( $S_1$ )
31: | else  $\triangleright$  for ties, prioritize boxes most likely to be invalid
32: | | return HIGH( $\square_{\min} J(S_0)$ )  $<$  HIGH( $\square_{\min} J(S_1)$ )
```

- Following [Johnen et al. 2014], we build the inclusion function over the Bezier form of the polynomial (massive reduction in depth).
- We precompute in rational both base changes and subdivision rules. At runtime, we use intervals to account for rounding errors.
- We use different split rules for space and time, see paper for details.



# Running Times



# Running Times

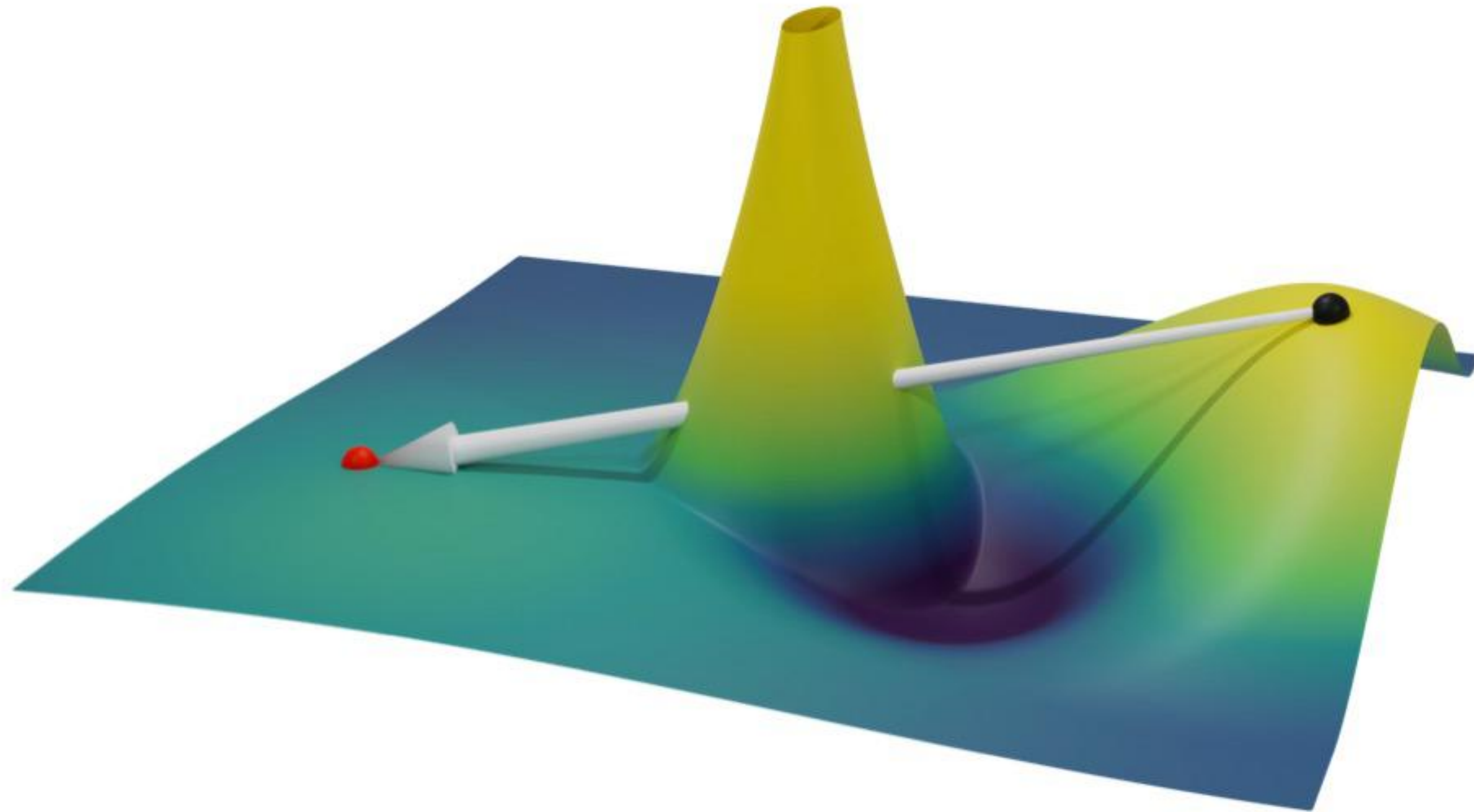
Dataset	Element	$n$	$s$	$p$ tot	# elements			time per element ( $\mu s$ )					
					valid	invalid	pre	avg	valid	invalid	max	std	
Kangaroo	Triangles	2	2	1	172800	146248	26552	9	3	1	21	29	4
				2	172800	145602	27198	12	12	9	28	189	7
				3	172800	145623	27177	33	52	47	80	295	12
				4	172800	118436	27187	89	187	177	240	1035	24
Bar	Quadrangles	2	1	1	130291	130244	47	16	8	8	21	73	1
				2	31879	31808	71	115	125	125	348	2021	22
Armadillo	Tetrahedra	3	3	1	54985	51605	3380	21	2	1	9	19	2
				2	54985	48599	6386	229	280	268	373	24197	142
				3	54985	47988	6997	1936	4720	4216	8173	176461	3357
Bunny	Tetrahedra	3	3	1	19800	19561	239	24	1	1	10	69	1
				2	19800	19058	742	235	277	270	451	7132	98
				3	19800	18954	846	1964	4290	4133	7811	84298	1731



# Running Times

Static algorithm	#val	#inv	#und	avg $\mu s$	med $\mu s$
Quadrature Points	48214	6771	-	16	16
Interval Bisection	46281	6561	2143	1400	23
FP Bézier (no optim.)	48050	6935	0	71	78
Ours (no optim.)	48050	6935	0	86	95
FP Bézier	48050	6935	0	5	5
<b>Ours</b>	48050	6935	0	8	8

# Elastic Potential Quadrature





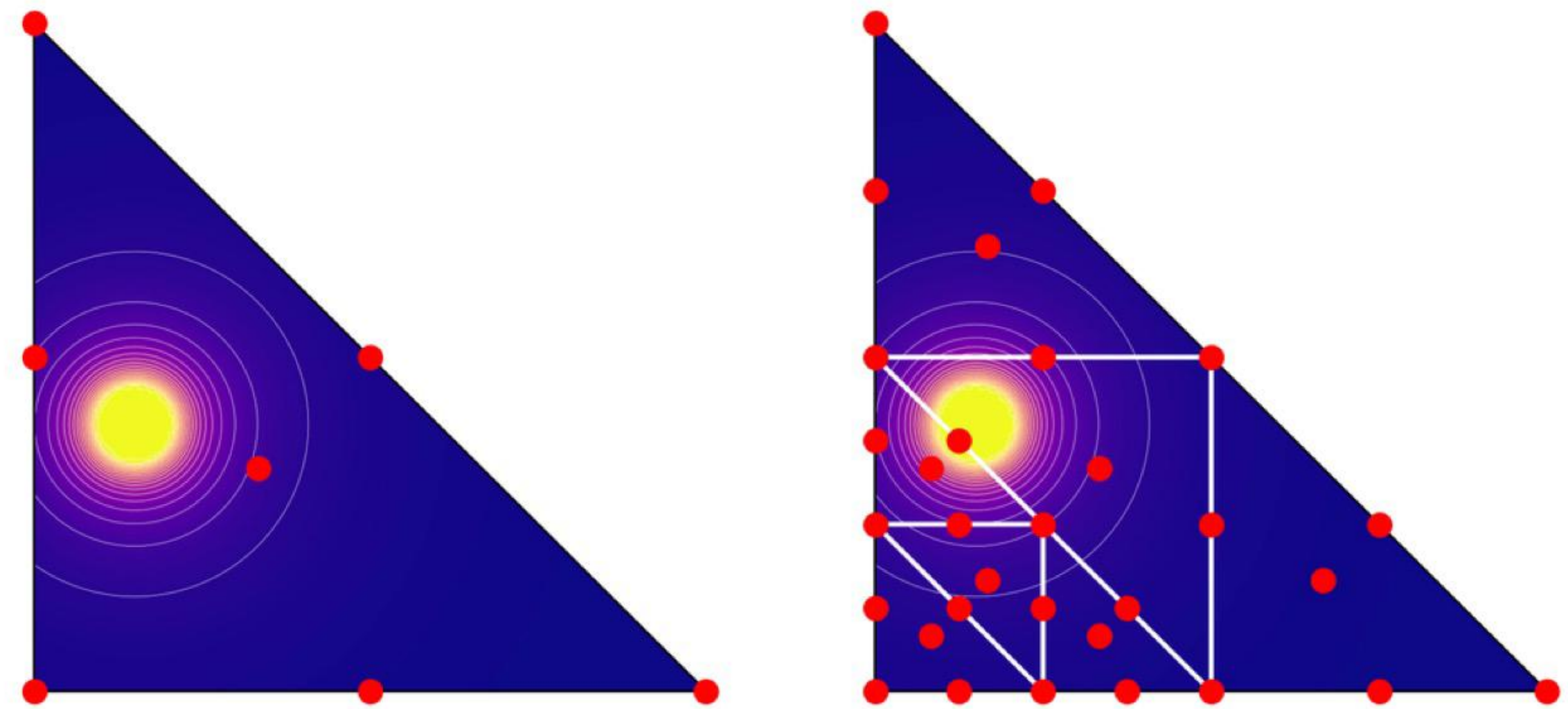
# Elastic Potential Quadrature

- Integrating the elastic potential requires care: if the potential diverges, the discrete quadrature should also diverge.
