Enabling high-fidelity aeronautic simulations with high-order methods: geometry, numerics, and physics

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Our current research at BSC

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  - A Huerta (UPC)
  - J Sarrate (UPC)
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  - J Peraire (MIT)
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  - Geuzaine & Remacle (UC Louvain)
  - Moxey, Peiró & Sherwin (Imperial College)
  - Evans (UC Boulder)
  - Mark Shephard (Rensselaer PI)
  - ….

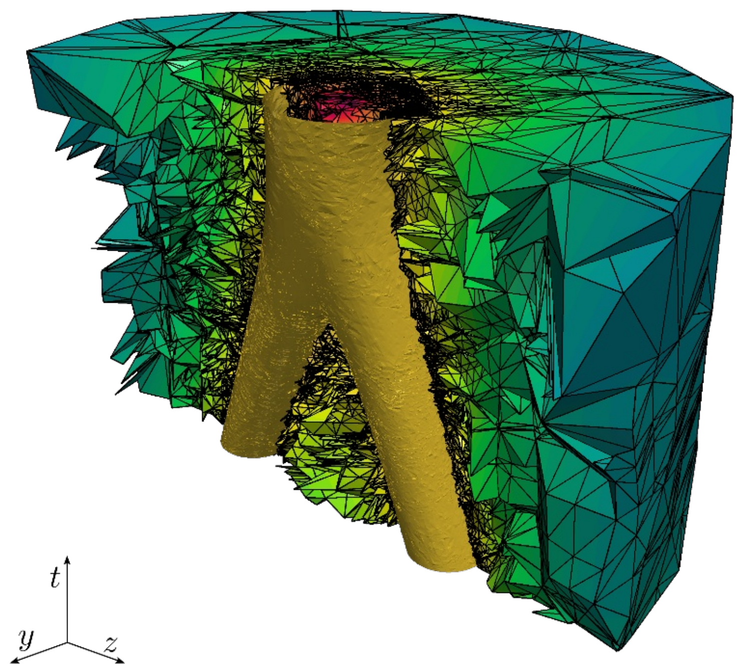
Aerodynamic & electromagnetic analyses of future aviation designs

- **Problem**: industrial geometry & simulation methods use low-order methods
- **So what**: without high-order methods, analyses might neglect key interactions

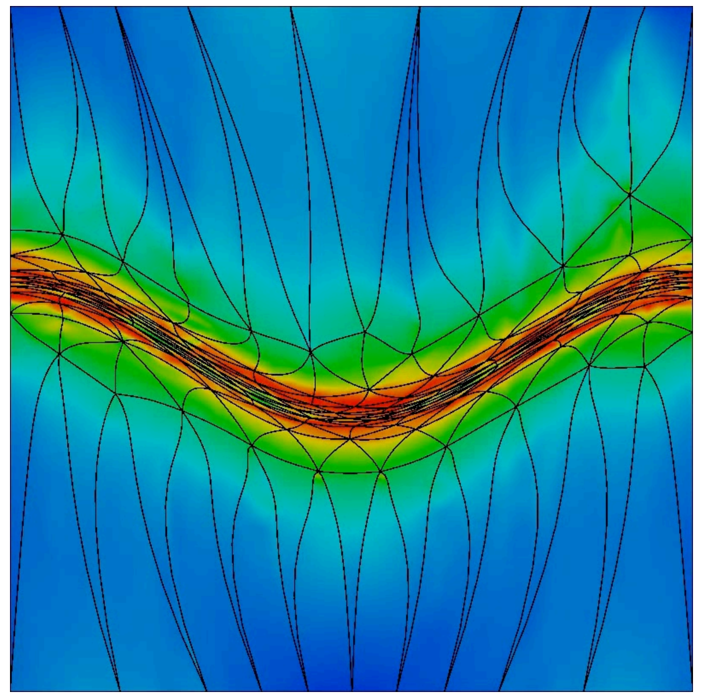
Fuel-efficient Aurora D8 (MIT / NASA)

From basic research to applied research

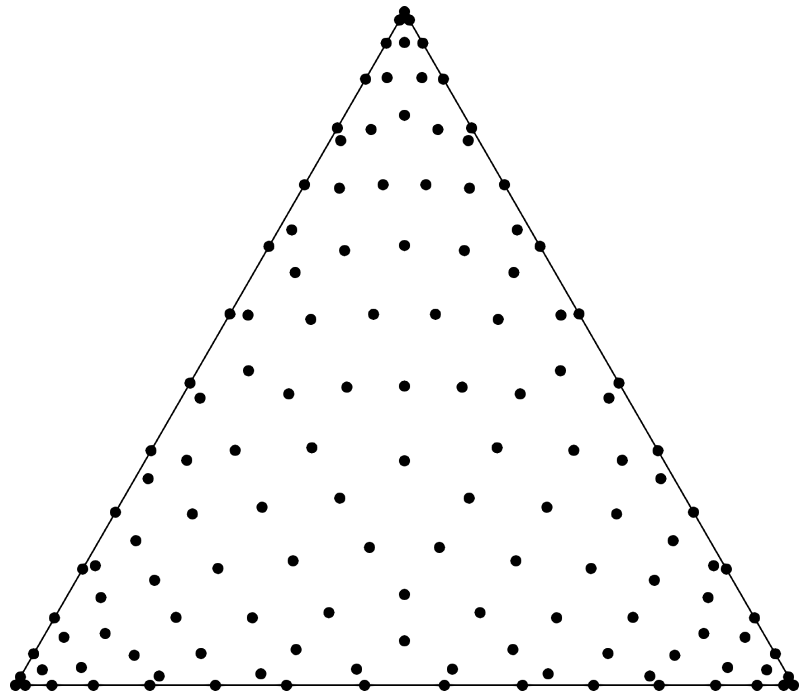
- **Our research**: devise high-order methods for HPC simulation
- **Benefits**: unprecedented geometrical, numerical & physical accuracy, yet efficiently



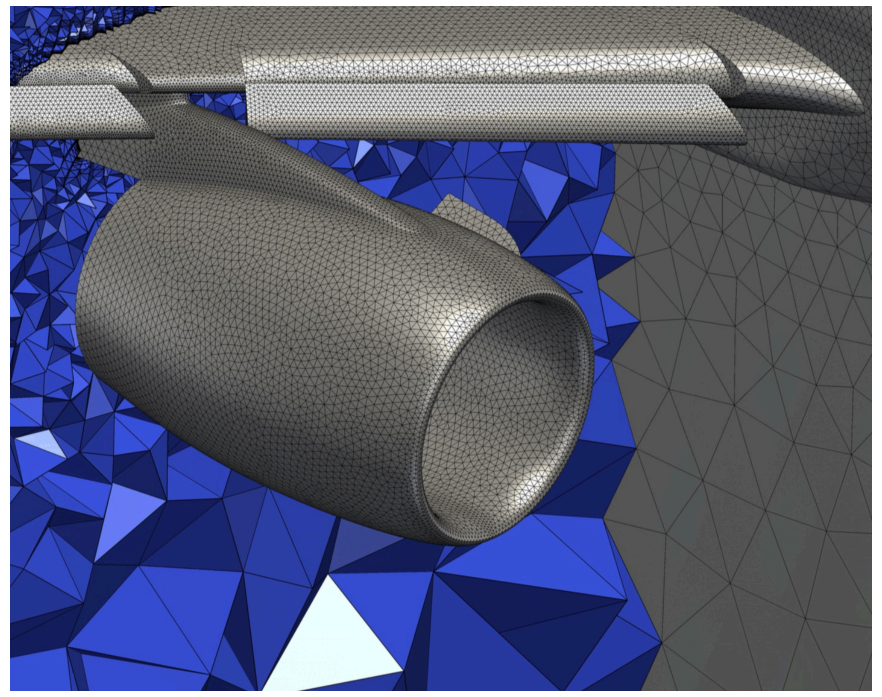
4D refinement
(Belda, Thesis’22)



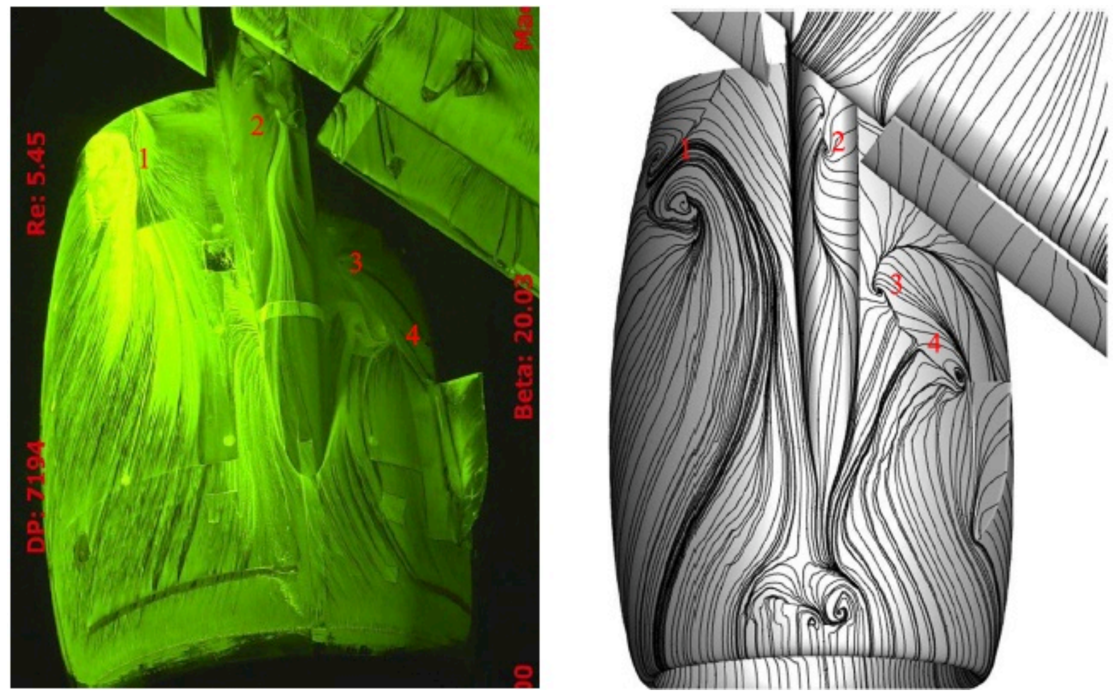
Curved adaptation
(Aparicio, Thesis’23)



Optimal interpolation
(Jiménez, Thesis’23)



Curved meshing
(with Ruiz, AIAA'22)



Experiment and simulation comparison
(Wang, AIAA’22) (with Ruiz, AIAA’22)

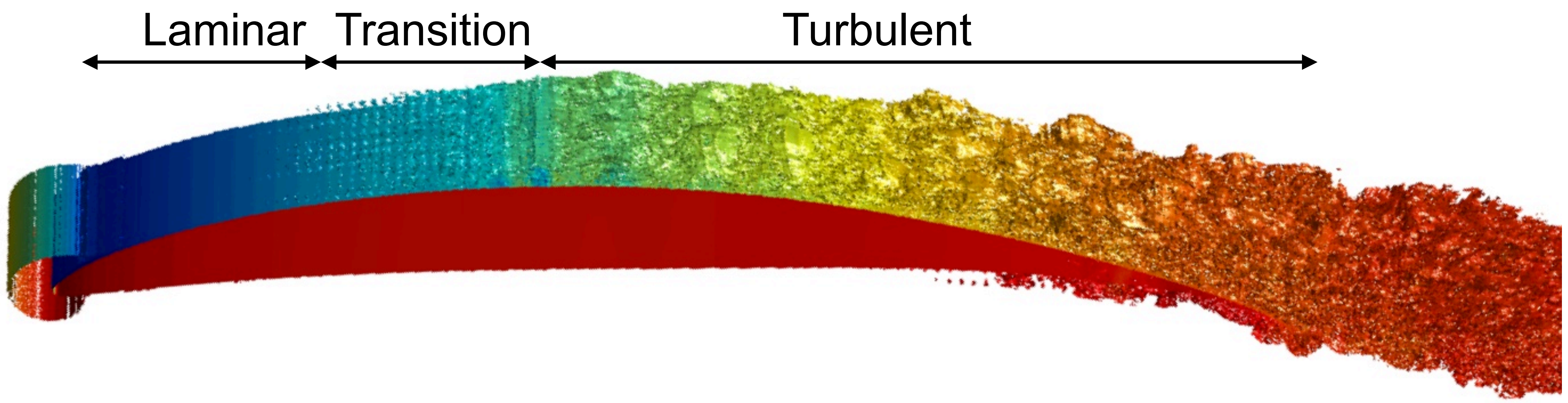
0 1 2 3 4 5

← **Basic research** → ← **Applied research** →

Technology Readiness Level

Example of combining geometry, numerics and physics

- **Problem.** Predicting transition from laminar to turbulent flow
- **Importance**. It is critical to obtain correct quantities of interest
- **Solution**. Entropy stable (physics) high-order HDG method (numerics) on curved meshes (geometry)
- **Benefits**. Accurate prediction of the transition region, so of the quantities of interest



Instantaneous Q-criterion isosurface colored by pressure in off-design condition
(Fernandez, Nguyen, Roca & Peraire, AIAA'16)

Outline: geometry, numerics, and physics for high-order methods

Just mention our previous works:

- **Geometry**. High-fidelity curved meshing (CFD)

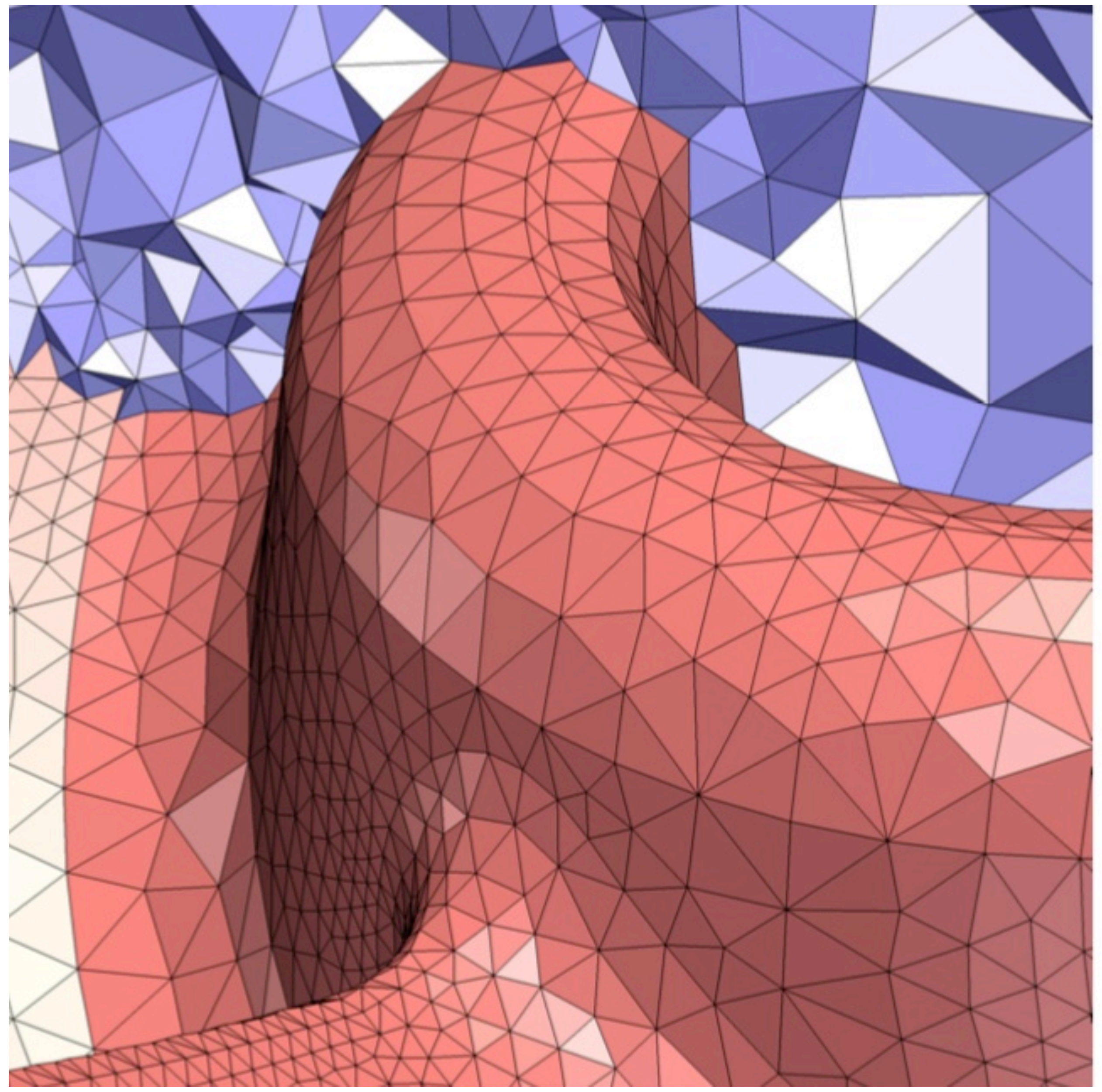
Present our preliminary results:

- **Numerics**. Low-order preconditioning for dG (solvers)
- **Physics**. Semi-discrete Poisson structure for dG (CEM)

