# The Development of the EM RF-Edge Interactions Mini-app "Stix" Using MFEM

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MIT PSFC Plasma Science and Fusion Center

RF-SciDAC

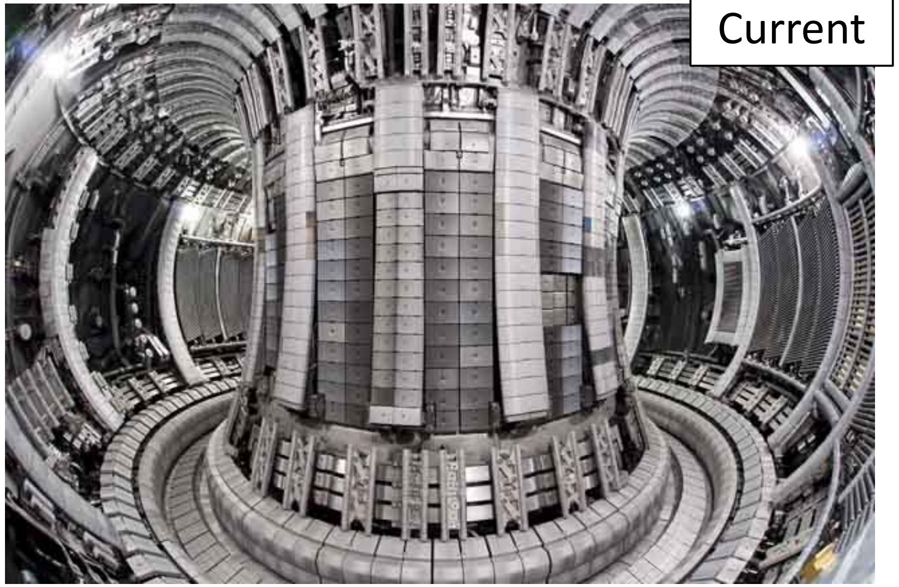
# Ion cyclotron radio frequency (ICRF) power is important in heating in fusion devices

Tokamaks need external heating to reach fusion relevant temperatures

- RF waves are common method to use
- Advantages of the ICRF regime (~ 20 – 100 MHz) include:
  - Efficient
  - Cost effective
  - Well established technology

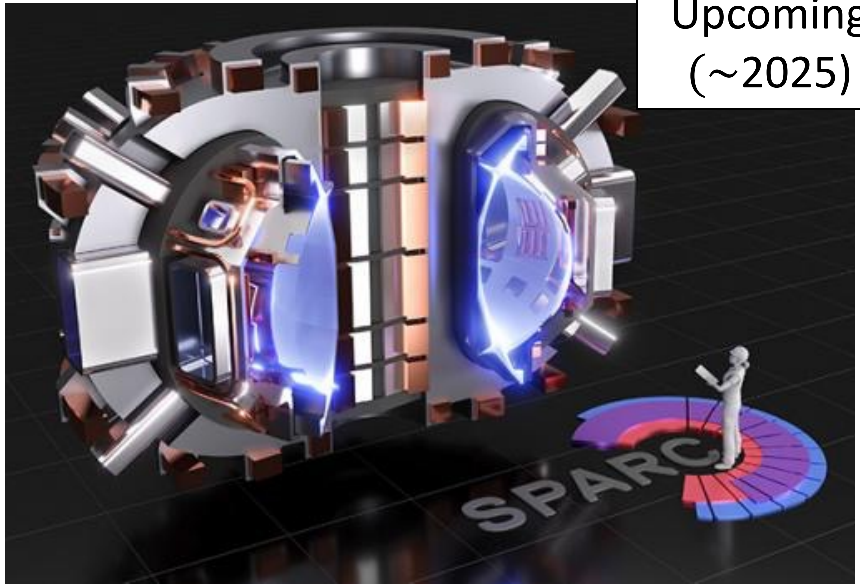
Current and future machines use significant ICRF heating