- Consider for example Neo-Hookean Elasticity:

$$\sigma[u] = \mu(F[u] - F[u]^{-T}) + \lambda \ln(\det F[u])F[u]^{-T}$$

- Or Mooney-Rivlin

$$\Psi[u] = c_1(\tilde{I}_1 - d) + c_2(\tilde{I}_2 - d) + \frac{k}{2} \ln^2(J)$$



We use the bisection hierarchy to define adaptive quadrature rules for each element.

# Software, Datasets, Benchmarks

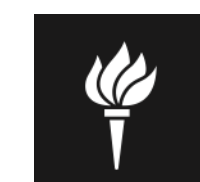
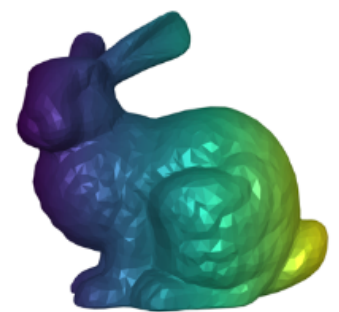
## Geometric Computing with Python

[Sebastian Koch](#), [Teseo Schneider](#), [Francis Williams](#), [Chencheng Li](#), [Daniele Panozzo](#),  
SIGGRAPH Course, 2019

[\[Website\]](#)

Hello World  

```
in [2]: %cd /tmp; python hello.py
```





# Wild Meshing (TetWild and TriWild)

Fast robust meshing

 wildmeshing  
3 Stars · 0 Forks

## Fast robust meshing

Home

TetWild

TriWild

Python ^

Home

[Notebook](#)

## Wildmeshing tutorial

This is a jupyter notebook. The “real” notebook can be found [here](#).

Wild meshing is a package that contains robust 2D and 3D meshing algorithms.

It has 4 main functions:

- `tetrahedralize`
- `triangulate`
- `triangulate_data`
- `triangulate_svg`



## Table of contents

File based API

numpy based API

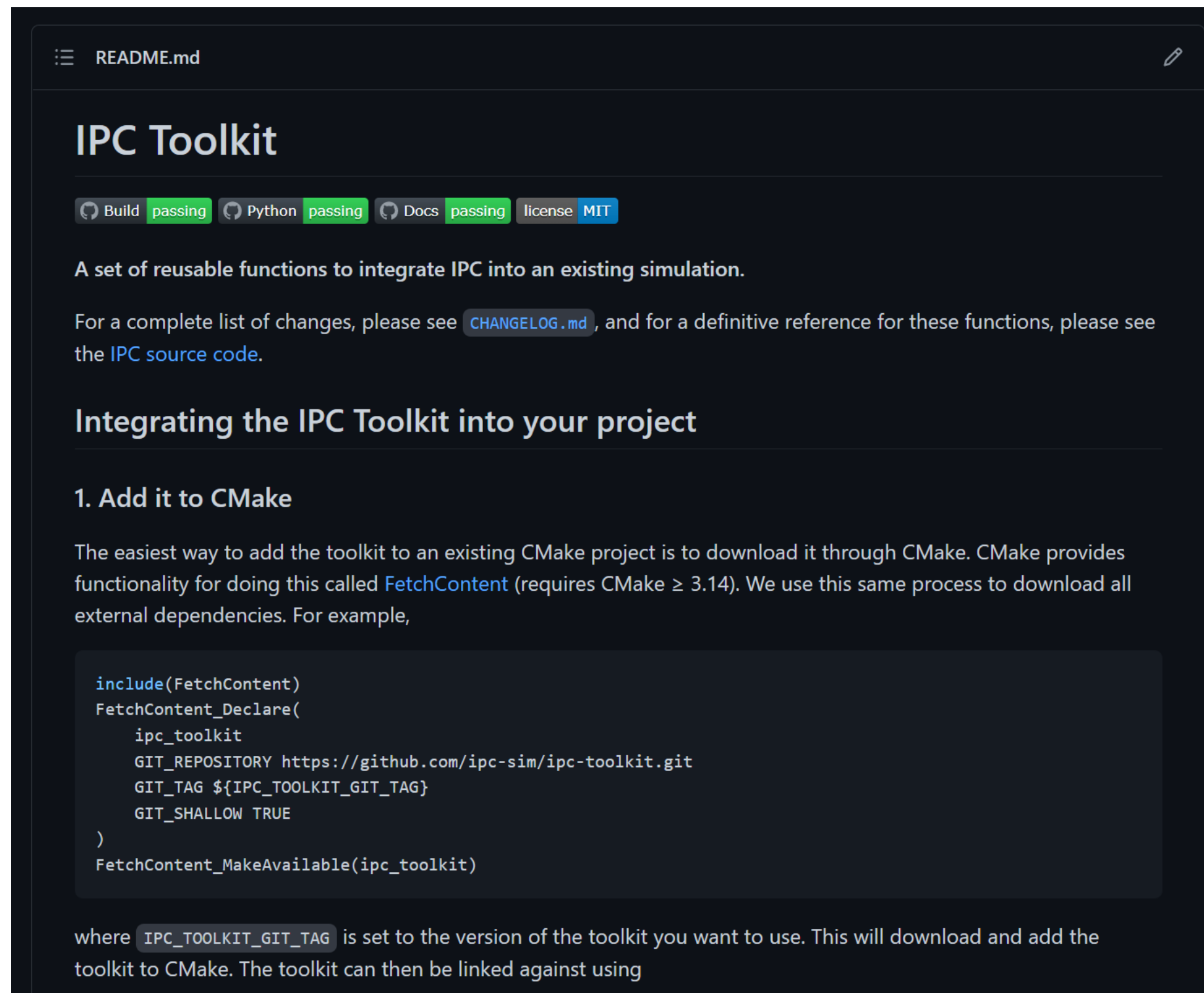
Common options

Triangulation

Tetrahedralization (alpha)

<https://wildmeshing.github.io>

# Collisions



README.md

## IPC Toolkit

Build passing Python passing Docs passing license MIT

A set of reusable functions to integrate IPC into an existing simulation.

For a complete list of changes, please see [CHANGELOG.md](#), and for a definitive reference for these functions, please see the [IPC source code](#).