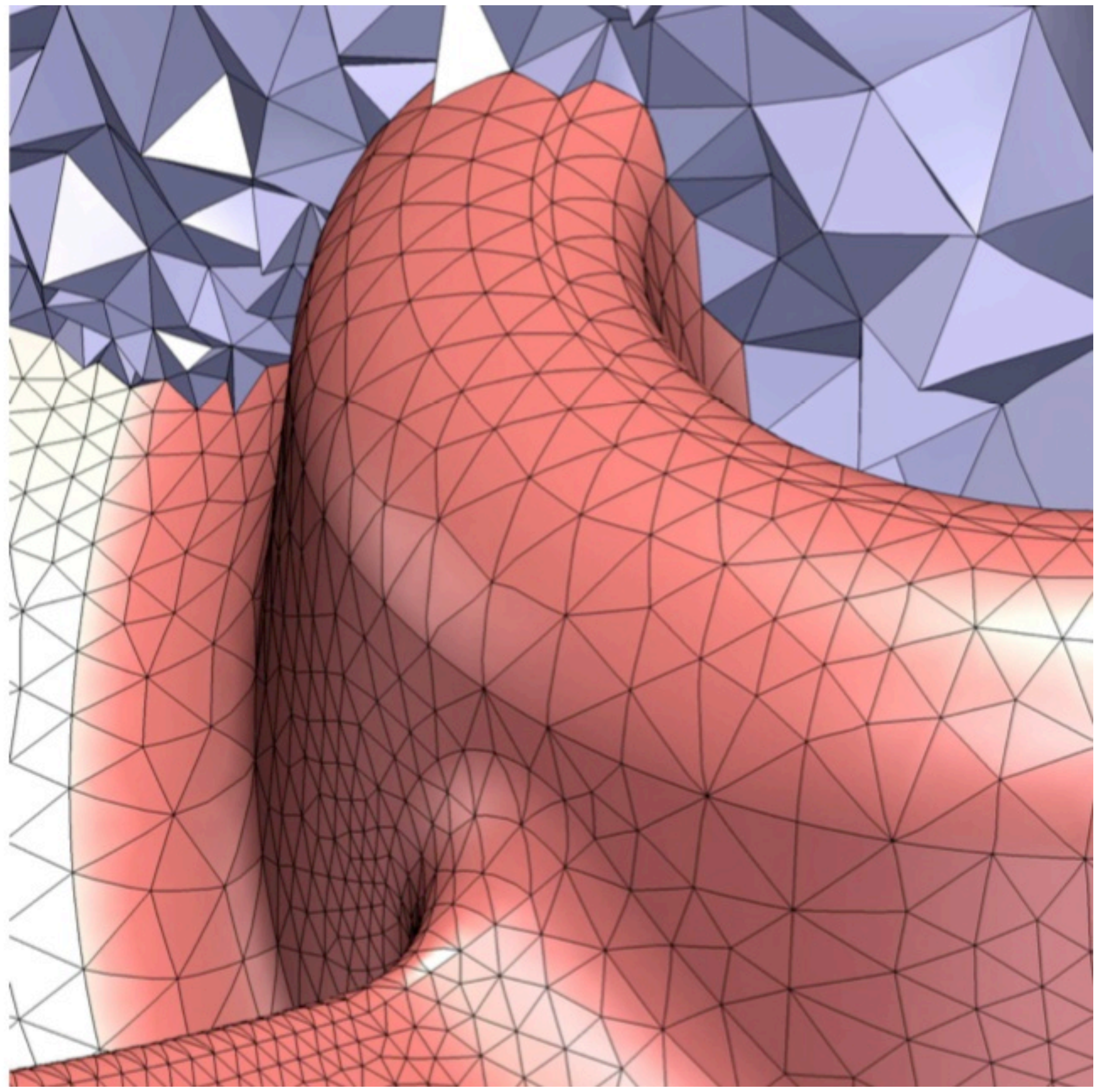
Geometry: high-fidelity curved meshing

Analyze the aerodynamic performance of future aviation designs

- **Problem**: at large-scale, standard meshing methods do not guarantee high-fidelity
- **So what**: without high-fidelity meshes, high-fidelity analyses are not guaranteed



Standard mesh: straight-edged polytopal decomposition

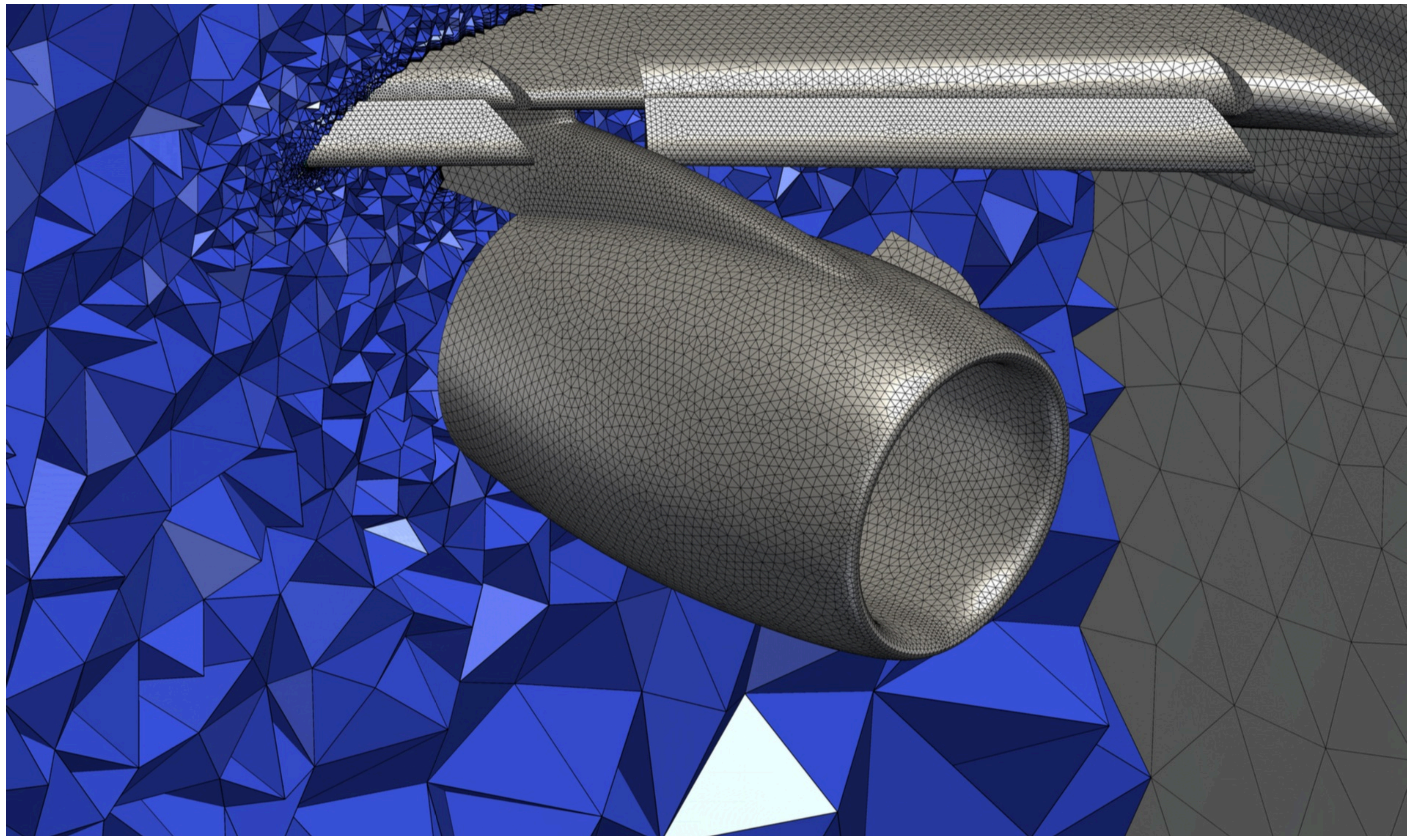


High-fidelity mesh: curved polytopal decomposition

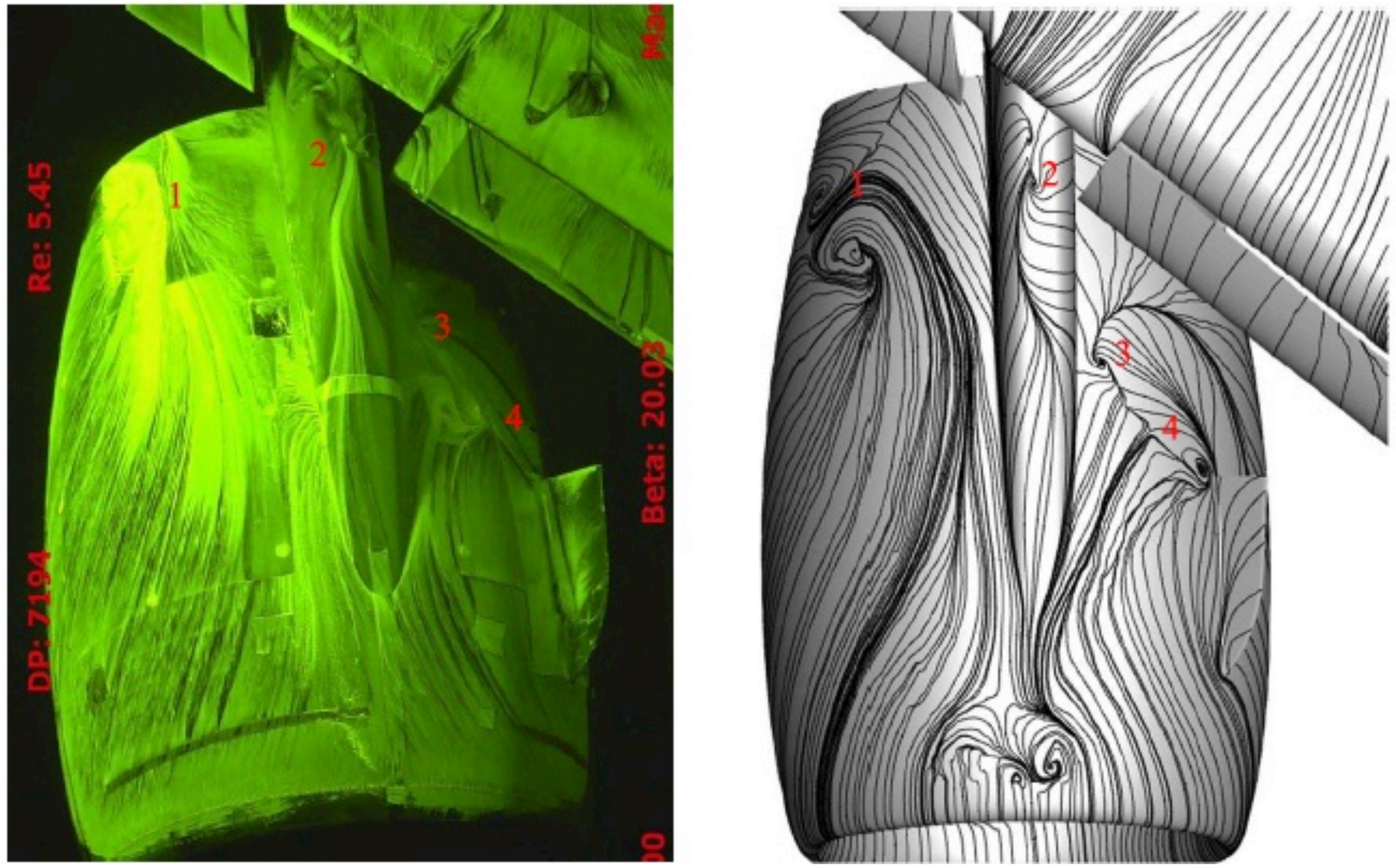
(Ruiz, Sarrate, Roca, CAD'16)

Analyze the aerodynamic performance of future aviation designs

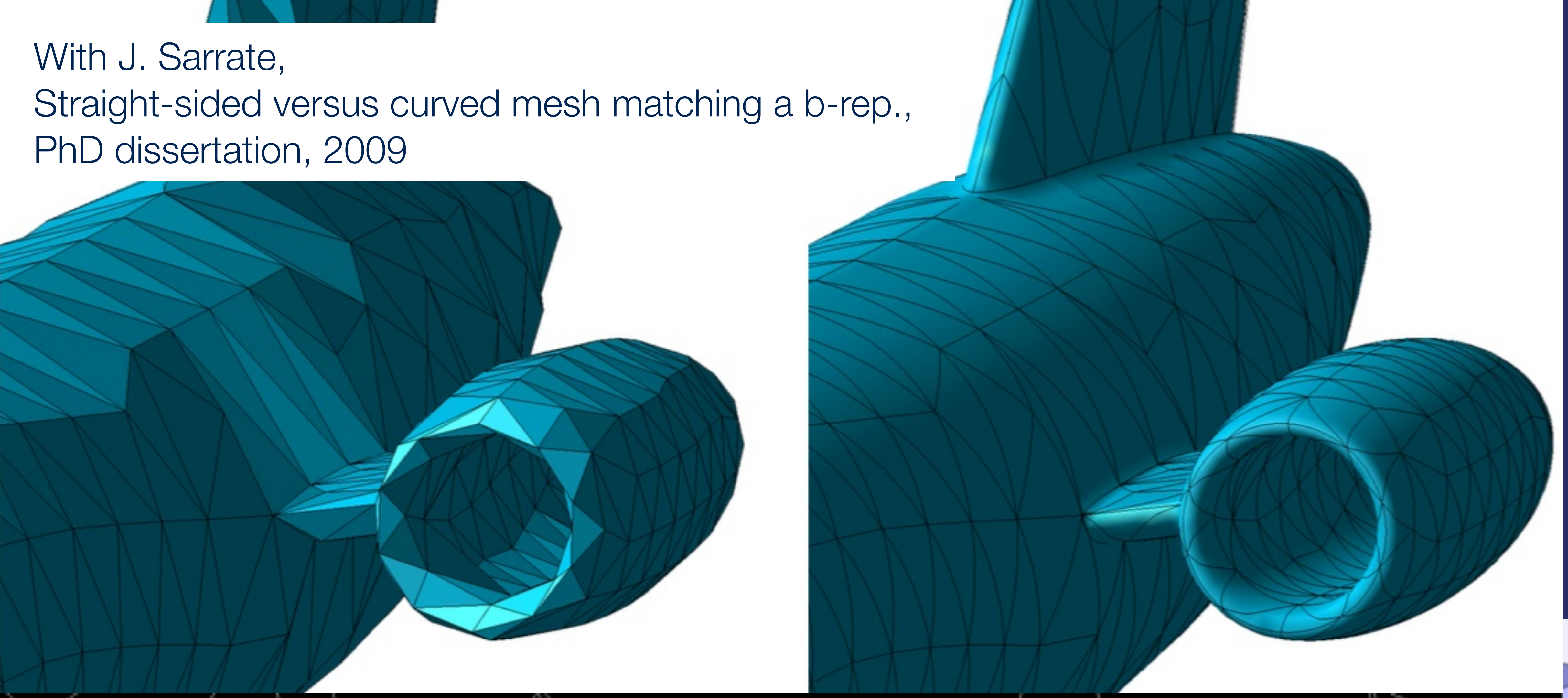
- **Solution**: unique large-scale high-order mesh curver guaranteeing high-fidelity
- **Benefits**: enable high-fidelity analyses in agreement with experiments



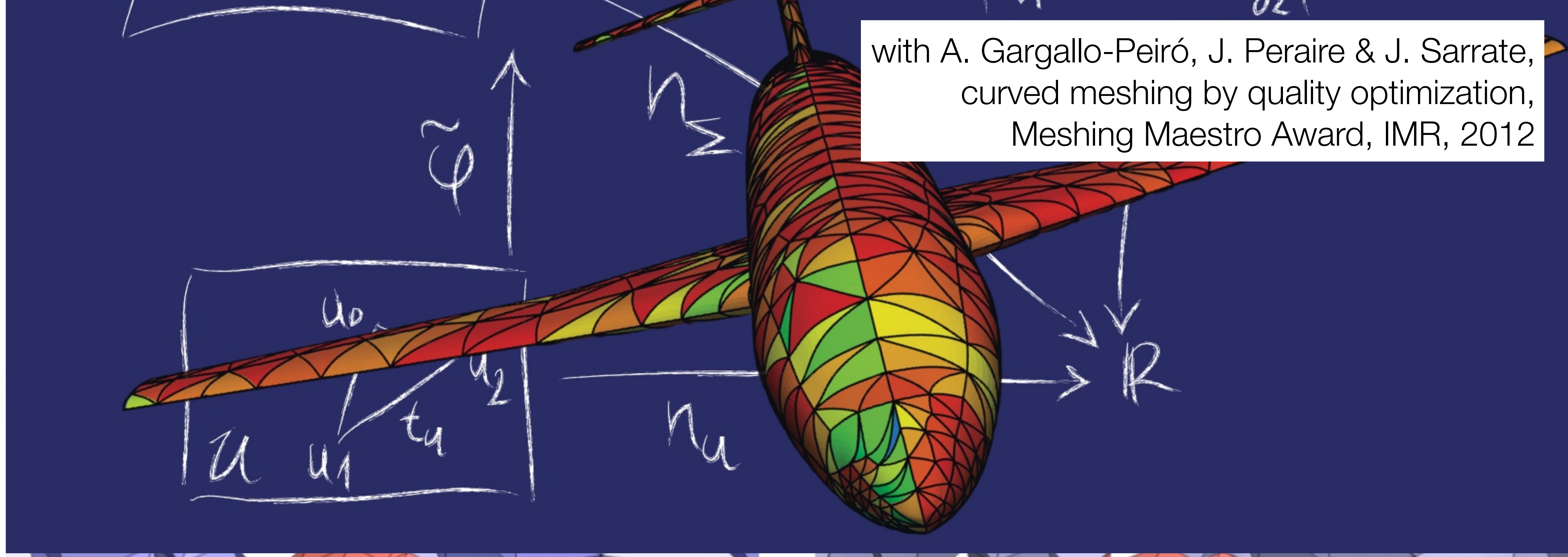
High-fidelity mesh: NASA High-lift Common Research Model (Ruiz, Roca, AIAA'22)