Current



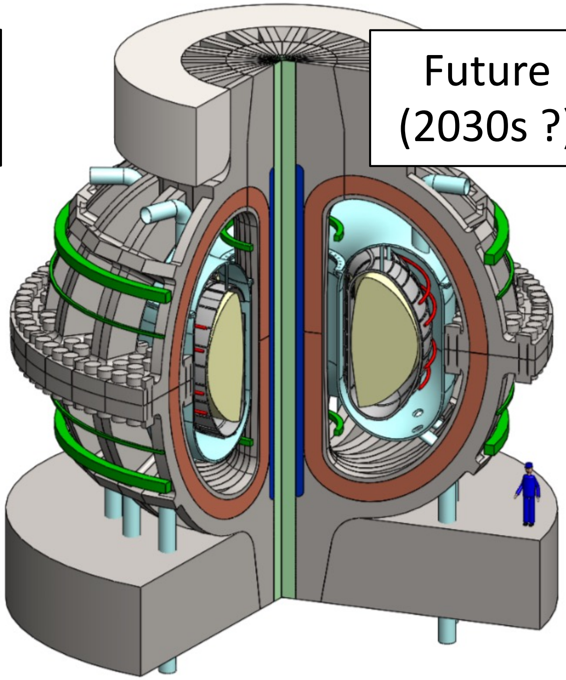
JET ~ 6 MW

Upcoming (~2025)




SPARC ~ 25 MW

Future (2030s ?)

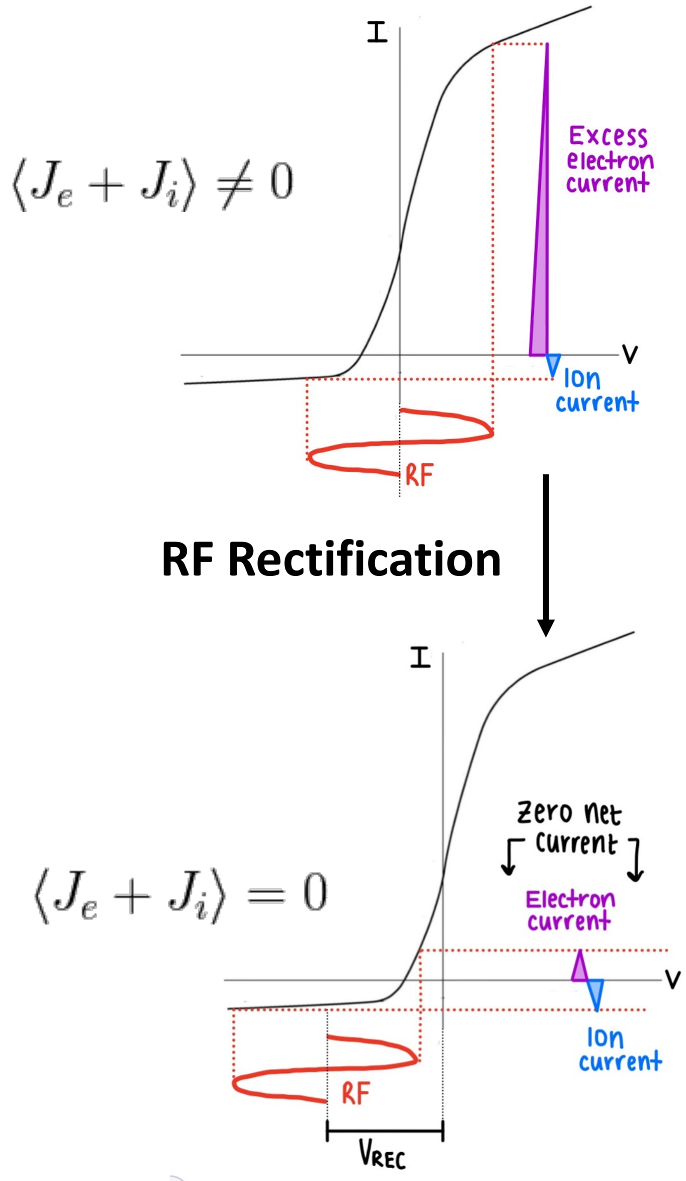


ARC ~ 13.6 MW

[B.N. Sorbom et al. 2015]