### Integrating the IPC Toolkit into your project

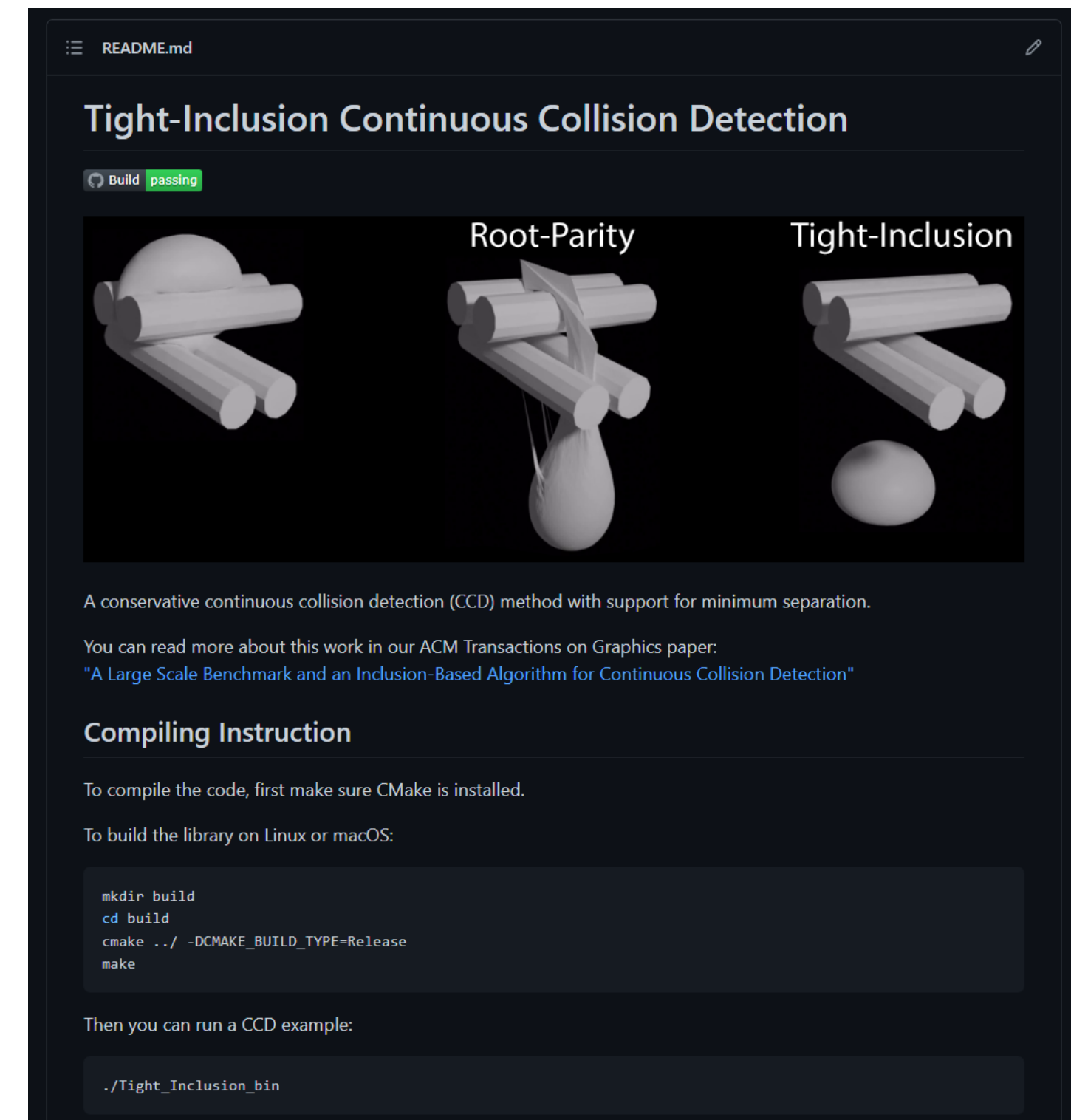
#### 1. Add it to CMake

The easiest way to add the toolkit to an existing CMake project is to download it through CMake. CMake provides functionality for doing this called [FetchContent](#) (requires CMake  $\geq$  3.14). We use this same process to download all external dependencies. For example,

```
include(FetchContent)
FetchContent_Declare(
  ipc_toolkit
  GIT_REPOSITORY https://github.com/ipc-sim/ipc-toolkit.git
  GIT_TAG ${IPC_TOOLKIT_GIT_TAG}
  GIT_SHALLOW TRUE
)
FetchContent_MakeAvailable(ipc_toolkit)
```

where `IPC_TOOLKIT_GIT_TAG` is set to the version of the toolkit you want to use. This will download and add the toolkit to CMake. The toolkit can then be linked against using