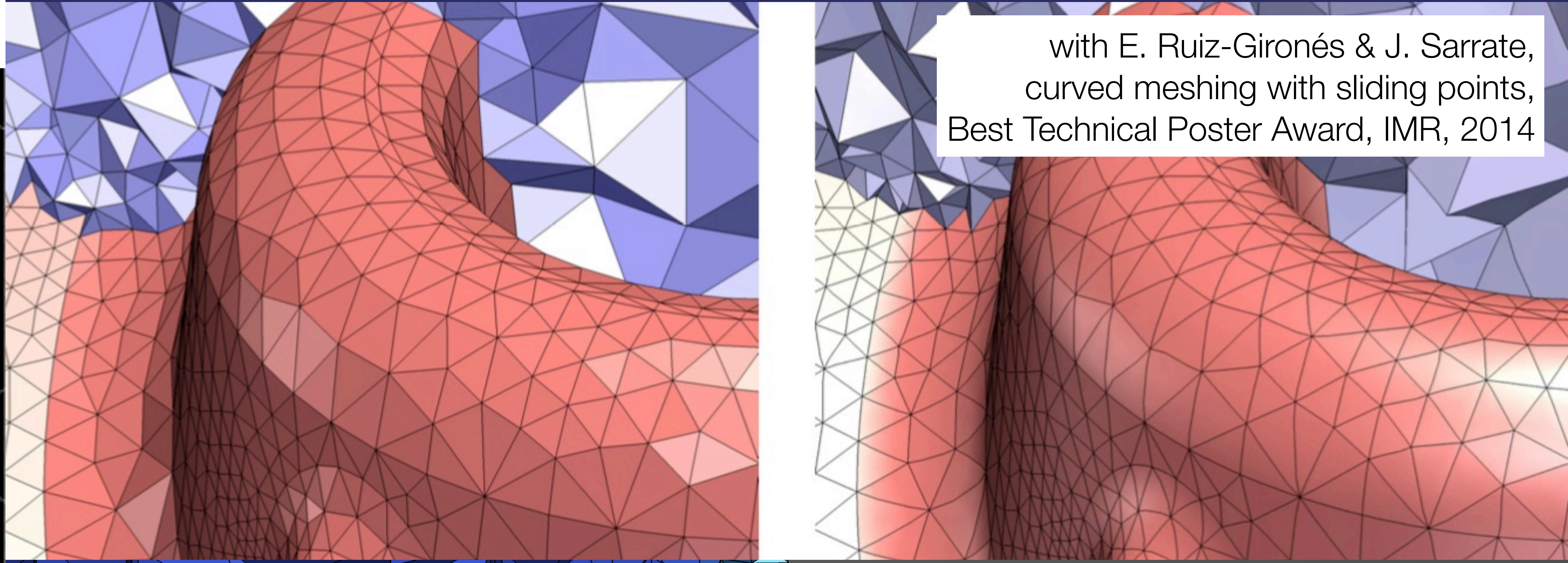
Oil flow comparison (Wang, AIAA'22) : (left) experiment & (right) simulation on **our mesh** (with Ruiz, AIAA'22)



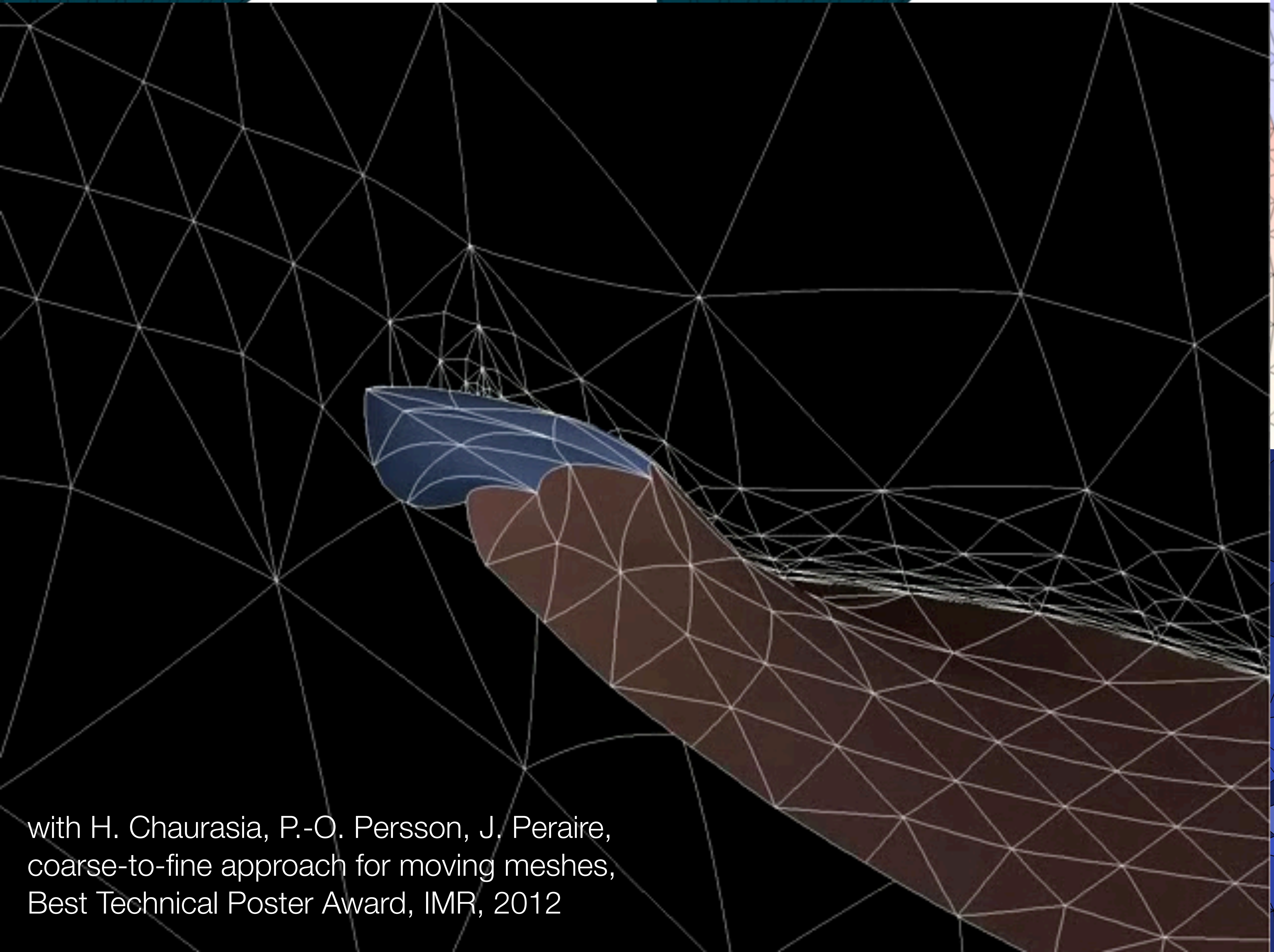
With J. Sarrate,
Straight-sided versus curved mesh matching a b-rep.,
PhD dissertation, 2009



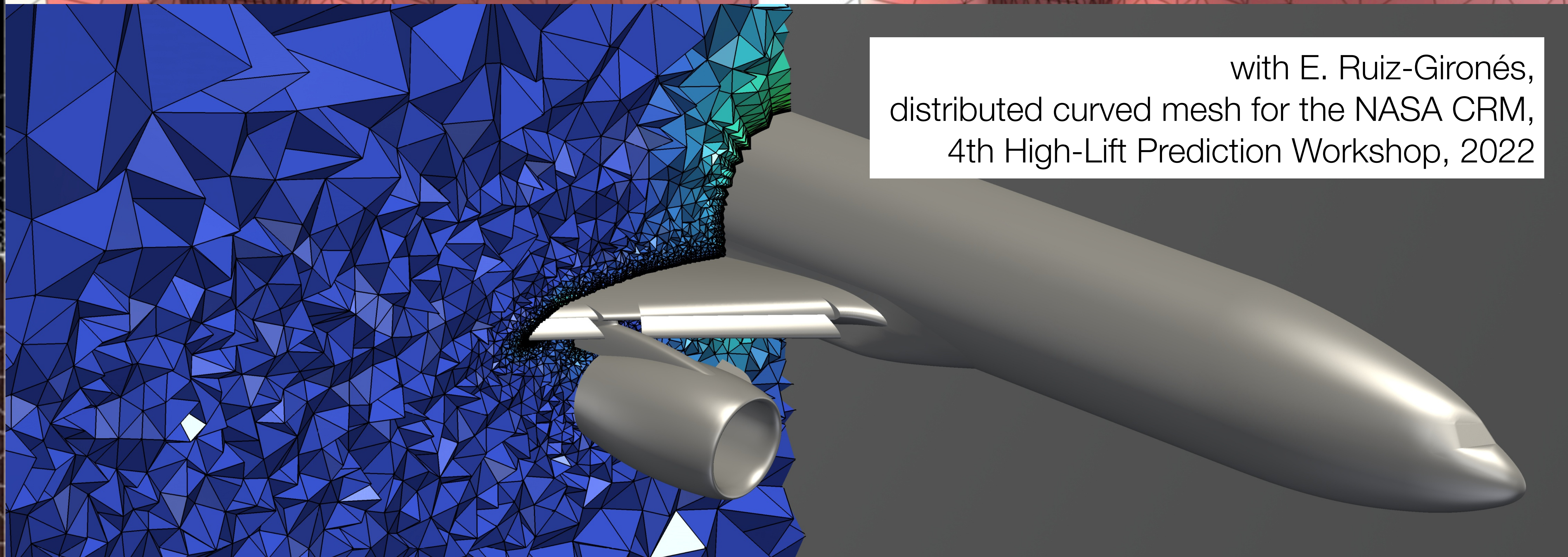
with A. Gargallo-Peiró, J. Peraire & J. Sarrate,
curved meshing by quality optimization,
Meshing Maestro Award, IMR, 2012



with E. Ruiz-Gironés & J. Sarrate,
curved meshing with sliding points,
Best Technical Poster Award, IMR, 2014



with H. Chaurasia, P.-O. Persson, J. Peraire,
coarse-to-fine approach for moving meshes,
Best Technical Poster Award, IMR, 2012



with E. Ruiz-Gironés,
distributed curved mesh for the NASA CRM,
4th High-Lift Prediction Workshop, 2022

Numerics: low-order preconditioning for dG

High-order methods: storage and conditioning

- Solving a Poisson problem in d dimensions with a high-order discretization:

$$
\begin{aligned}
-\Delta u &= f \qquad \text{in } \Omega \\
u &= 0 \qquad \text{on } \partial\Omega\,.
\end{aligned}
$$

- **Problem**. For system matrix $\mathbf{A}_p$, *storage and conditioning worsen with degree* p:
  - *number of nonzeros per row is* $O(p^d)$ (Huerta, Angeloski, Roca, Peraire, IJNME'13)
  - *e.g., for SIP, the condition number is* $O\left(p^4 h^{-2}\right)$ (Pazner, Kolev, ComAppComp'20)

SIP: storage and conditioning solution

- For LGL-cubes, solved in Pazner, Kolev, and Dohrmann, SIAMSciComp'23

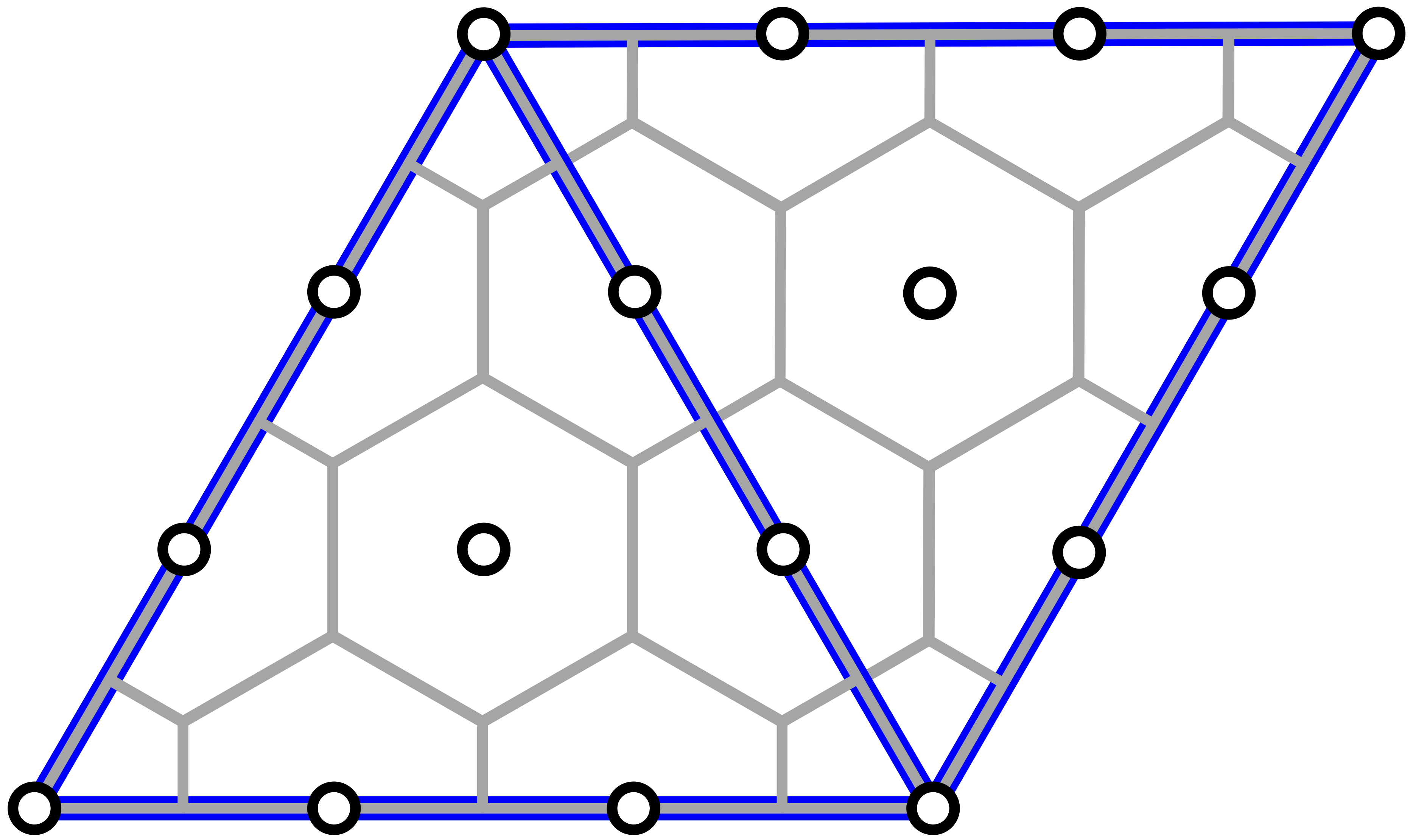
- Using sub-cubes bound by LGL($p+1$) points, get a spectrally equivalent matrix $\mathbf{A}_0$
  - *Given p, the number of preconditioned iterations is constant under h-refinement*
  - *Given h, the number of preconditioned iterations is constant under p-refinement*

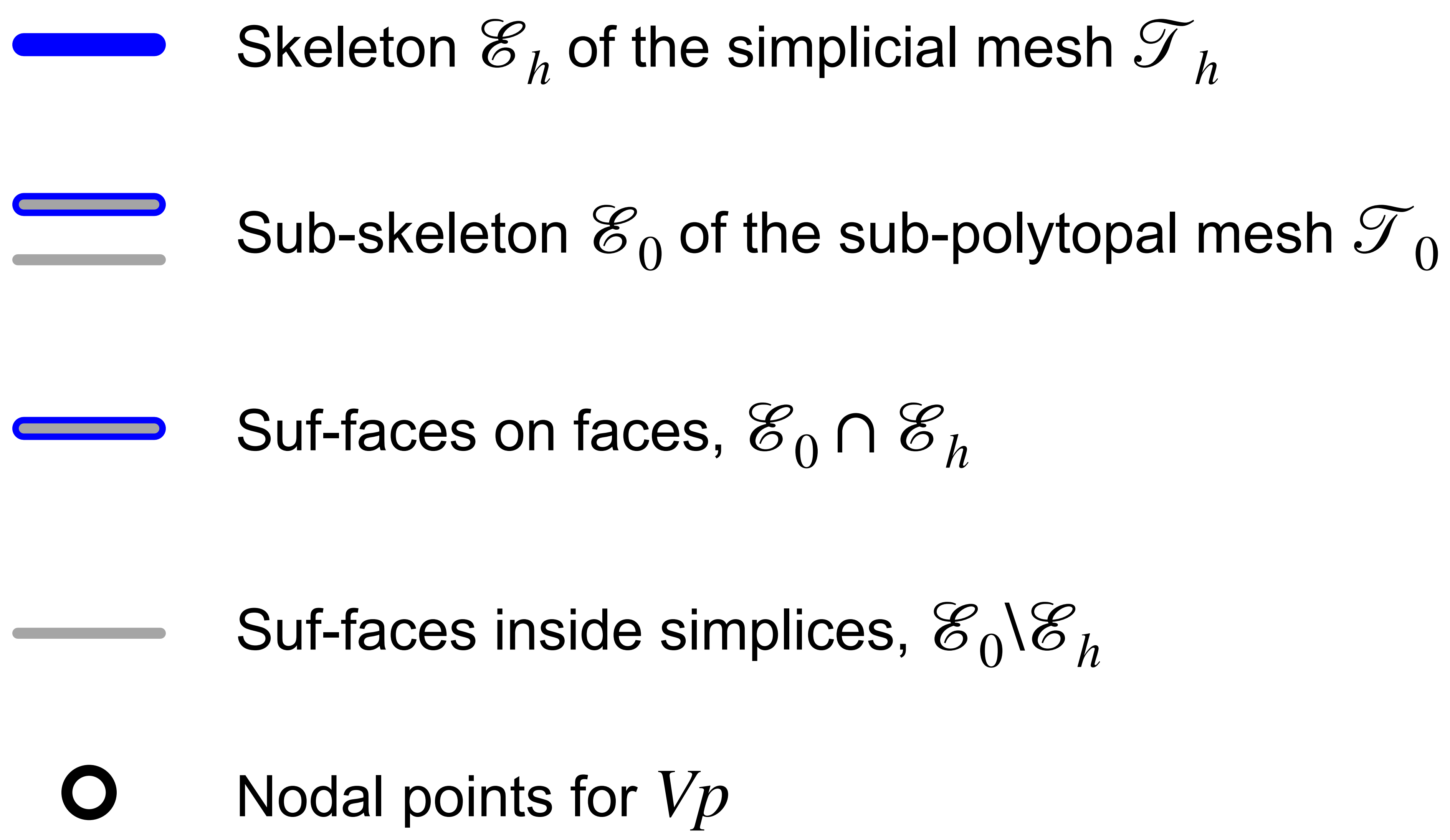
- Store $\mathbf{A}_0$ for preconditioning, and use matrix-free actions of $\mathbf{A}_p$ in the Krylov solver
  - *Affordable storage because the number of nonzeros per row of $\mathbf{A}_0$ is independent of p*