# ICRF power produces large rectified potentials that influence adverse effects

- **Problem:** ICRF power produces rectified sheath potentials ($\sim 100$s of Volts) that damage PFCs due to impurity generation and hot spots
- Path to longer pulsed fusion makes these effects increasingly important to predict and model
- Numerically modeling rectified sheaths will help to understand:
  - **Where** they form
  - What factors **influence** larger rectification
  - Ways to engineer methods of **mitigation** of sputtering and hot spots



# Motivation for "Stix" Mini-app

- Need a RF simulation model that incorporates physics of RF sheath in global RF code
  - Because $\lambda_{ICRF}$ >> $\Delta$ (sheath thickness), sheath can be approximated as a BC
    (~ cm) (~ mm)
  - In the form of J. Myra et al. 2015:

$$E_t = \nabla_t (V_{sh}) = \nabla_t \left( \frac{\omega}{i} D_n z_{sh} \right)$$

[J. Myra et al. PoP 2015]

- Want a robust code framework to test different solvers and have the possibility of an **integrated model via impurity generation (RustBCA) and transport (MAPS) codes**
  - Result: cold plasma finite element RF solver - Stix1D and Stix2D mini-app built off the MFEM library

# Physics Involved: the RF sheath BC

- The J. Myra et al. 2015 RF sheath boundary condition is given as:

Oscillating RF sheath potential (AC) → $V_{sh}$

$$E_t = \nabla_t \left( V_{sh} \right) = \nabla_t \left( \frac{\omega}{i} D_n z_{sh} \right)$$

[J. Myra et al. PoP 2015]

- This BC couples the macro-scale global RF code to the micro-scale sheath physics encapsulated in the complex sheath impedance, $z_{sh}$

- $z_{sh}$ is found through J. Myra 2017 parameterization code:
  - Inputs: $V_{sh},\ \omega_{pi},\ \Omega_i,\ \hat{b}_n$

$V_{sh}$: RF sheath potential

$\omega_{pi}$: Ion plasma frequency

$\Omega_i$: Ion cyclotron frequency

$\hat{b}_n$: Magnetic field angle

- **$V_{sh}$ is dependent on $D_n$ making this BC non-linear**

# What is the Stix mini-app?

- Originally adapted from an EM mini-app solver

Solves for the magnetic field, $\vec{H}$, in the frequency domain using H(Curl) basis functions:

$$\nabla \times \vec{\vec{\varepsilon}}^{\,-1}(\nabla \times \vec{H}) - \omega^2 \mu_0 \vec{H} = \nabla \times \vec{\vec{\varepsilon}}^{\,-1} \vec{J}_{\text{ext}}$$

From $\vec{H}$, Stix computes: $\vec{E}$, $\vec{D}$, and $\vec{S}$ (Poynting flux)

- Uses a cold plasma dielectric tensor:
  - There is the option to include a temperature profile that adds artificial dissipation into $\bar{\bar{\varepsilon}} \rightarrow \frac{i\nu}{\omega}$
- Is a 3D code but is run in pseudo-1D (Stix1D) and pseudo-2D (Stix2D)
  - 1D and 2D meshes are extruded into y-z and z directions
  - With periodicity in the respective extruded directions: set $k_y$, $k_z$ in 1D and $k_z$ in 2D
- Non-linear sheath BC implemented in a fixed-point iteration
  - The Minimal Polynomial Extrapolation (MPE) technique was also developed
  - Shown to converge in less iterations to the sheath solution
- Currently solves using direct solvers: SuperLU and MUMPS

# Implementation of RF Sheath BC in Stix: $E_t = \nabla_t \left(V_{sh}\right) = \nabla_t \left(\frac{\omega}{i} D_n z_{sh}\right)$