<https://github.com/ipc-sim/ipc-toolkit>



README.md

## Tight-Inclusion Continuous Collision Detection

Build passing



Root-Parity      Tight-Inclusion

A conservative continuous collision detection (CCD) method with support for minimum separation.

You can read more about this work in our ACM Transactions on Graphics paper:  
["A Large Scale Benchmark and an Inclusion-Based Algorithm for Continuous Collision Detection"](#)

### Compiling Instruction

To compile the code, first make sure CMake is installed.

To build the library on Linux or macOS:

```
mkdir build
cd build
cmake ../ -DCMAKE_BUILD_TYPE=Release
make
```

Then you can run a CCD example:

```
./Tight_Inclusion_bin
```

<https://github.com/Continuous-Collision-Detection/Tight-Inclusion>



# PolyFEM

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polyfem/polyfem  
22 Stars · 3 Forks

**polyfem**

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Citation

Acknowledgements & Funding

<https://polyfem.github.io>

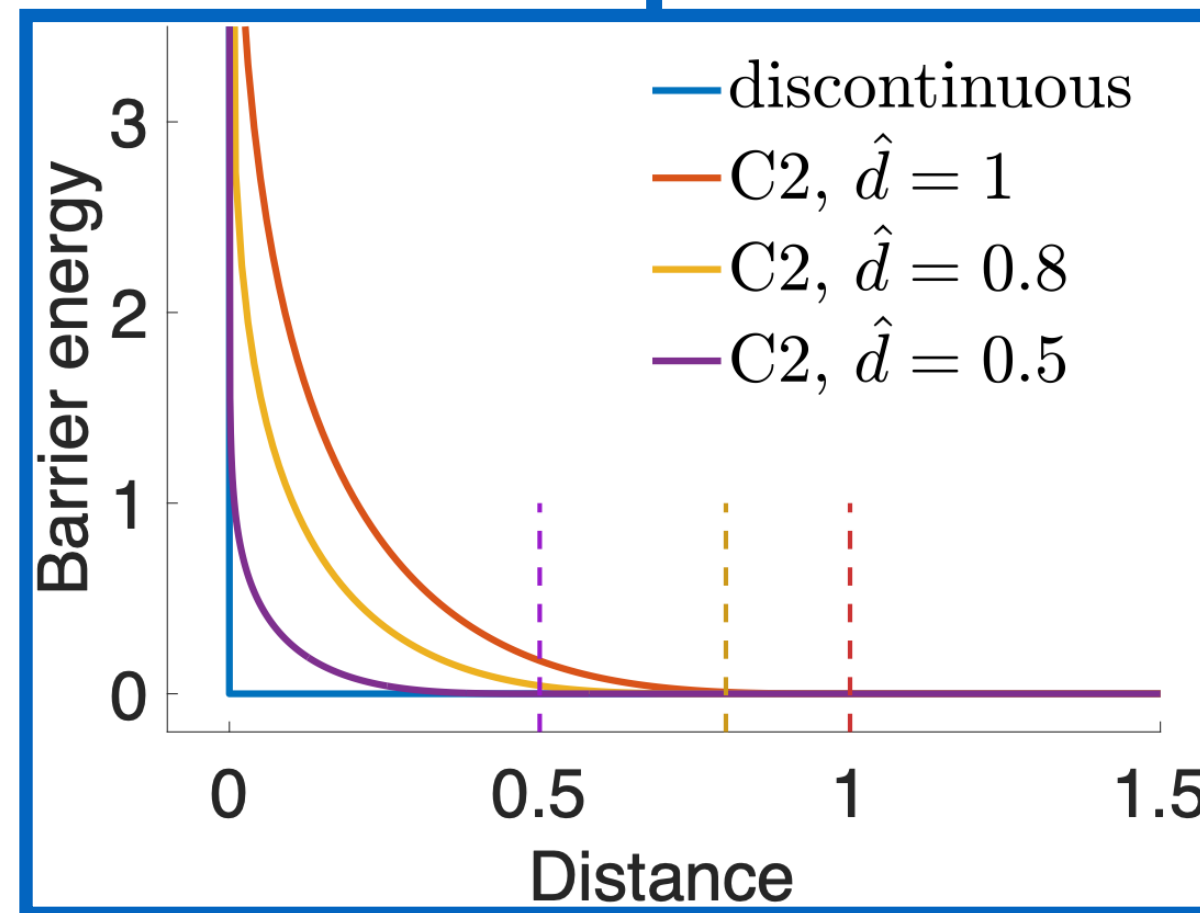
# Concluding Remarks



# Incremental Potential Formulation

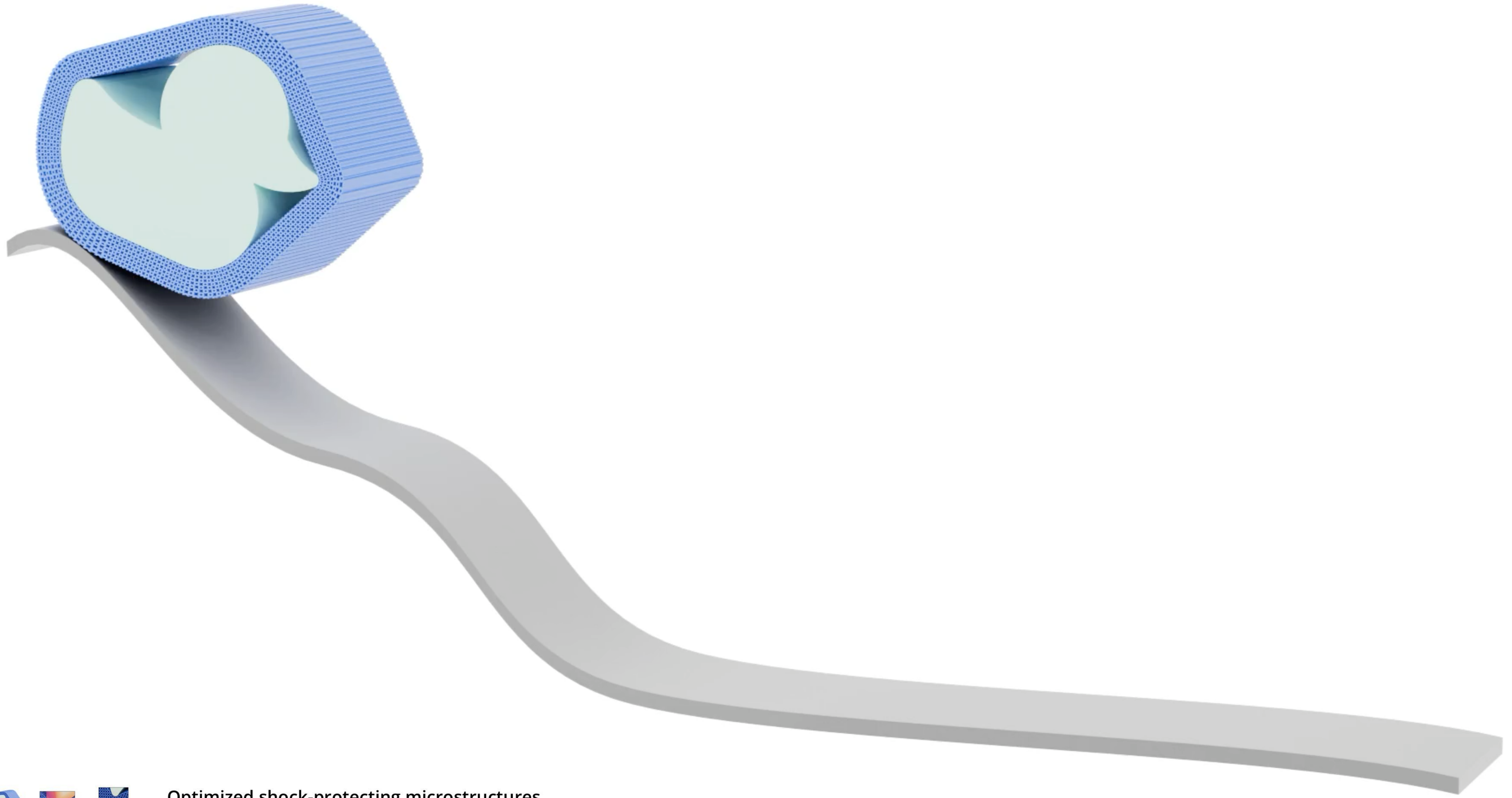
$$x^{t+1} = \underset{x}{\operatorname{argmin}} E(x) + B(x, \hat{d}) + D(x)$$

Inertia &  
Elasticity



Friction

Minimize using Newton's method, with line search to ensure lack of constraints violation

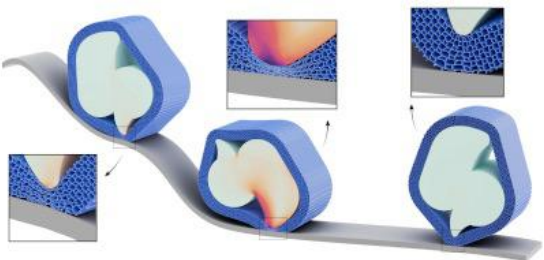