What about low-order dG preconditioning on simplices?

with Aparicio-Estrems, Ruiz-Gironés

- We are solving analogously, yet different:
  - **Equidistributed** and warped **symmetric** points on **simplices**
  - SIP and **local dG** (LDG)
- Using sub-**polytopes dual to nodal** points, get a spectrally h**-equivalent** matrix $\mathbf{A}_0$





Sub-polytopes: equidistributed and warped simplicial points

with Aparicio-Estrems, Ruiz-Gironés

- **Master element** and **sub-elements** in 2D and 3D, *e.g.*, polynomial degree $p = 6$
- **Elemental couplings**: *one-to-all for points vs. one-to-neighbours for sub-polytopes*

28 nodal points on an equilateral triangle (V_p) and 28 dual sub-polygons (V_0)

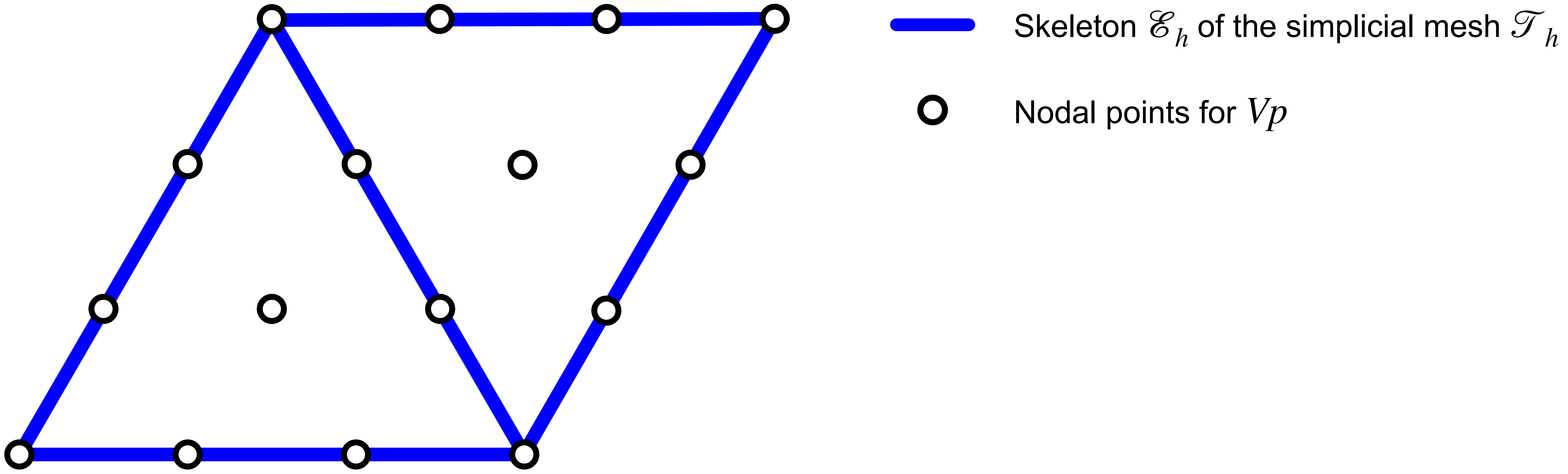
84 nodal points on an equilateral tetrahedron (V_p) and 84 dual sub-polyhedrons (V_0)

SIP: standard weak form

- Let V_p be the broken space of polynomials of degree $p > 0$ on a **simplicial** mesh $\mathcal{T}_h$ with skeleton $\mathcal{E}_h$, we seek u in V_p such that for all v in V_p:

$$a_p(u, v) = (\nabla u, \nabla v)_{\mathcal{T}_h} - (\{\!\{\nabla u\}\!\}, [\![v]\!])_{\mathcal{E}_h} - ([\![u]\!], \{\!\{\nabla v\}\!\})_{\mathcal{E}_h} + (\sigma_p [\![u]\!], [\![v]\!])_{\mathcal{E}_h} = 0,$$

where $\{\!\{\cdot\}\!\}$ is the average, $[\![\cdot]\!]$ is the jump, and σ_p is the corresponding penalty.

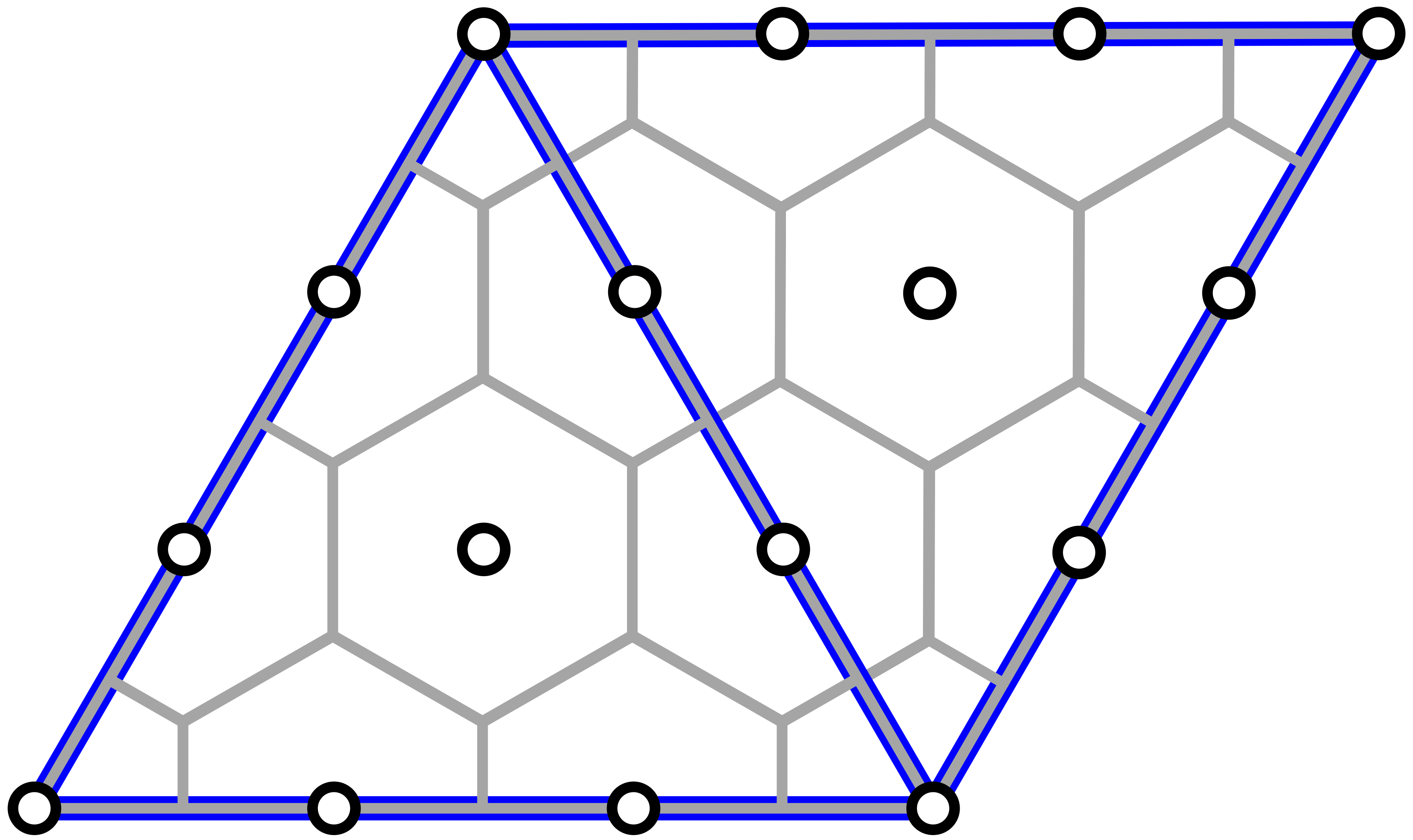


Low-order spectrally equivalent SIP problem for h-refinement

with Aparicio-Estrems, Ruiz-Gironés

- Let V_0 be the broken space of constants on a sub-**polytopal** mesh $\mathcal{T}_0$ with skeleton $\mathcal{E}_0$, we seek u in V_0 such that for all v in V_0:

$$a_0(u,v) = (\sigma_0[\![u]\!],[\![v]\!])_{\mathcal{E}_0} = (\sigma_0[\![u]\!],[\![v]\!])_{\mathcal{E}_0\setminus\mathcal{E}_h} + (\sigma_0[\![u]\!],[\![v]\!])_{\mathcal{E}_0\cap\mathcal{E}_h} = 0.$$



Suf-faces on faces, $\mathcal{E}_0 \cap \mathcal{E}_h$

Suf-faces inside simplices, $\mathcal{E}_0 \setminus \mathcal{E}_h$

Nodal points for Vp

Low-order spectrally equivalent SIP problem for h-refinement

with Aparicio-Estrems, Ruiz-Gironés

- In our **preliminary results**, we are considering a penalty term σ_0 such *that*:

$$(\sigma_0[\![u]\!], [\![v]\!])_{\mathscr{E}_0 \setminus \mathscr{E}_h} = \left(w \frac{[\![u]\!]}{[\![\mathbf{x}]\!]}, \frac{[\![v]\!]}{[\![\mathbf{x}]\!]} \right)_{\mathscr{E}_0 \setminus \mathscr{E}_h} \approx (\nabla u, \nabla v)_{\mathscr{T}_h}$$

$$(\sigma_0[\![u]\!], [\![v]\!])_{\mathscr{E}_0 \cap \mathscr{E}_h} \approx (\sigma_p[\![u]\!], [\![v]\!])_{\mathscr{E}_h}$$

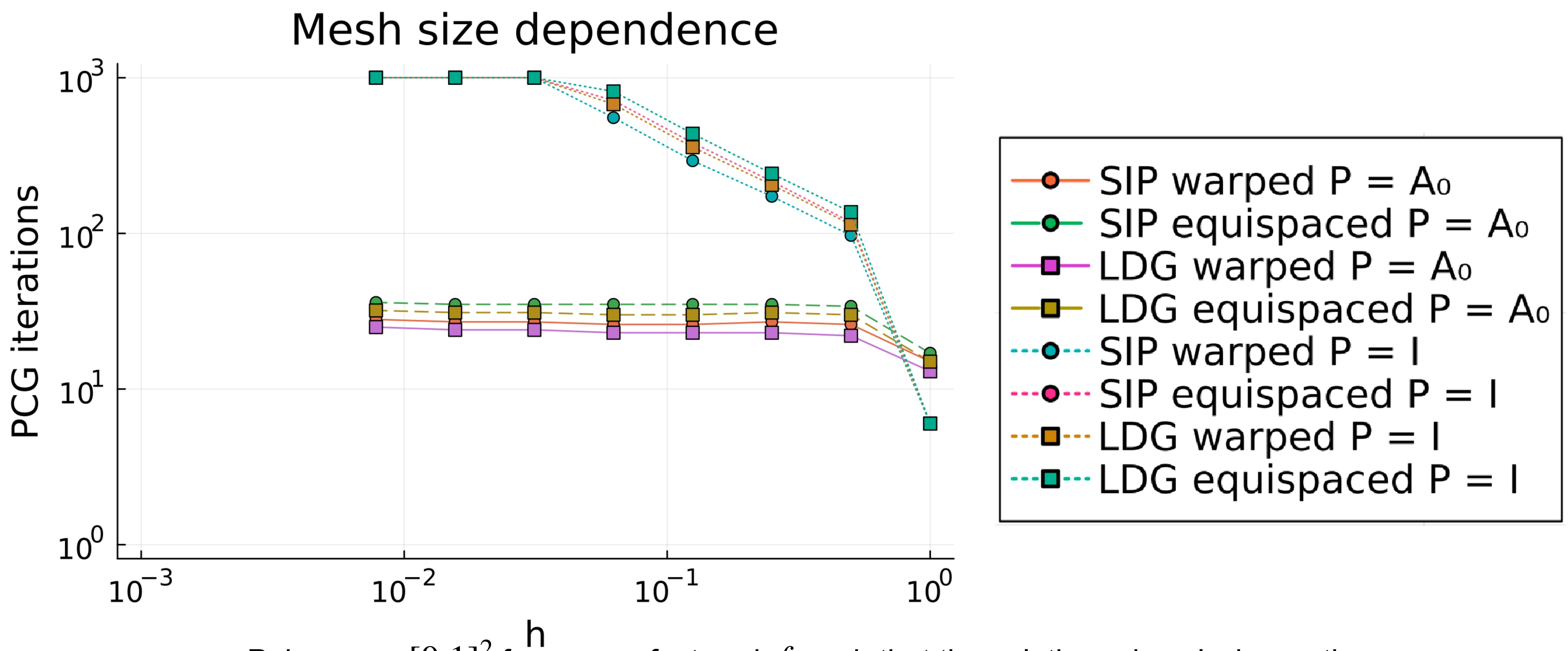
- **For LDG**, we tune the numerical fluxes of a dG scheme for the first-order system:

$$\begin{aligned} \mathbf{q} &= \nabla u && \text{in } \Omega \\ -\nabla \cdot \mathbf{q} &= f && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega\,. \end{aligned}$$

Low-order preconditioning results for simplices: h-refinement

with Aparicio-Estrems, Ruiz-Gironés

- For SIP and **LDG** on **simplices** with **equidistributed** and warped points:
  - *Given p, the number of preconditioned iterations is constant under h-refinement*

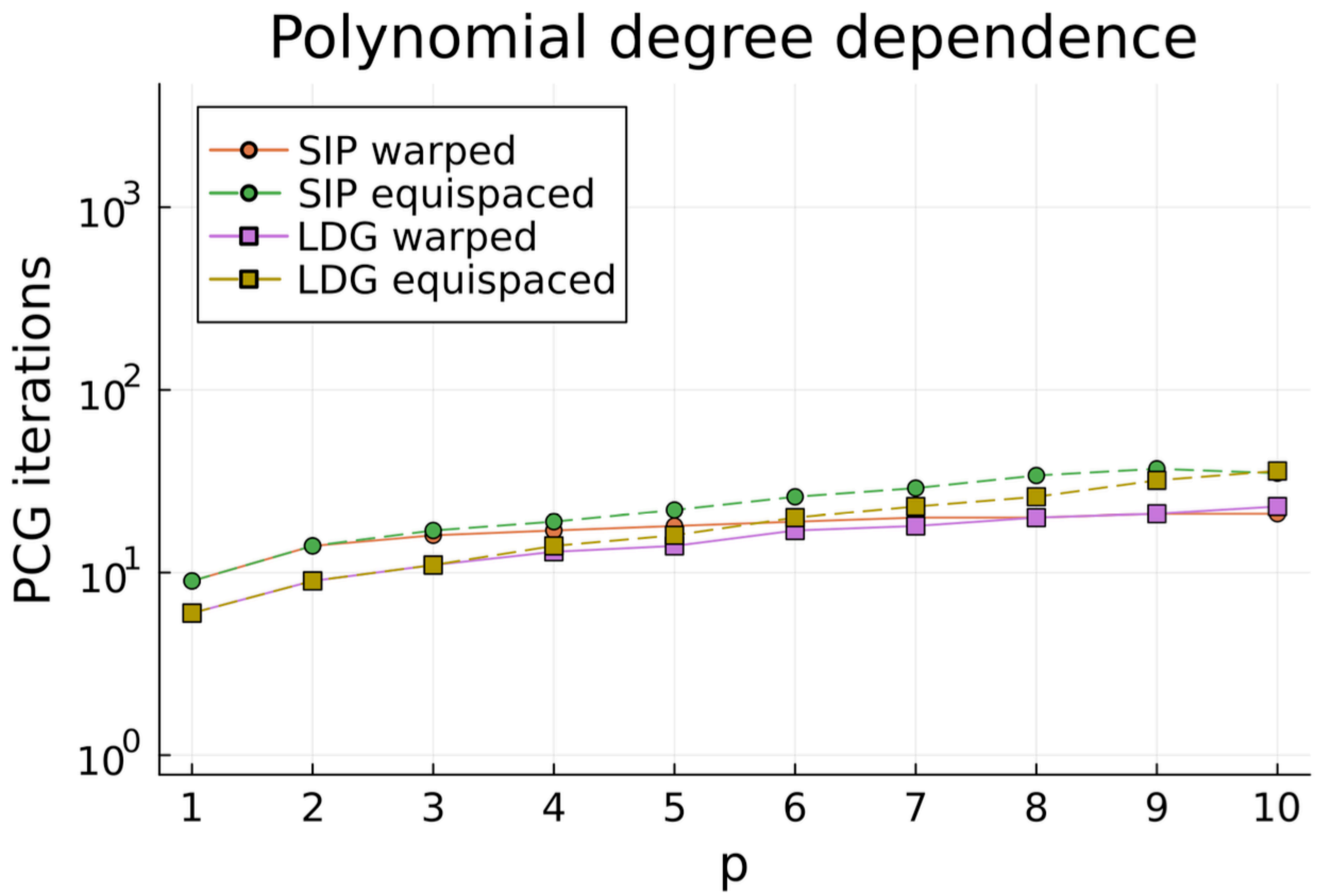


Poisson on $[0,1]^2$ for a manufactured f such that the solution u is polychromatic
(benchmark proposed in Kolev *et al.*, CEED report'22)
(maximum PCG iterations set to 1000)