- Idea is to have the non-linear sheath BC be solved at the same time as the H field:

  **Solved simultaneously are the**:

  1) Wave question

$$\sum_j \left\{ \int_\Omega \left(\nabla \times \vec{W}_i\right) \cdot \left(\vec{\vec{\varepsilon}}^{\,-1} \nabla \times \vec{W}_j\right) d\Omega - \omega^2 \mu_0 \int_\Omega \vec{W}_i \cdot \vec{W}_j \; d\Omega + \int_{\partial\Omega} \vec{W}_i \cdot \left(\hat{n} \times \underbrace{\vec{\vec{\varepsilon}}^{\,-1} \nabla \times \vec{W}_j}_{= E_t = \nabla_t (V_{sh})}\right) d\Gamma \right\} \vec{H}_j = -i\omega \int_\Omega \vec{W}_i \cdot \left(\nabla \times \vec{\vec{\varepsilon}}^{\,-1} \vec{J}_{\mathrm{ext}}\right) d\Omega$$

  **and**

  2) $V_{RF}$ potential

$$V_{sh} = -i\omega D_n z_{sh} = -i\omega \; \hat{n} \cdot \left(\frac{\nabla \times \vec{H}}{-i\omega}\right) z_{sh} = \hat{n} \cdot \left(\nabla \times \vec{H}\right) z_{sh}$$

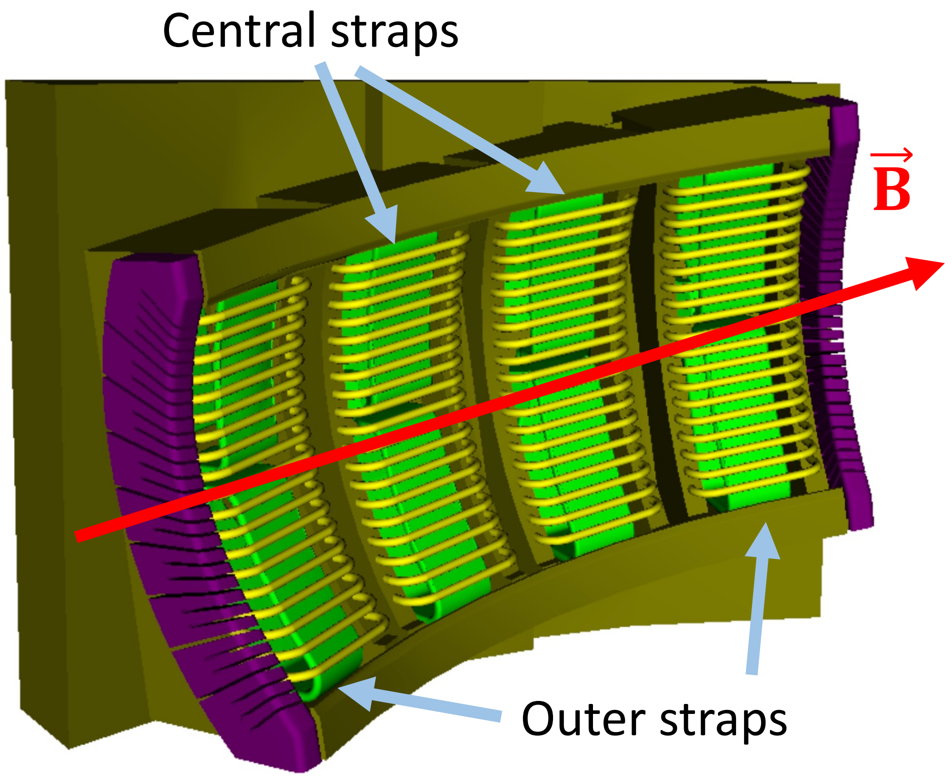
Found using J. Myra parameterization code [2]