### Optimized shock-protecting microstructures

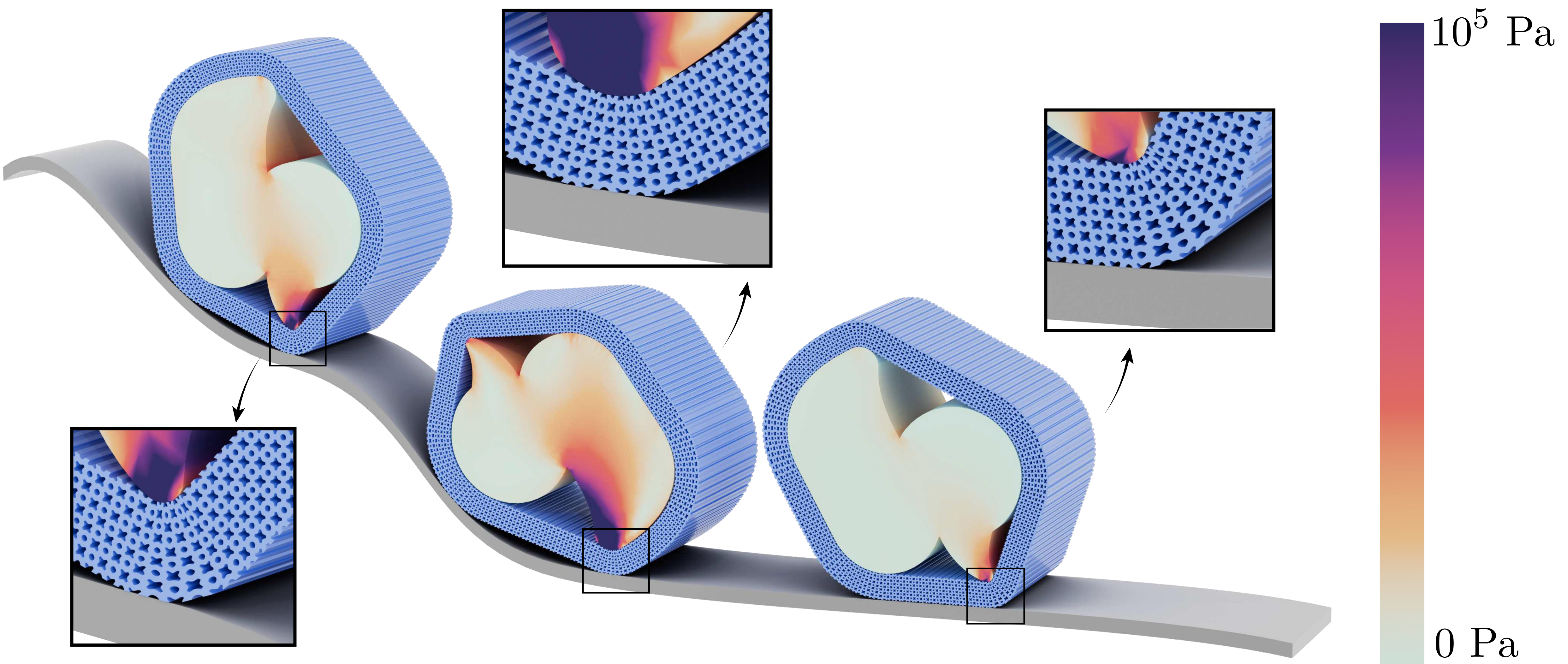
[Zizhou Huang](#), [Daniele Panozzo](#), [Denis Zorin](#),

ACM Transaction on Graphics (SIGGRAPH Asia), 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)



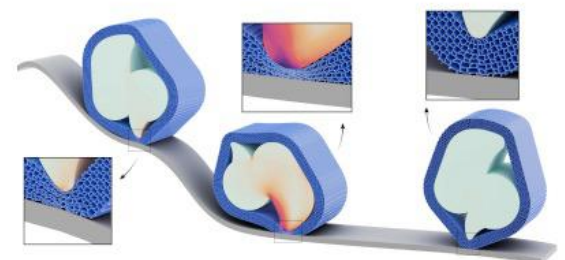




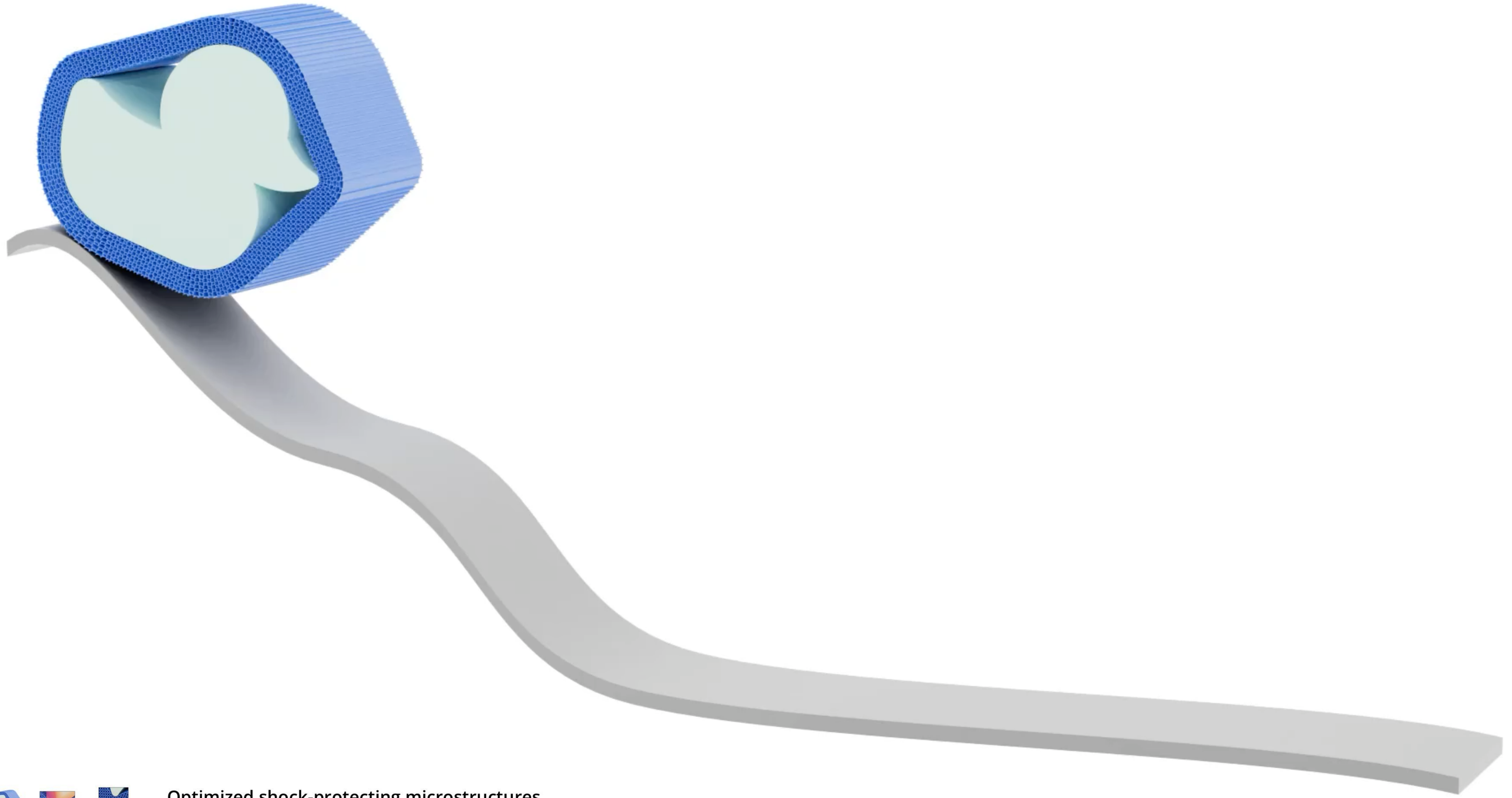
**Optimized shock-protecting microstructures**

[Zizhou Huang](#), [Daniele Panozzo](#), [Denis Zorin](#),  
ACM Transaction on Graphics (SIGGRAPH Asia), 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)





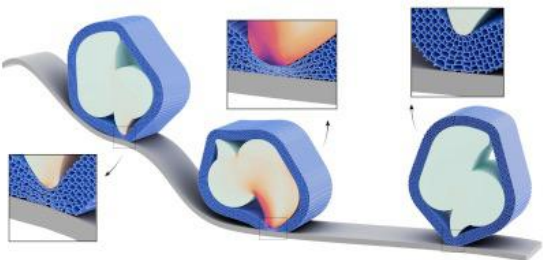


**Optimized shock-protecting microstructures**

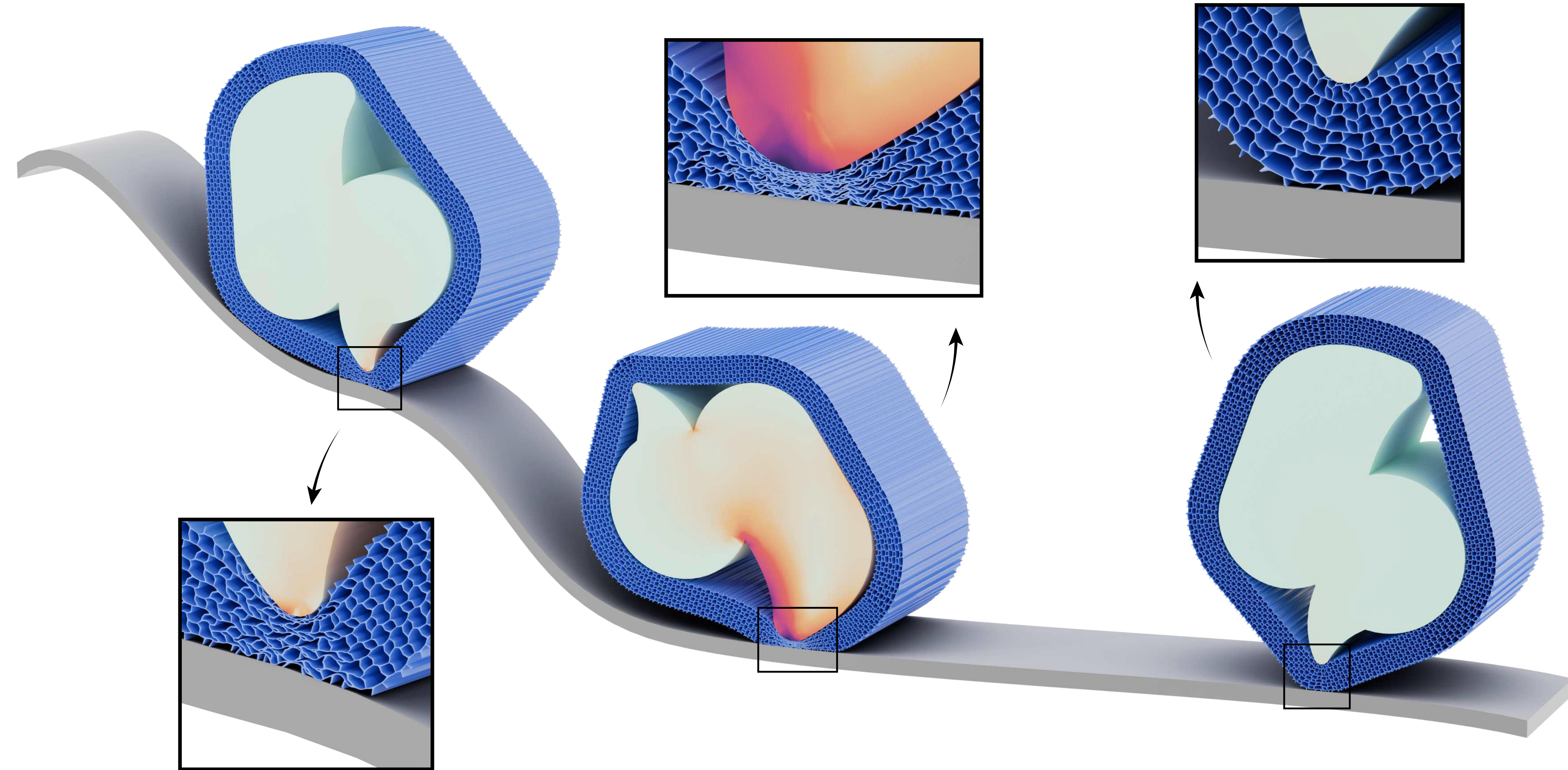