Low-order preconditioning results for simplices: p-refinement

with Aparicio-Estrems, Ruiz-Gironés

- For SIP and **LDG** on **simplices** with **equidistributed** and warped points:
  - *Given h, the number of preconditioned iterations **mildly grows** under p-refinement*

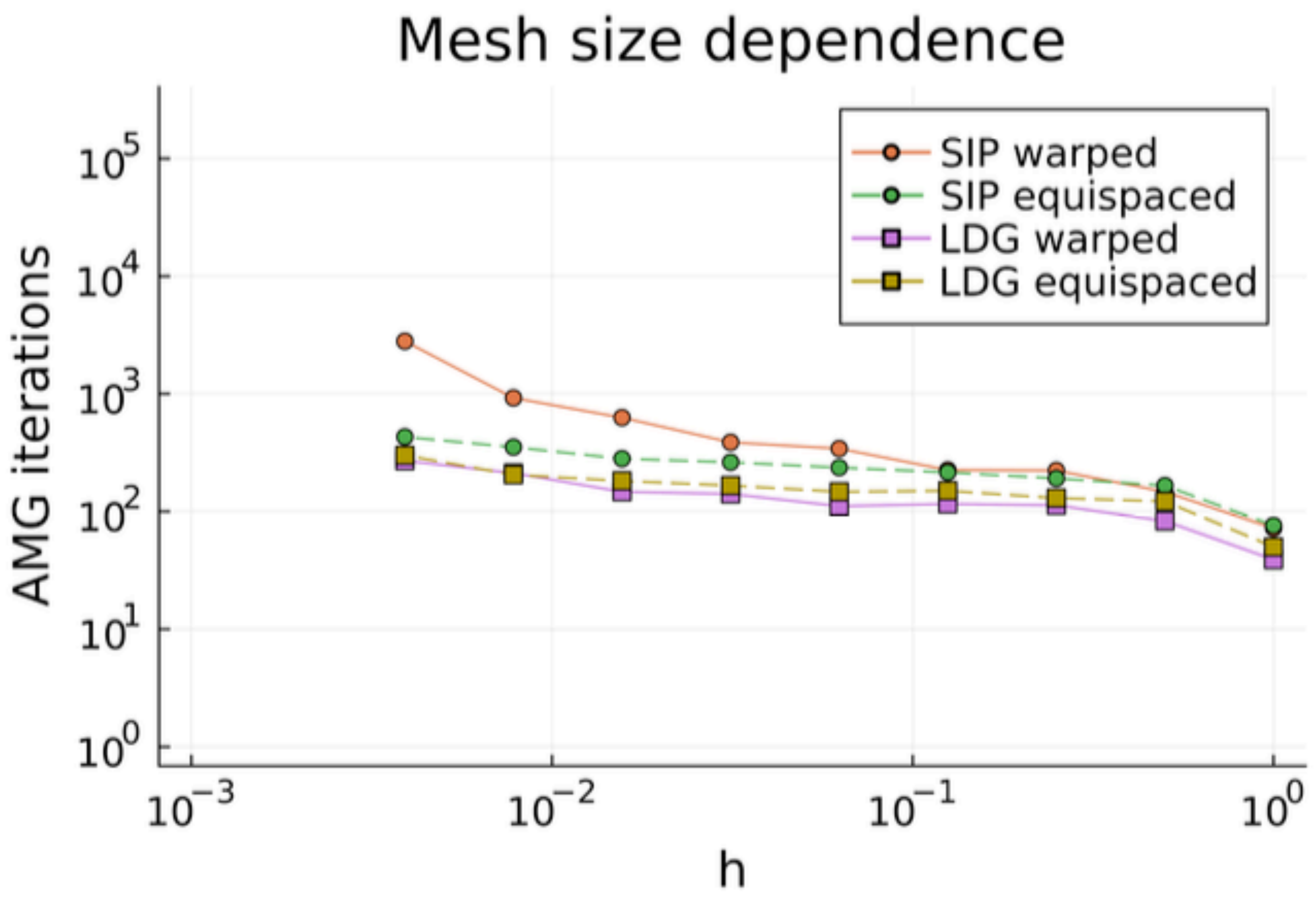


Poisson on $[0,1]^2$ for a manufactured f such that the solution u is polychromatic
(benchmark proposed in Kolev *et al.*, CEED report'22)

Low-order preconditioning for simplices: next steps

with Aparicio-Estrems, Ruiz-Gironés

- For simplices, can we improve the conditioning under p-refinement?
- Polish and review the proof of spectral h-equivalence for SIP and LDG
- Tune and compare different solvers for the low-order preconditioning problem



h-refinement: accumulated AMG iterations to compute the action of $\mathbf{A}_0^{-1}$

Low-order preconditioning for simplices: summary

with Aparicio-Estrems, Ruiz-Gironés

- Using sub-**polytopes dual to nodal** points, we get a preconditioning matrix $\mathbf{A}_0$
- For SIP and **LDG** on **simplices** with **equidistributed** and warped points

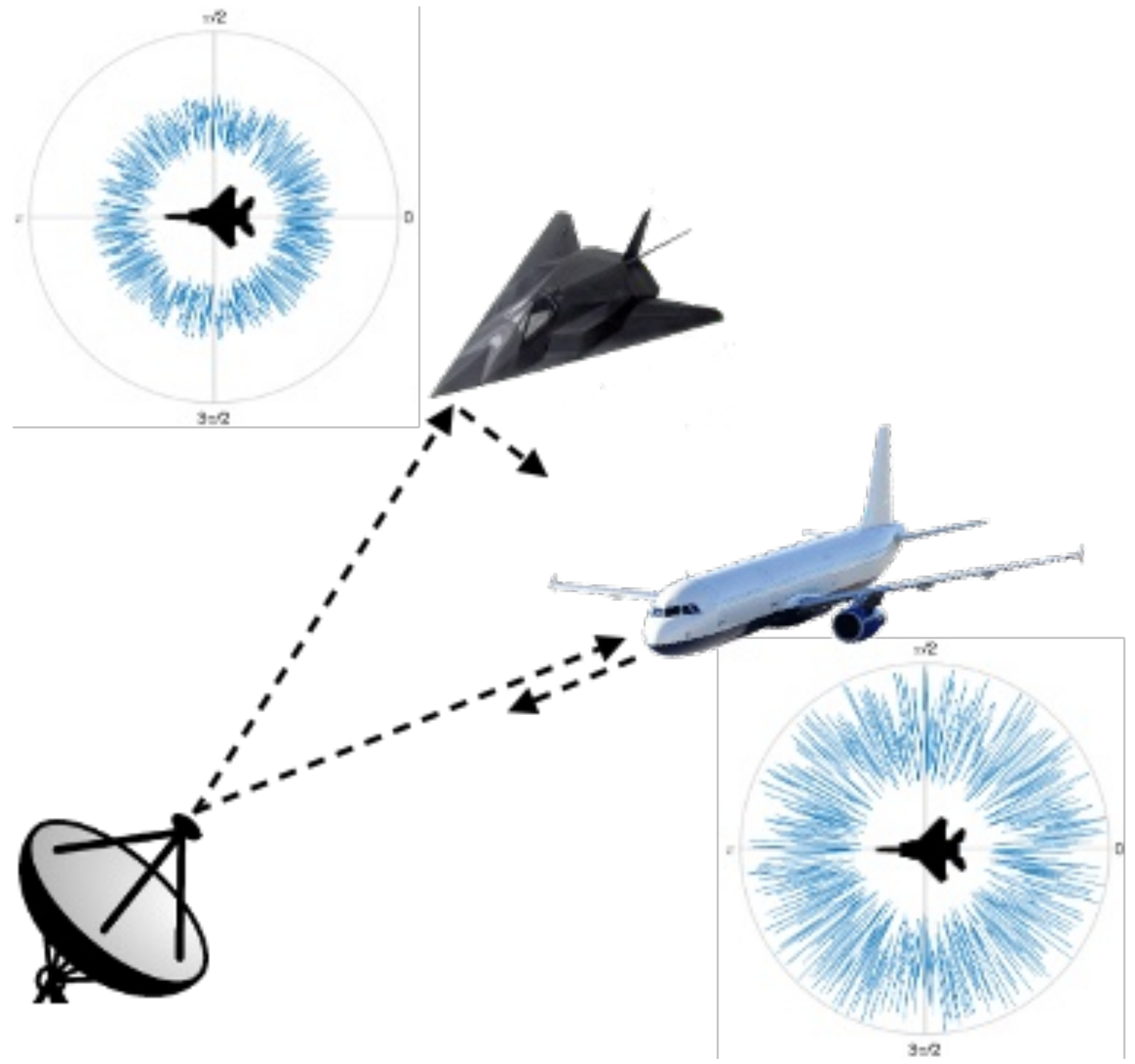
- *Affordable storage because the number of nonzeros per* $\mathbf{A}_0$ *row is independent of* p
- *The number of preconditioned iterations is constant under* h*-refinement*
- *The number of preconditioned iterations* ***mildly grows*** *under* p*-refinement*

Physics: semi-discrete Poisson structure for dG

Analyze the electro-magnetic observability of future aviation designs

with Ruiz, Farnós, de la Puente

- **Problem**: for large-scale high-fidelity analyses, some standard methods do not scale
- **So what**: without scalable & accurate methods, design & analyses cycles are too slow

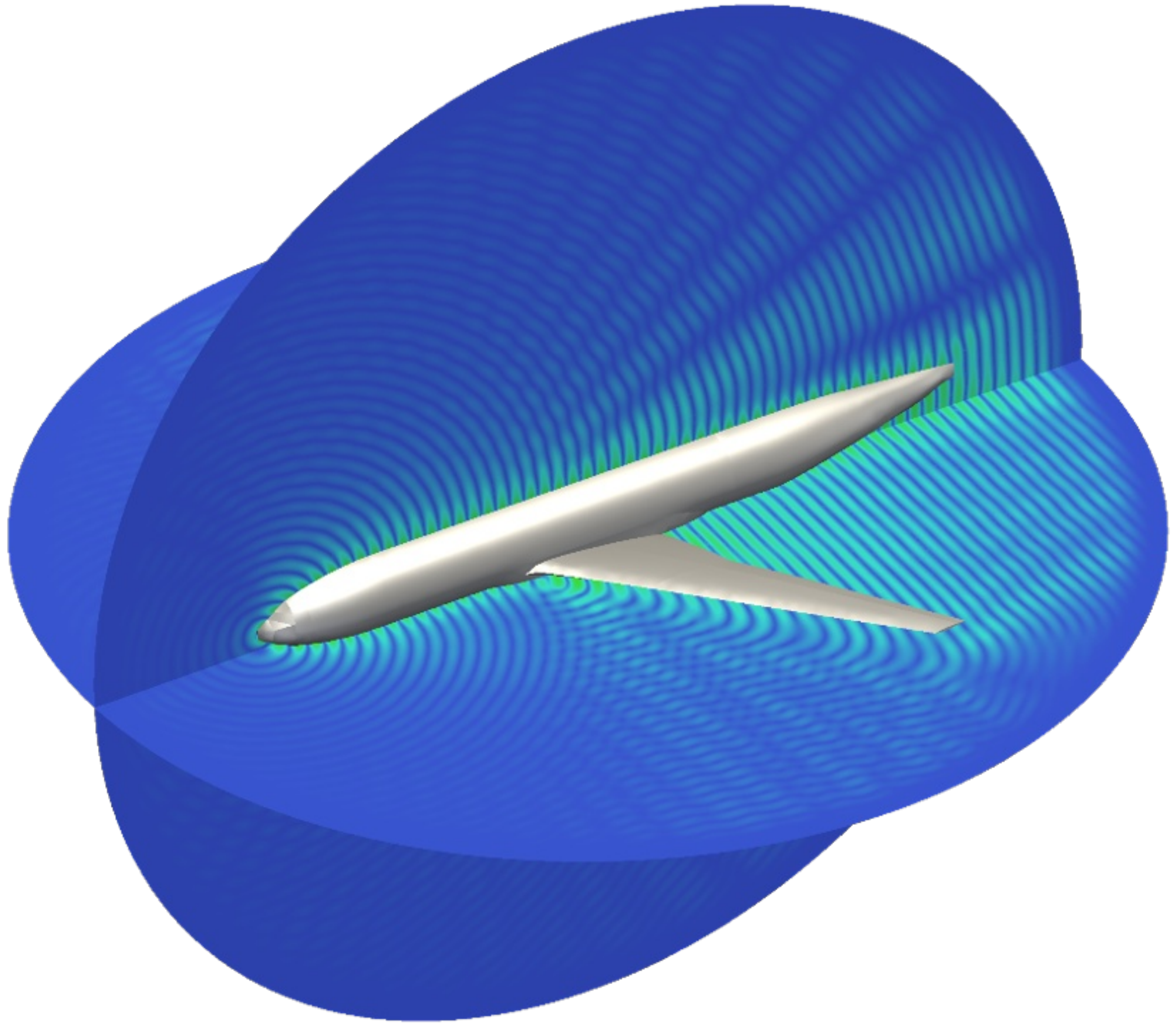


Electromagnetic observability: a radar emits EM waves; aircrafts scatter them; a radar detects part of the scattered waves

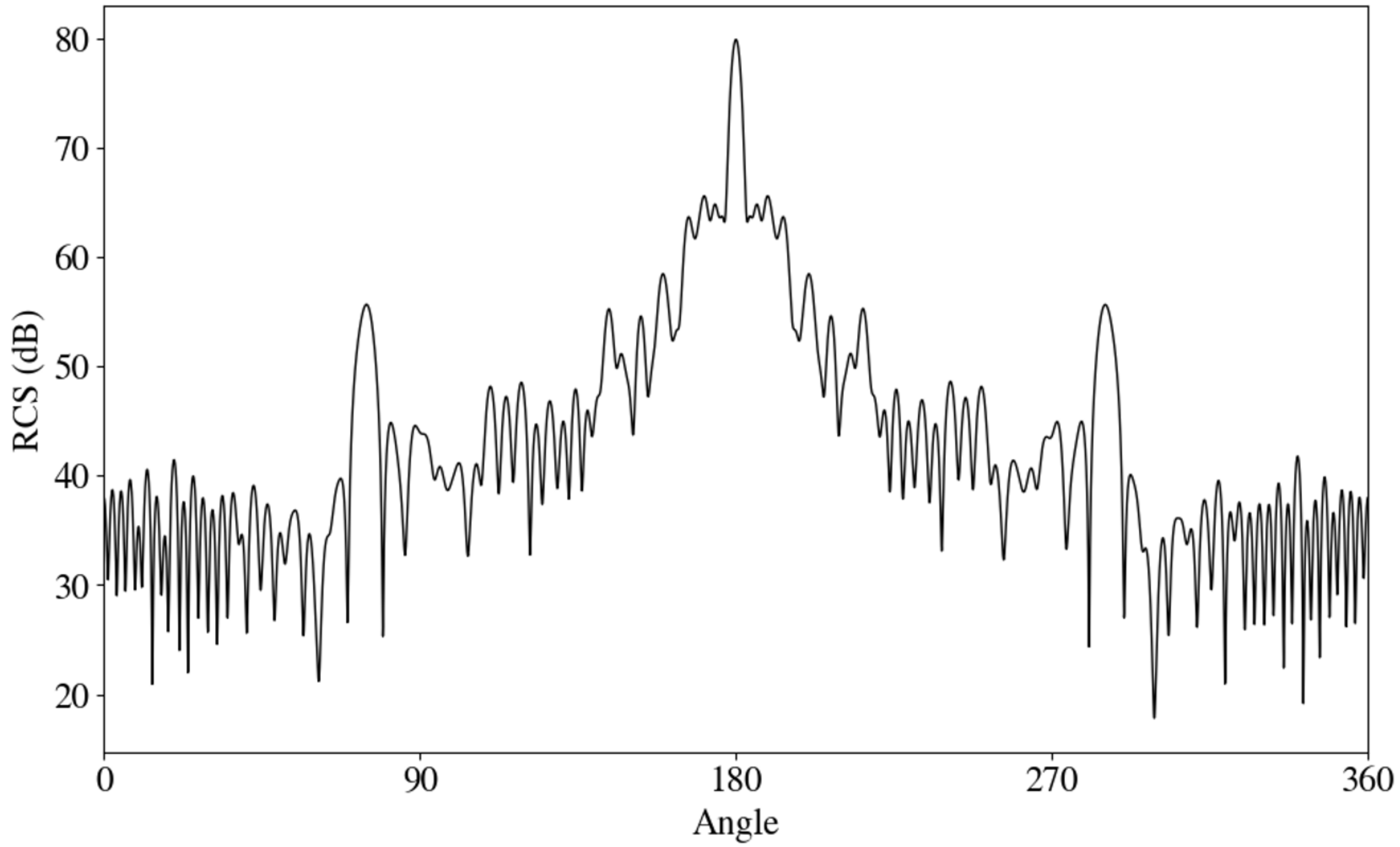
Analyze the electro-magnetic observability of future aviation designs

with Ruiz, Farnós, de la Puente

- **Solution**: numerically and physically accurate large-scale high-order solver
- **Benefits**: enable fast high-fidelity analyses agreeing with observability benchmarks



Scattered electric field:
4 hours on 14000 processors



Radar cross section:
scattered signal in different directions

Simplified Maxwell equations and problem

- For the sake of clarity*, we consider Maxwell equations without currents and charges

$$\epsilon\dot{\mathbf{E}} = \nabla \times \mathbf{H} \quad \text{in } \Omega \times (0,T]$$
$$\mu\dot{\mathbf{H}} = -\nabla \times \mathbf{E} \quad \text{in } \Omega \times (0,T],$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, ϵ and μ material parameters, Ω is a periodic domain, T is the final time, and the initial conditions

$$\epsilon\mathbf{E}(\cdot,0) = \epsilon\mathbf{E}_0(\,\cdot\,) \quad \text{(electric displacement field)}$$
$$\mu\mathbf{H}(\cdot,0) = \mu\mathbf{H}_0(\,\cdot\,) \quad \text{(magnetic flux density)}$$

are divergence free.