- A block matrix is created using these equations
  - Using SuperLU the Schur complement is found for the wave equation block which is then used to solve the entire matrix iteratively using GMRES
- The code then iterates on the sheath impedance ($z_{sh}$) as a fixed-point iteration
- Once convergence criterion is met, currently set to $|\phi_n - \phi_{n+1}| < 10^{-3}$:
  - Code stops and writes out $V_{RF}$ and $V_{REC}$ (along with the EM fields)

# Bridging simulation with experiment: Alcator C-Mod power phasing study

- Looked at how to mitigate impurities by optimizing the ICRF antenna design
- Ratio of power of central straps to the total power of the 4 straps ($P_{cent}/P_{total}$) was varied from ~ 0 to 1
  - Phasing: $[0\ \pi\ 0\ \pi]$
- Impurities and plasma potential on/close to the antenna were measured
- Motivation to use an integrated model approach for simulations:
  - Stix (potentials) + RustBCA (impurity fluxes)

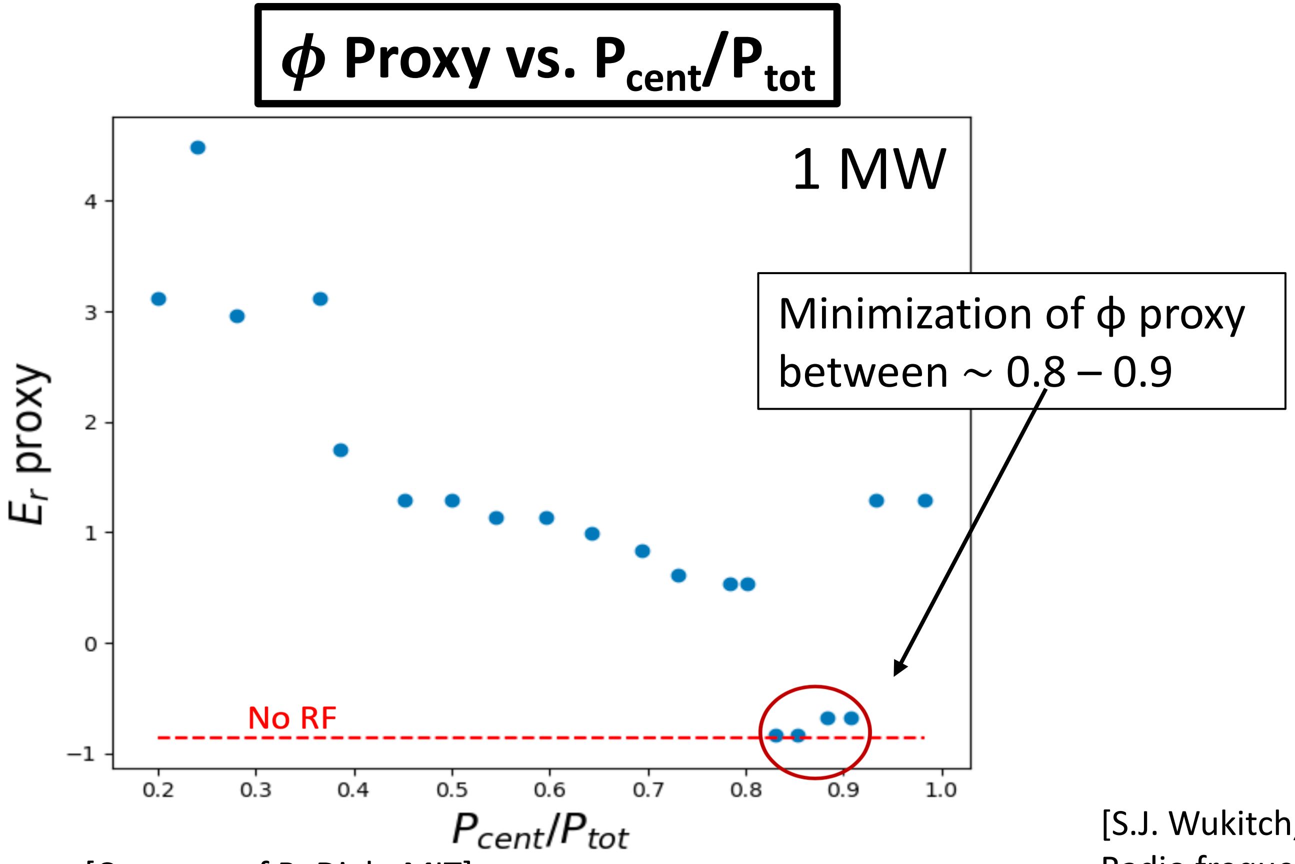
**C-Mod's field aligned 4-strap ICRF antenna**