[Zizhou Huang](#), [Daniele Panozzo](#), [Denis Zorin](#),

ACM Transaction on Graphics (SIGGRAPH Asia), 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)





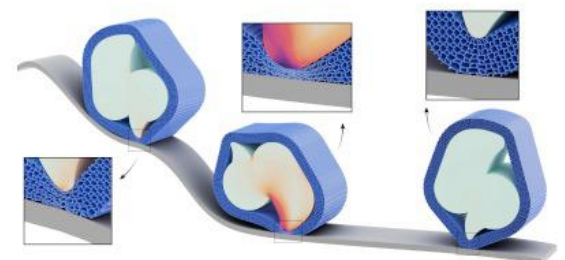


Stress is reduced by 72%

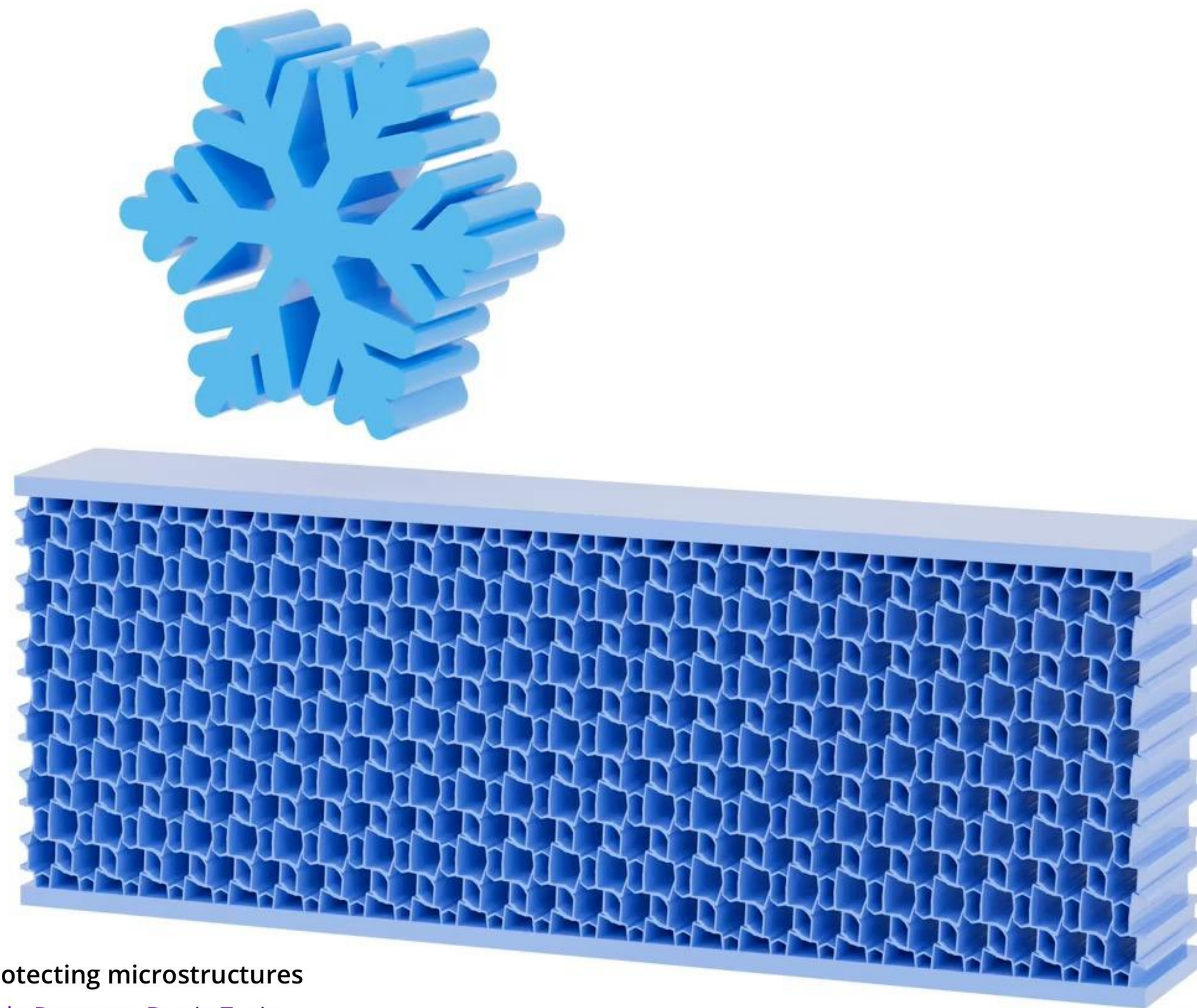
**Optimized shock-protecting microstructures**

[Zizhou Huang](#), [Daniele Panozzo](#), [Denis Zorin](#),  
ACM Transaction on Graphics (SIGGRAPH Asia), 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)





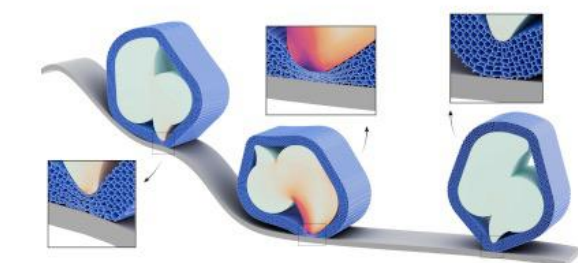


### Optimized shock-protecting microstructures

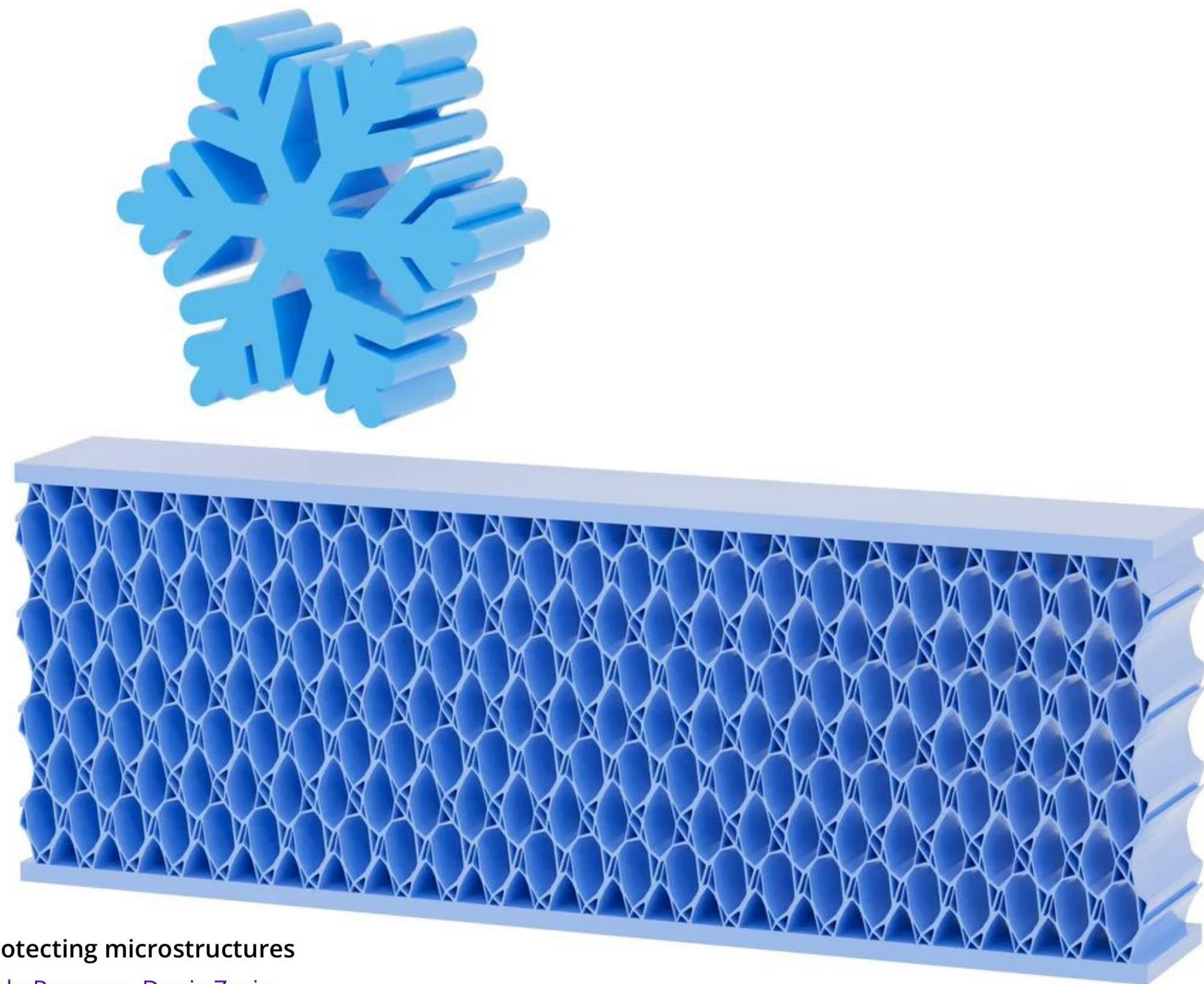
[Zizhou Huang](#), [Daniele Panozzo](#), [Denis Zorin](#),

ACM Transaction on Graphics (SIGGRAPH Asia), 2024