**In our work, we consider boundary conditions (BCs) and currents.*

Standard dGs semi-discrete weak form: upwdinded fluxes

- For the vectorial broken space $\mathbf{V}_p$ of degree p, we seek $(\mathbf{E}, \mathbf{H})$ in $\mathbf{V}_p \times \mathbf{V}_p$ such that:

$$\left(\epsilon\dot{\mathbf{E}}, \mathbf{v}\right)_{\mathcal{T}_h} = (\mathbf{H}, \nabla \times \mathbf{v})_{\mathcal{T}_h} + (\mathbf{n} \times \mathbf{H}^*, \mathbf{v})_{\partial\mathcal{T}_h}$$

$$\left(\mu\dot{\mathbf{H}}, \mathbf{w}\right)_{\mathcal{T}_h} = -(\mathbf{E}, \nabla \times \mathbf{w})_{\mathcal{T}_h} - (\mathbf{n} \times \mathbf{E}^*, \mathbf{w})_{\partial\mathcal{T}_h}$$

for all $(\mathbf{v}, \mathbf{w})$ in $\mathbf{V}_p \times \mathbf{V}_p$, where $(\,\cdot\,,\,\cdot\,)_{\mathcal{T}_h}$ and $(\,\cdot\,,\,\cdot\,)_{\partial\mathcal{T}_h}$ are the inner products.

- *To stabilize, we can use upwinded numerical fluxes* $\mathbf{n} \times \cdot^*$
- Additional integration by parts leads to the strong weak form (Hesthaven & Warburton)

Standard DGs with upwinded fluxes: pros and cons

- Combined with high-order **explicit** time integration, *e.g.*, ERK($p+1$):
  - *Low dispersion and dissipation* (Ainsworth, JCP'04)
  - *Well suited for distributed parallelization on CPUs and GPUs* (Klöckner *et al.*, JCP'09)

- However, it does not preserve the Poisson structure of Maxwell equations:
  - *For long-time simulations, conserved quantities drift* (Sánchez *et al.*, CMAME'22)
  - *e.g., energy* $\mathscr{E} = \frac{1}{2}(\epsilon \mathbf{E}, \mathbf{E})_{\mathscr{T}_h} + \frac{1}{2}(\mu \mathbf{H}, \mathbf{H})_{\mathscr{T}_h}$

Standard DG: solution to preserve the Poisson structure

- Studied in Sánchez, Du, Nguyen, Cockburn, and Peraire, CMAME'22
- *Using centered or alternating fluxes, the semi-discrete standard dG is equivalent to a semi-discrete Poisson system*

$$\dot{F}_{\mathbf{E}} = \{F_{\mathbf{E}}, \mathscr{H}\}_h, \text{ for } F_{\mathbf{E}} = (\epsilon \mathbf{E}, \mathbf{v})_{\mathscr{T}_h}$$

$$\dot{F}_{\mathbf{H}} = \{F_{\mathbf{H}}, \mathscr{H}\}_h, \text{ for } F_{\mathbf{H}} = \left(\mu \mathbf{H}, \mathbf{w}\right)_{\mathscr{T}_h},$$

where $\mathscr{H}$ is the Hamiltonian, and $\{\,\cdot\,,\,\cdot\,\}_h$ is a non-canonical discrete bracket.

- *The discrete energy, and the electric and magnetic charge follow a conservation law*
- *The discrete versions of other conserved quantities remain remarkably no-drifting*

Poisson preservation prove

- Start from the semi-discrete weak form of standard dG
- Parameterizing fluxes (dG) and traces (Poisson bracket)
- Manipulate volume and inner and boundary skeleton terms through equivalences
- Reach the semi-discrete Poisson system

- *However, the prove for standard dGs is inherently intricate.* **What about other dGs?**

Our dG semi-discrete weak form: centered fluxes*

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- *To facilitate the preservation prove, we required a form closer to the Poisson system*
- *To facilitate the implementation, we required a form with unmodified test functions*
- We got the following semi-discrete form, seek $(\mathbf{E},\mathbf{H})$ in $\mathbf{V}_p \times \mathbf{V}_p$ such that:

$$\left(\epsilon\dot{\mathbf{E}},\mathbf{v}\right)_{\mathcal{T}_h} = +(\nabla\times\mathbf{H},\mathbf{v})_{\mathcal{T}_h} - \left(\mathbf{n}\times\frac{1}{2}\left[\mathbf{H}\right],\mathbf{v}\right)_{\partial\mathcal{T}_h}$$

$$\left(\mu\dot{\mathbf{H}},\mathbf{w}\right)_{\mathcal{T}_h} = -(\mathbf{E},\nabla\times\mathbf{w})_{\mathcal{T}_h} - \left(\mathbf{n}\times\{\mathbf{E}\},\mathbf{w}\right)_{\partial\mathcal{T}_h}$$

for all $(\mathbf{v},\mathbf{w})$ in $\mathbf{V}_p \times \mathbf{V}_p$, where $\{\,\cdot\,\}$ is the average, and $[\,\cdot\,]$ is the jump.

**In our work, we also consider alternating fluxes, boundary conditions, and currents*

Proving Poisson-structure preservation: to skew-symmetry

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- Using equivalences on the skeleton terms, **our semi-discrete dG weak form**

$$\left(\epsilon\dot{\mathbf{E}},\mathbf{v}\right)_{\mathcal{T}_h} = +(\nabla\times\mathbf{H},\mathbf{v})_{\mathcal{T}_h} - \left(\mathbf{n}\times\frac{1}{2}[\mathbf{H}],\mathbf{v}\right)_{\partial\mathcal{T}_h}$$

$$\left(\mu\dot{\mathbf{H}},\mathbf{w}\right)_{\mathcal{T}_h} = -(\mathbf{E},\nabla\times\mathbf{w})_{\mathcal{T}_h} - \left(\mathbf{n}\times\{\mathbf{E}\},\mathbf{w}\right)_{\partial\mathcal{T}_h}$$

- can be transformed in **fewer steps** into a skew-symmetric system

$$\left(\epsilon\dot{\mathbf{E}},\mathbf{v}\right)_{\mathcal{T}_h} = +(\nabla\times\mathbf{H},\mathbf{v})_{\mathcal{T}_h} - \left(\mathbf{n}\times\mathbf{H},\hat{\mathbf{v}}\right)_{\partial\mathcal{T}_h}$$

$$\left(\mu\dot{\mathbf{H}},\mathbf{w}\right)_{\mathcal{T}_h} = -(\mathbf{E},\nabla\times\mathbf{w})_{\mathcal{T}_h} + \left(\hat{\mathbf{E}},\mathbf{n}\times\mathbf{w}\right)_{\partial\mathcal{T}_h}$$

, where the numerical trace $\hat{\cdot}$ is just $\{\,\cdot\,\}$ for centered fluxes and periodic boundaries*

** In our work, we also account for alternating fluxes and general BCs*

Proving Poisson-structure preservation: to Poisson system

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- Closing the prove, this skew-symmetric system

$$\left(\epsilon\dot{\mathbf{E}},\mathbf{v}\right)_{\mathcal{T}_h} = +(\nabla\times\mathbf{H},\mathbf{v})_{\mathcal{T}_h} - \left(\mathbf{n}\times\mathbf{H},\hat{\mathbf{v}}\right)_{\partial\mathcal{T}_h}$$

$$\left(\mu\dot{\mathbf{H}},\mathbf{w}\right)_{\mathcal{T}_h} = -(\mathbf{E},\nabla\times\mathbf{w})_{\mathcal{T}_h} + \left(\hat{\mathbf{E}},\mathbf{n}\times\mathbf{w}\right)_{\partial\mathcal{T}_h}$$

- is just the semi-discrete system in Poisson form

$\dot{F}_{\mathbf{E}} = \{F_{\mathbf{E}},\mathcal{H}\}_h$, for $F_{\mathbf{E}} = (\epsilon\mathbf{E},\mathbf{v})_{\mathcal{T}_h}$

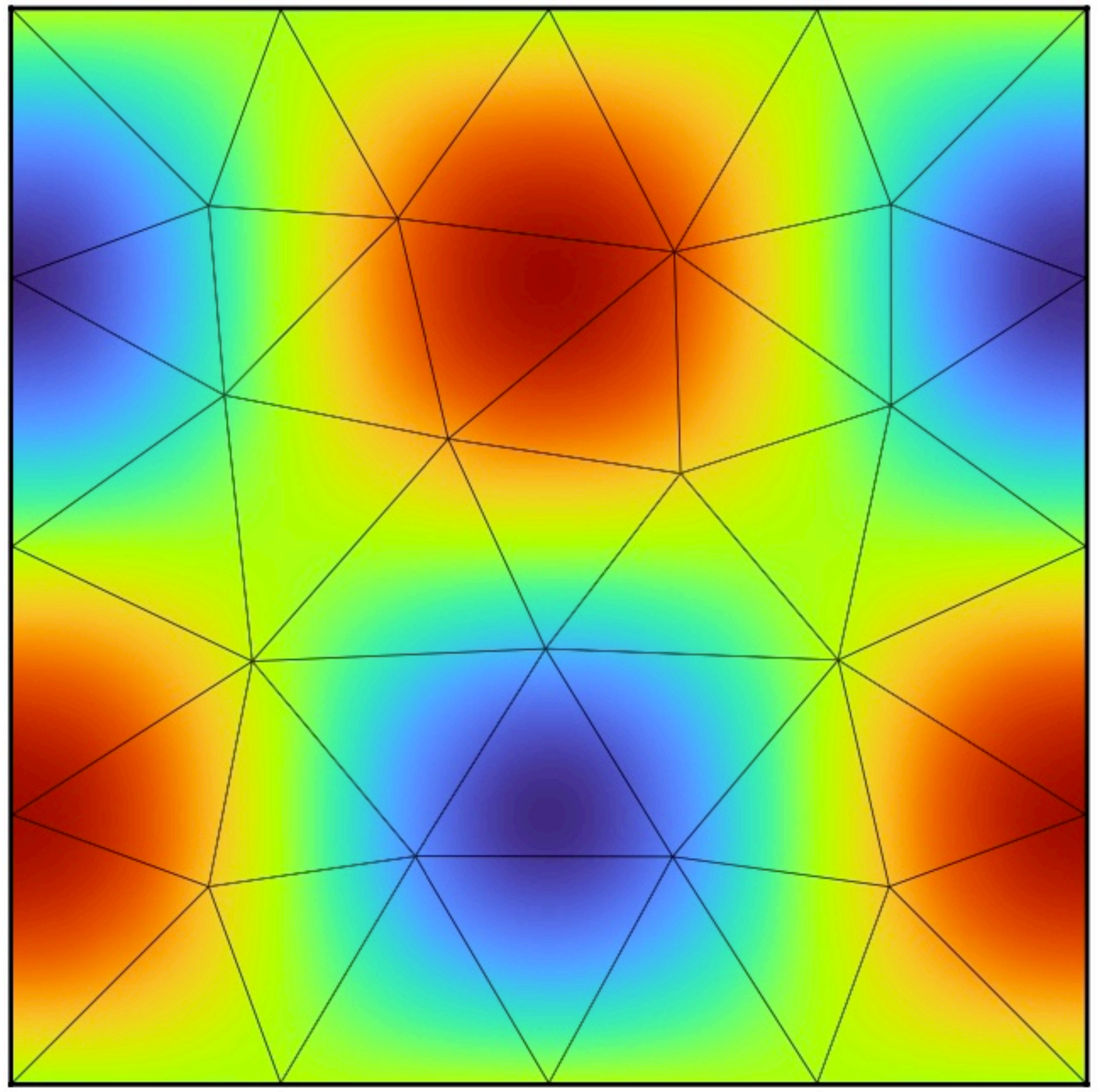
$\dot{F}_{\mathbf{H}} = \{F_{\mathbf{H}},\mathcal{H}\}_h$, for $F_{\mathbf{H}} = \left(\mu\mathbf{H},\mathbf{w}\right)_{\mathcal{T}_h}$,

where $\mathcal{H}$ is the Hamiltonian, and $\{\,\cdot\,,\,\cdot\,\}_h$ is the non-canonical discrete bracket.

Results: space-time convergence (1/2)

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- Cavity mode on a vacuum square for a Perfect Electric Conductor boundary
- In space (h), our Poisson-preserving dG of degree p
- In time (Δt), $T = 100$ periods, Explicit Symplectic Partitioned RK of order $p + 1$,



Standing wave: for $p = 4$, y-axis component of the $\mathbf{H}_h$ field

Results: space-time convergence (2/2)

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- $\mathbf{E}_h$ as $O(h^{p+1})$, but
- $\mathbf{H}_h$ as $O(h^p)$ since $\mu \mathbf{H}_h = \nabla \times \mathbf{A}_h$ ($\mathbf{A}_h$ is the vector potential)

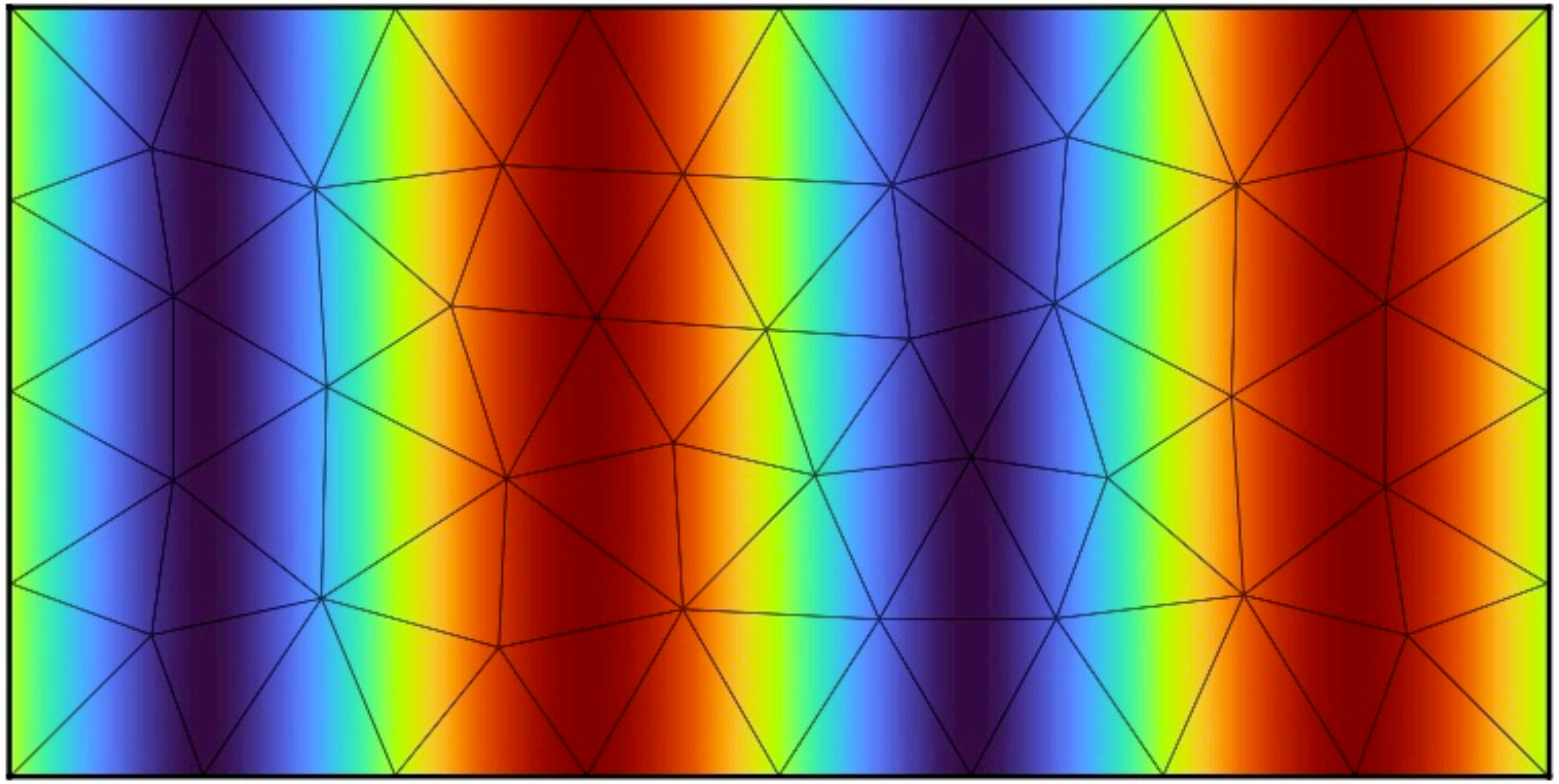
p	h	Δt	$\mathbf{H}_h$ $\|\|\mathbf{e}_{\mathbf{H}_h}\|\|_{L^\infty(0,T;L^2(\mathcal{T}_h))}$	$\mathcal{O}_{\mathbf{H}_h}$	$\mathbf{E}_h$ $\|\|\mathbf{e}_{\mathbf{E}_h}\|\|_{L^\infty(0,T;L^2(\mathcal{T}_h))}$	$\mathcal{O}_{\mathbf{E}_h}$
1	3.715×10^{-2}	9.287×10^{-3}	1.720×10^{-1}	-	1.677×10^{-1}	-
	1.857×10^{-2}	4.643×10^{-3}	4.409×10^{-2}	1.96	4.275×10^{-2}	1.97
	9.287×10^{-3}	2.322×10^{-3}	1.851×10^{-2}	1.25	1.076×10^{-2}	1.99
	4.644×10^{-3}	1.161×10^{-3}	9.249×10^{-3}	1.00	2.695×10^{-3}	2.00
2	7.430×10^{-2}	1.857×10^{-2}	1.658×10^{-2}	-	1.332×10^{-2}	-
	3.715×10^{-2}	9.287×10^{-3}	2.901×10^{-3}	2.52	1.159×10^{-3}	3.52
	1.857×10^{-2}	4.643×10^{-3}	6.758×10^{-4}	2.10	1.060×10^{-4}	3.45
	9.287×10^{-3}	2.322×10^{-3}	1.681×10^{-4}	2.01	1.070×10^{-5}	3.31
4	1.486×10^{-1}	3.714×10^{-2}	1.177×10^{-3}	-	6.262×10^{-4}	-
	7.430×10^{-2}	1.857×10^{-2}	6.383×10^{-5}	4.20	1.490×10^{-5}	5.39
	3.715×10^{-2}	9.287×10^{-3}	3.954×10^{-6}	4.01	4.077×10^{-7}	5.19
	1.857×10^{-2}	4.643×10^{-3}	2.487×10^{-7}	3.99	1.178×10^{-8}	5.11

$(h, \Delta t)$-refinement: errors for $\mathbf{H}_h$ and $\mathbf{E}_h$, and convergence orders

Results: discrete conservation and no-drifting (1/2)

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- Traveling planar wave on a vacuum periodic rectangle
- In space (h), our Poisson-preserving dG(p)
- In time (Δt), $T = 300$ periods, ESPRK($p+1$)

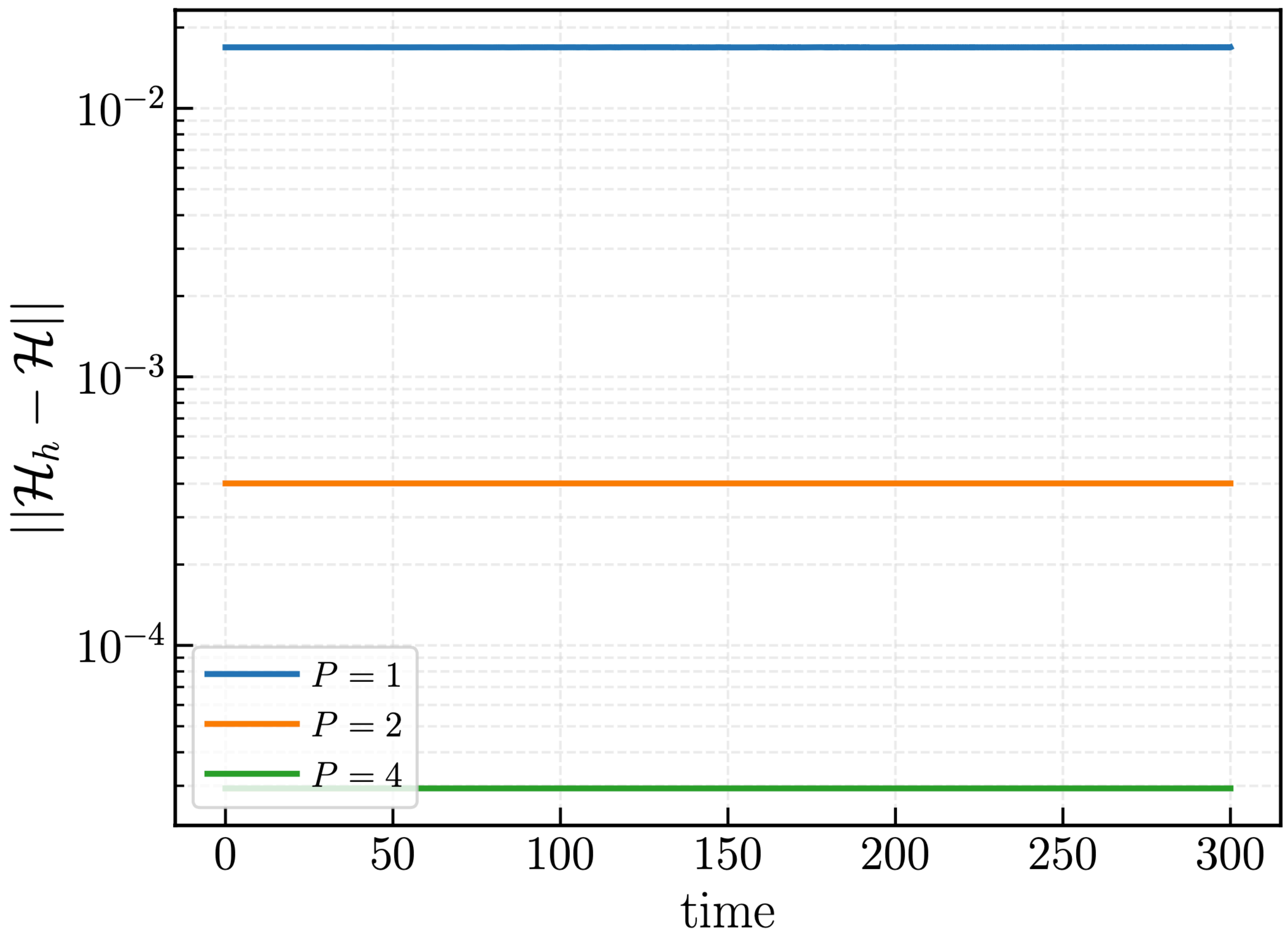


Travelling planar wave: for $p = 3$, z-axis component of the $\mathbf{E}_h$ field

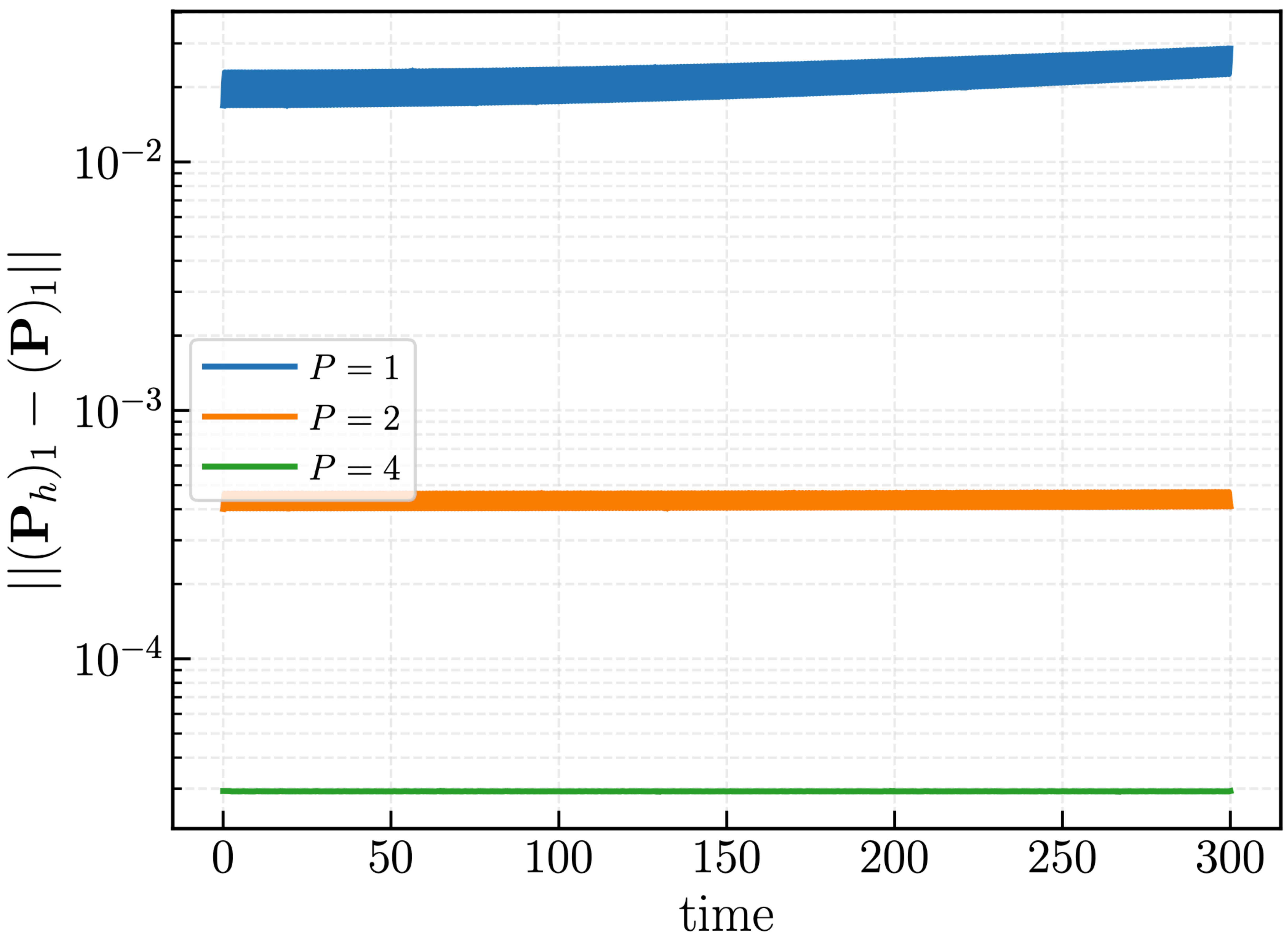
Results: discrete conservation and no-drifting (2/2)

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- Planar wave on a periodic square, for Explicit Symplectic Partitioned RK $(p+1)$



Discrete energy error: conservation



Discrete linear momentum error: remain no-drifting

Preserving a semi-discrete Poisson structure: summary (1/2)

with Circuns-Duxans, Ruiz-Gironés, de la Puente

- We derived a **new dG method** preserving a semi-discrete Poisson structure
- *Our method is devised to facilitate the **implementation** and the **preservation prove***
- Accordingly, it is not directly a Galerkin projection from the Maxwell equations
- *Using ESPRK($p+1$), $\mathbf{E}$ converges as $O(h^{p+1})$, and $\mathbf{H}$ as $O(h^p)$ ($\mu \mathbf{H}_h = \nabla \times \mathbf{A}_h$)*

Preserving a semi-discrete Poisson structure: summary (2/2)

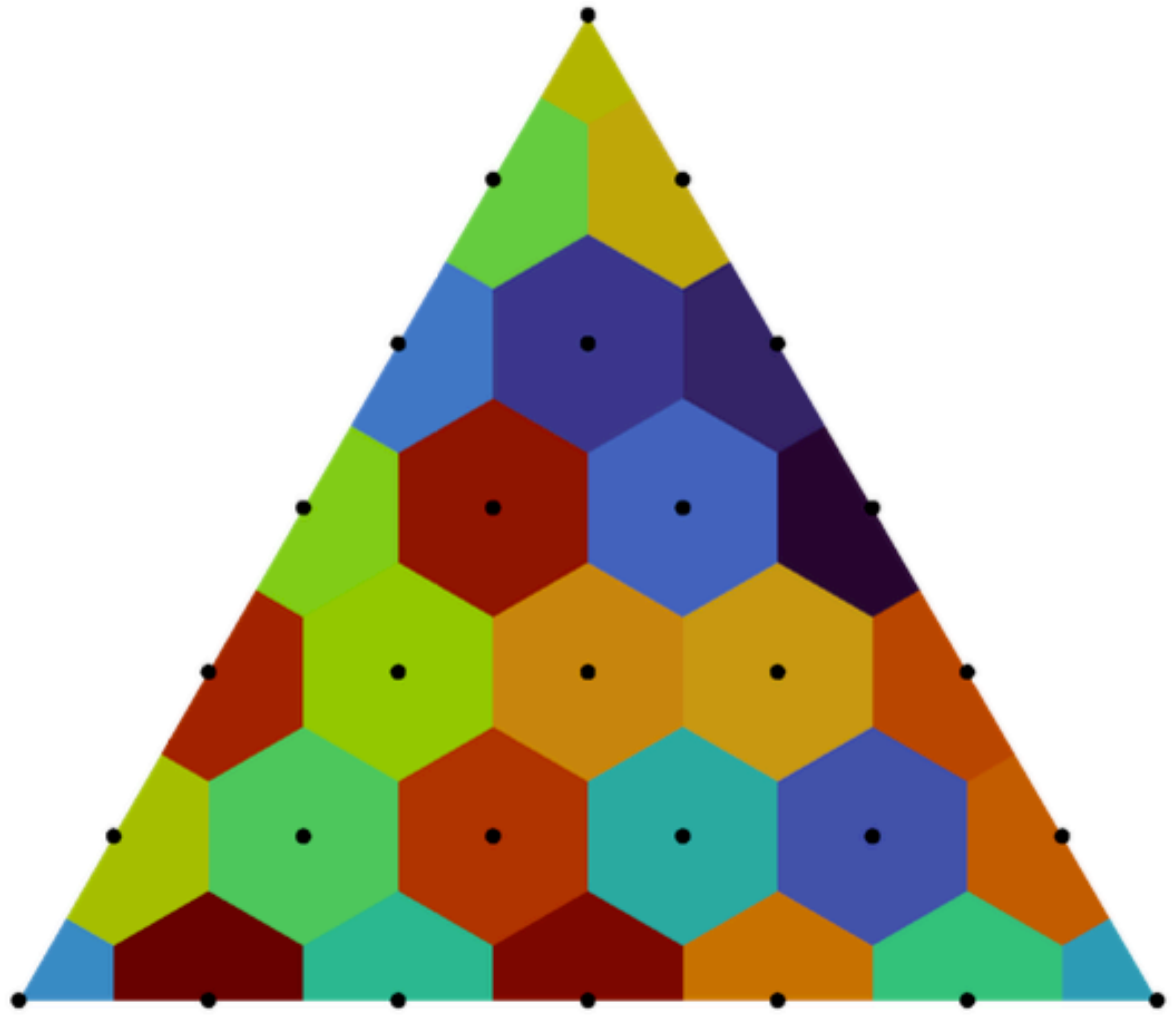
with Circuns-Duxans, Ruiz-Gironés, de la Puente

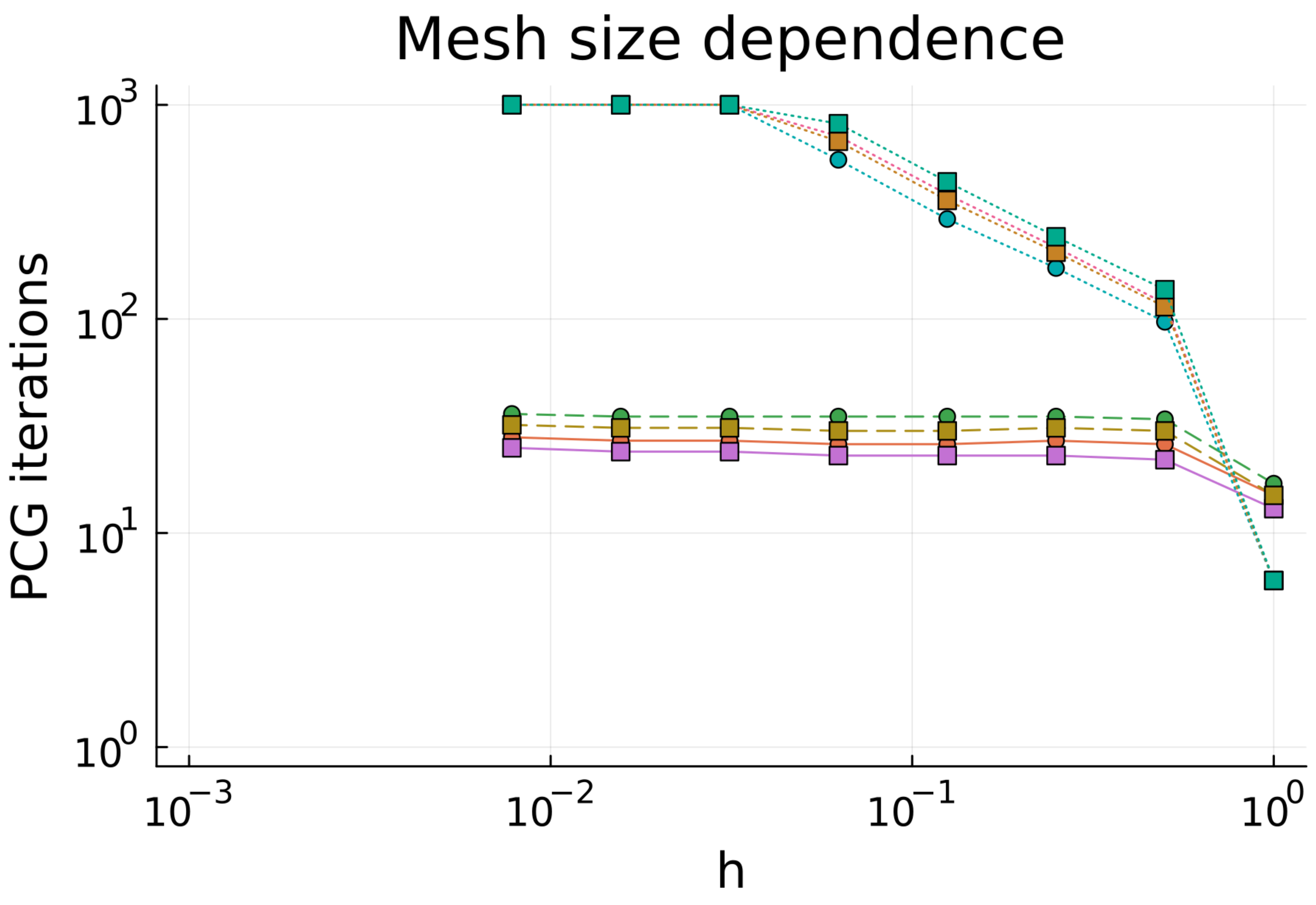
- *The discrete energy, and the electric and magnetic charge follow a conservation law*
- *The discrete versions of other conserved quantities remain no-drifting for long periods*
- Current steps, 3D results, and dissipation, dispersion, and reversibility benchmarks
- Next steps, use it in our electro-magnetic observability applications

Concluding remarks

Concluding remarks: preliminary results on numerics (1/2)