[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)





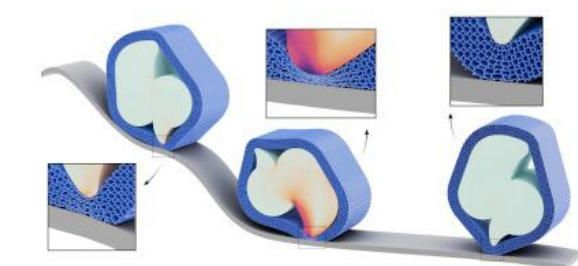


### Optimized shock-protecting microstructures

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ACM Transaction on Graphics (SIGGRAPH Asia), 2024

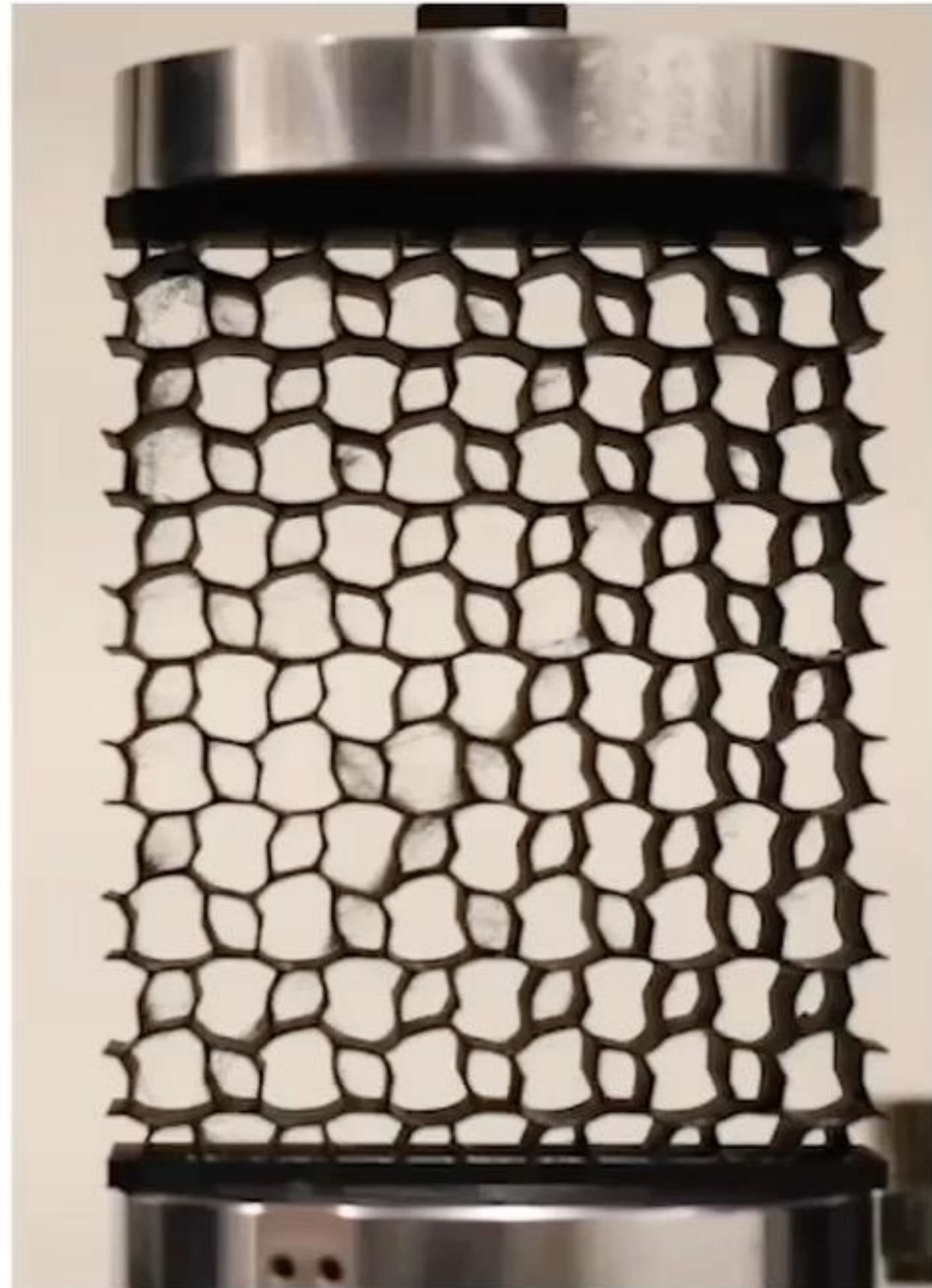
[\[Paper\]](#) [\[Video\]](#) [\[Code\]](#)



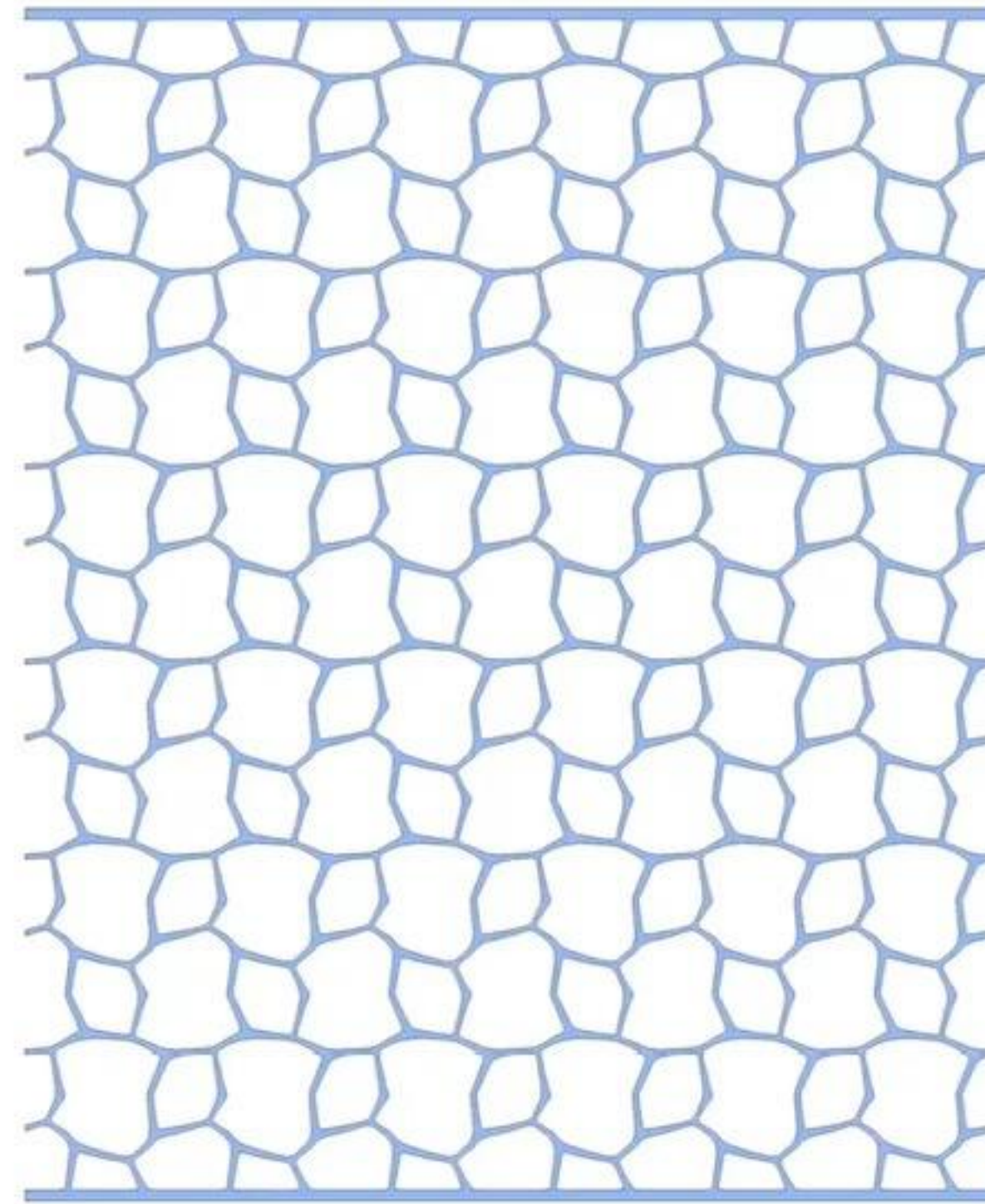


Video accelerated

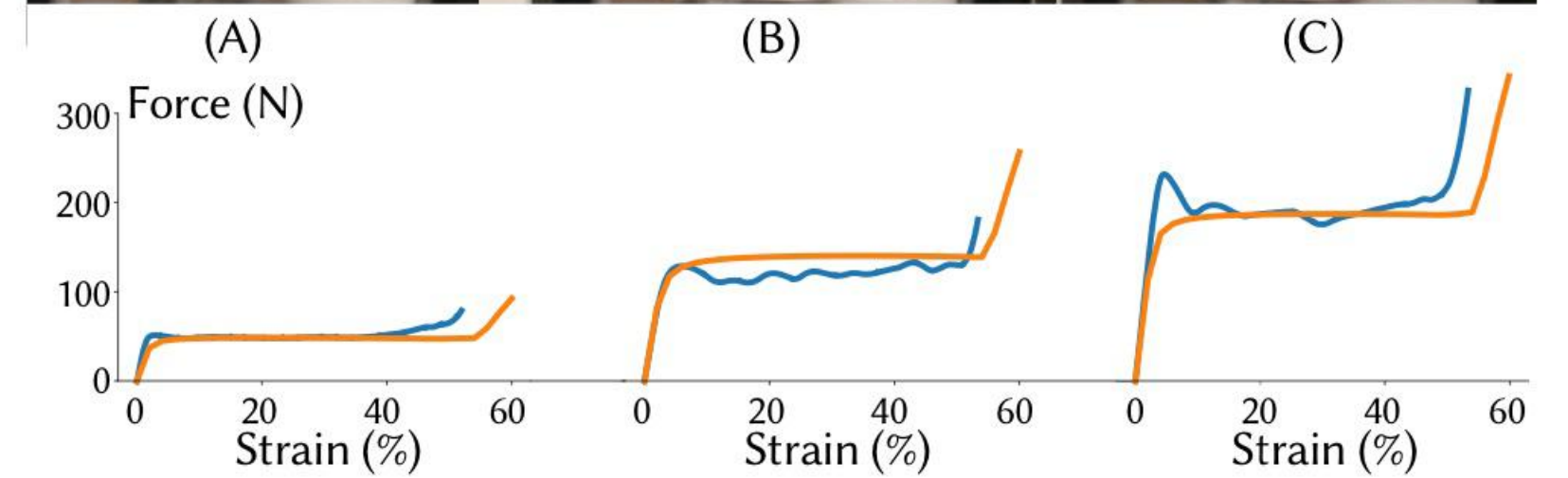
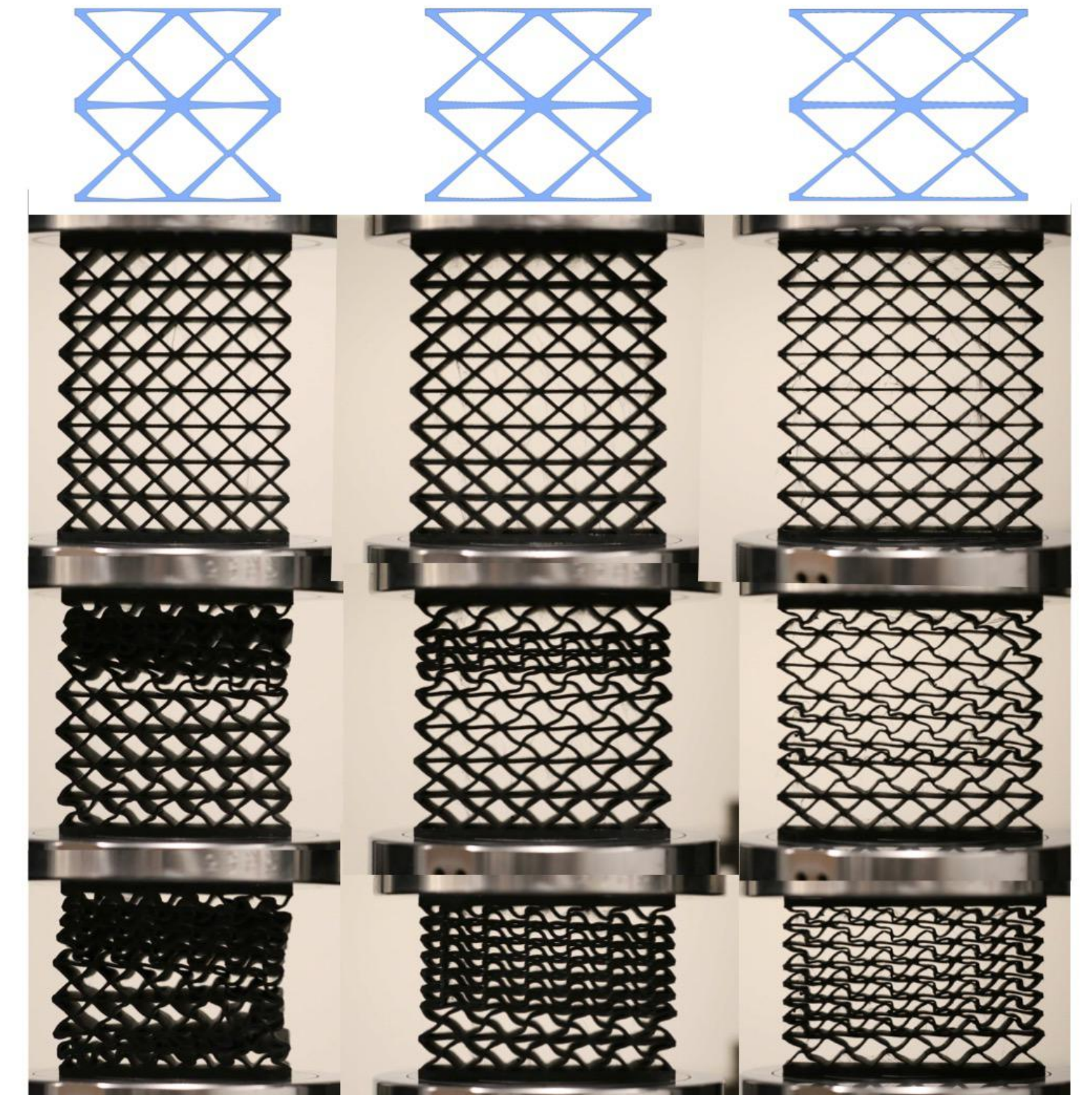
# Compression test



Experiment



Full simulation





# Geometric Predicates for Unconditionally Robust Elastodynamics Simulation

Daniele Panozzo  
NYU Geometric Computing Lab



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MATHEMATICAL SCIENCES