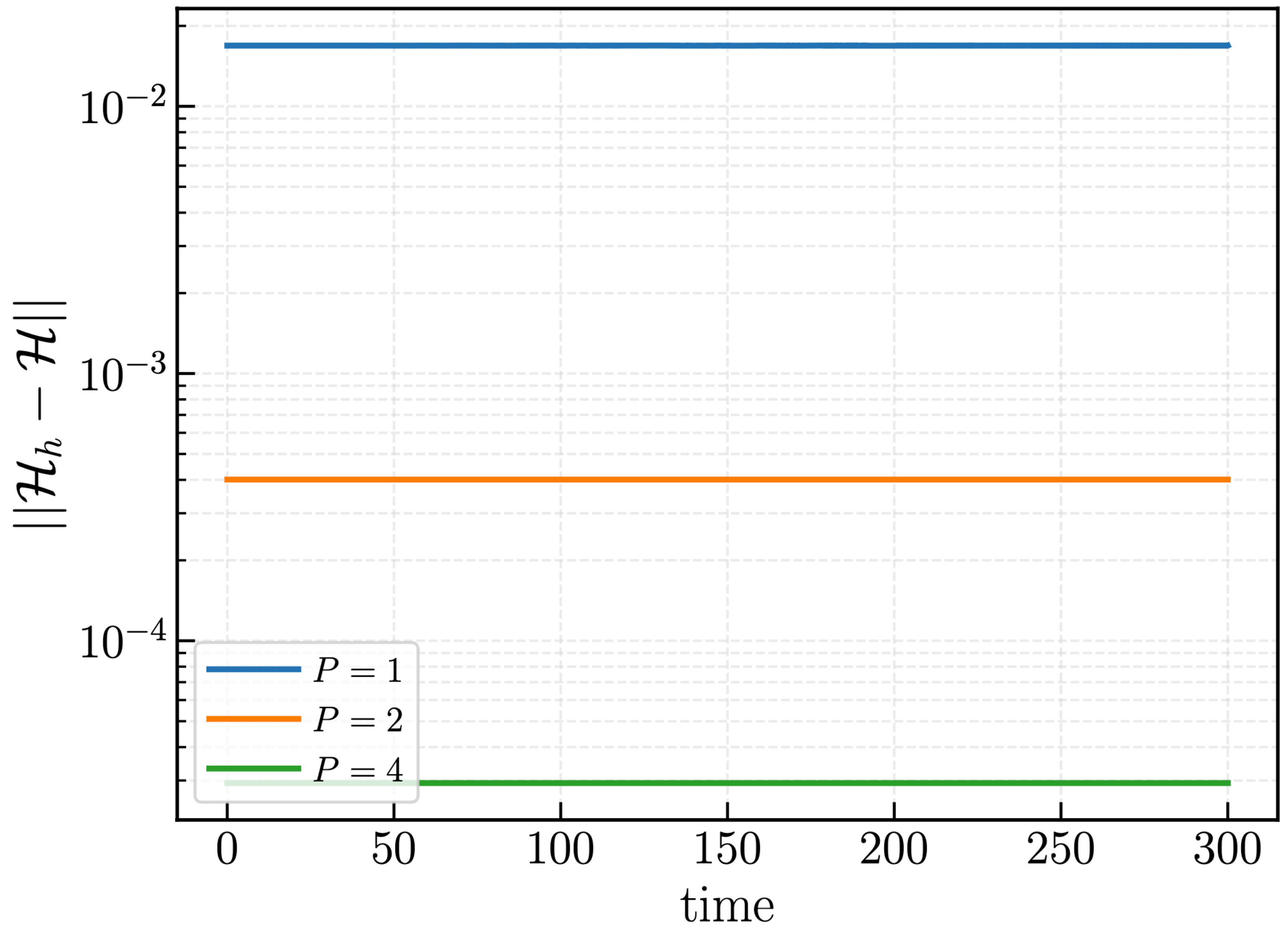
- Low-order preconditioning for DG on simplices with symmetric points
  - Considering dual sub-polytopal meshes on a master equilateral element
  - The number of iterations remains constant under h-refinement, and
  - It mildly grows under p-refinement

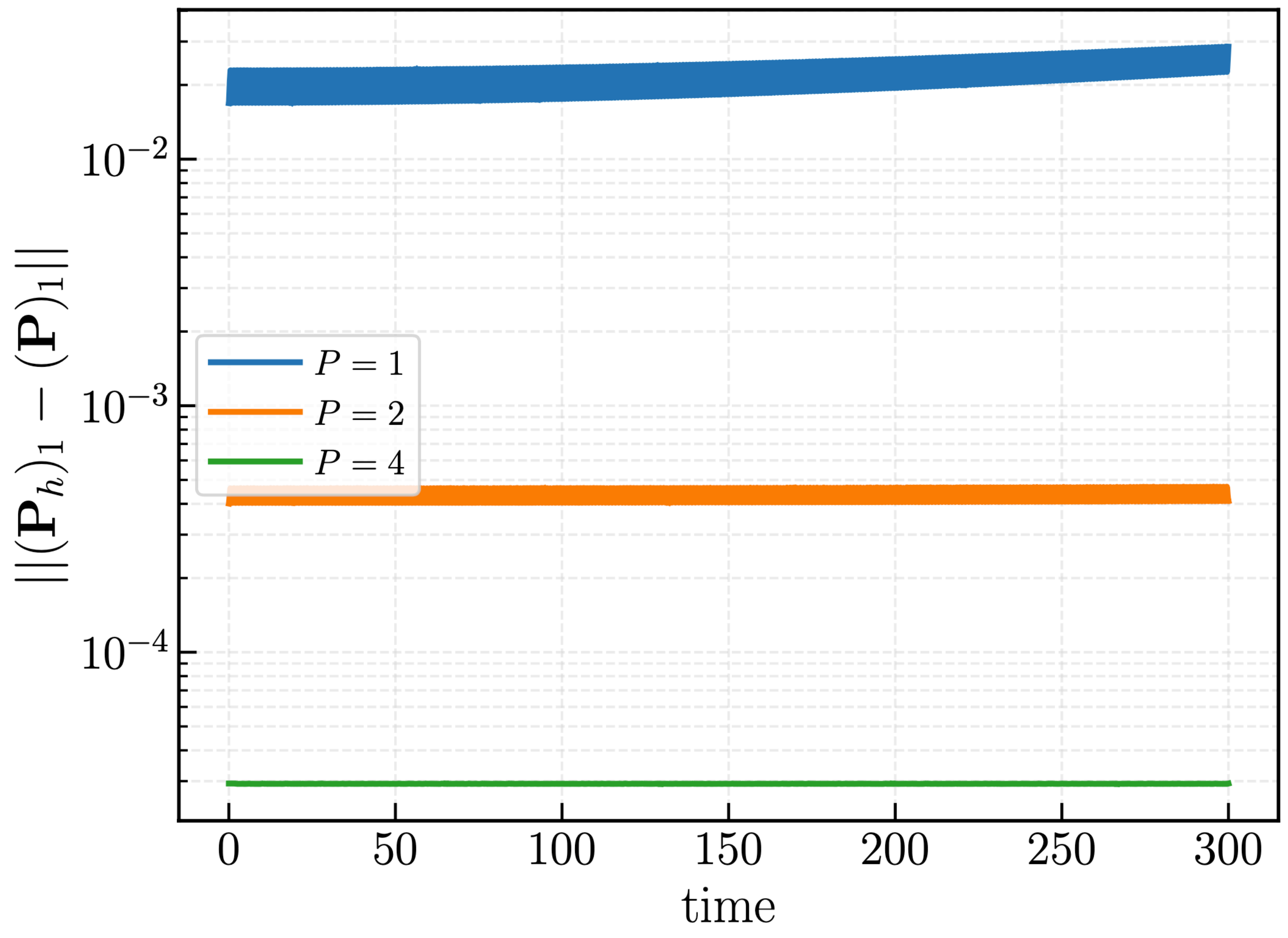




Concluding remarks: preliminary results on physics (2/2)

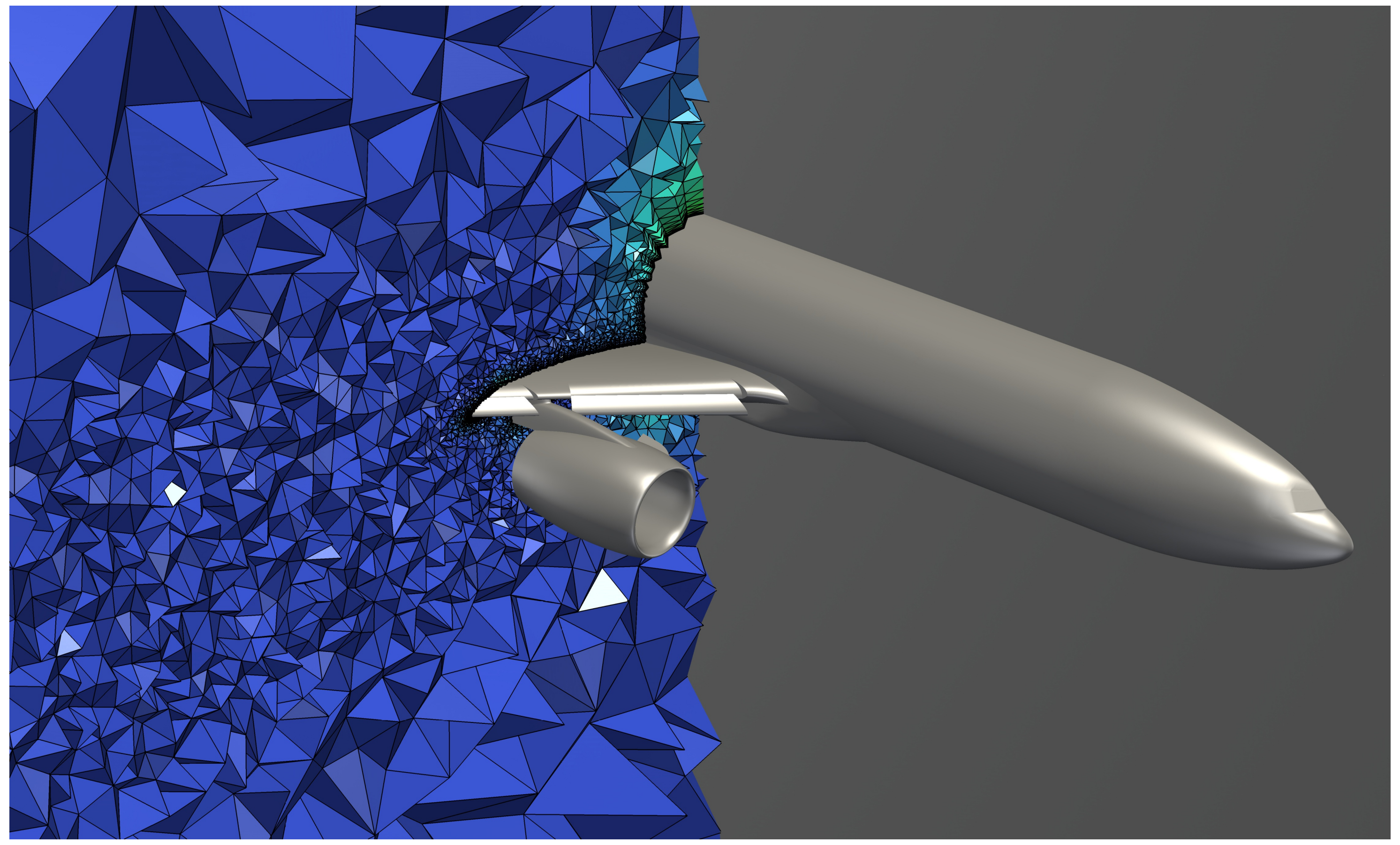
- A explicit dG method preserving the Poisson structure for electro-magnetics
  - Closer to a semi-discrete Poisson system, yet implementable as a standard DG
  - Energy, and electric and magnetic charge follow a conservation law
  - Other conserved quantities remain remarkably no-drifting



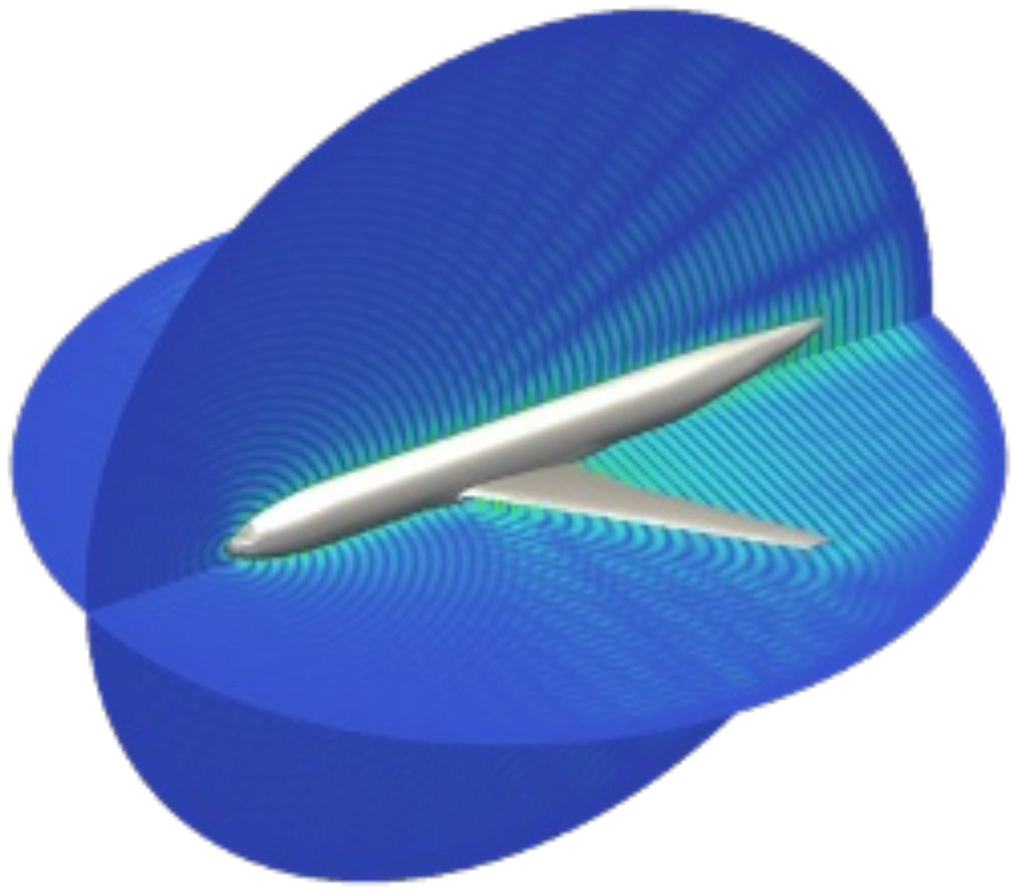


Concluding remarks: high-fidelity analysis (2/2)

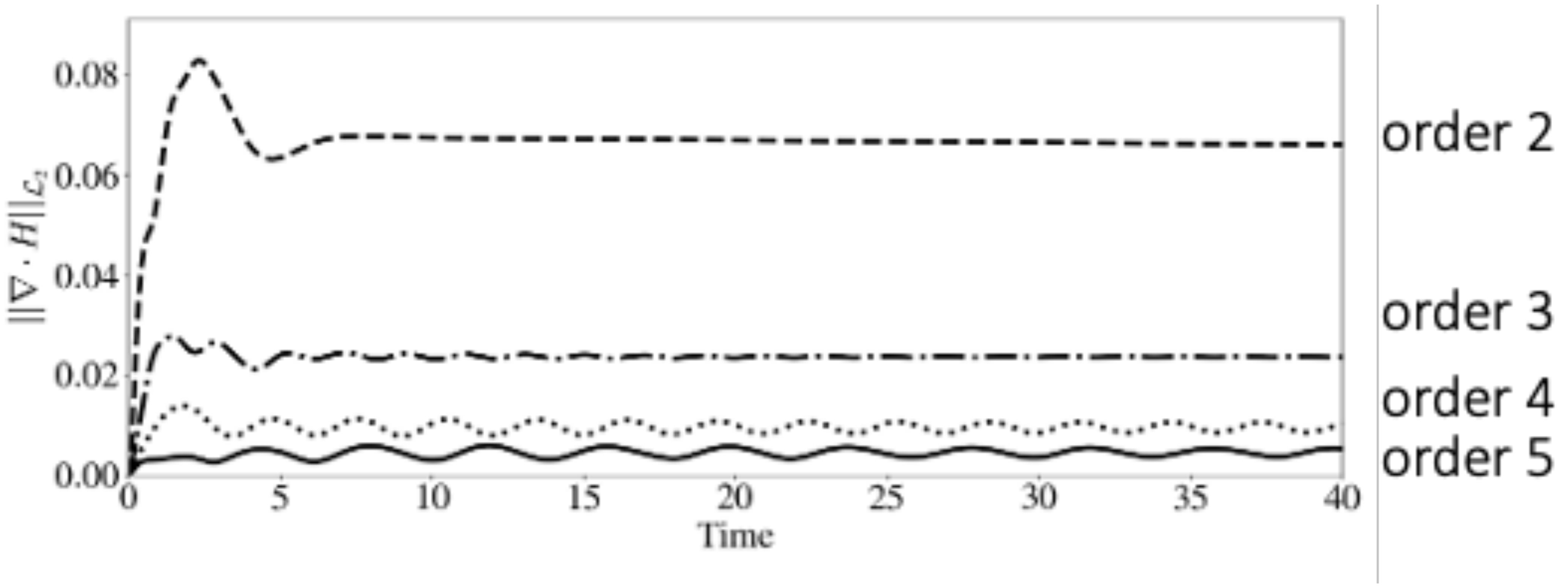
- We devise computational methods to enable high-fidelity analysis of aviation designs
- We are combining geometrical, numerical & physical aspects of high-order methods
- We want that our methods help to analyze future aviation designs



Geometric accuracy for **NASA/AIAA** and **Large-Scale CFD:** curved meshing (with Muela, Lehmkuhl, Ruiz)



Numerical accuracy for **Dual Technologies:** high-fidelity observability (with Ruiz, Farnós, de la Puente)



Physical accuracy for **Wave Phenomena**: high-fidelity electromagnetics (with Circuns, Ruiz, de la Puente)

Thank you

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