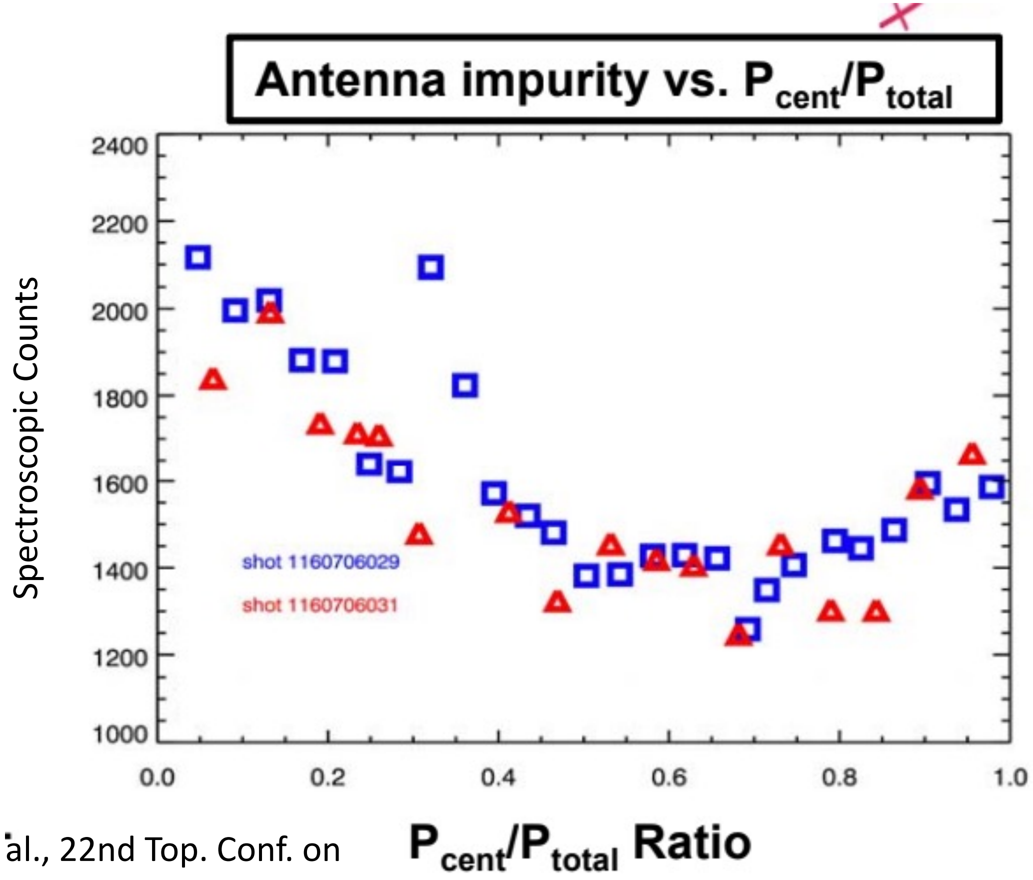

[Courtesy of Tom Jenkins, TechX]

# Experiment found minimization of both impurities and potentials

- Antenna impurities were minimized for $P_{cent}/P_{total} \sim$ 0.5 – 0.9 for 1 MW
- Sheath potential were minimized for $P_{cent}/P_{total} \sim$ 0.8 – 0.9 for 1 MW
- Behavior believed to be due to the **image current cancelation** on the antenna box



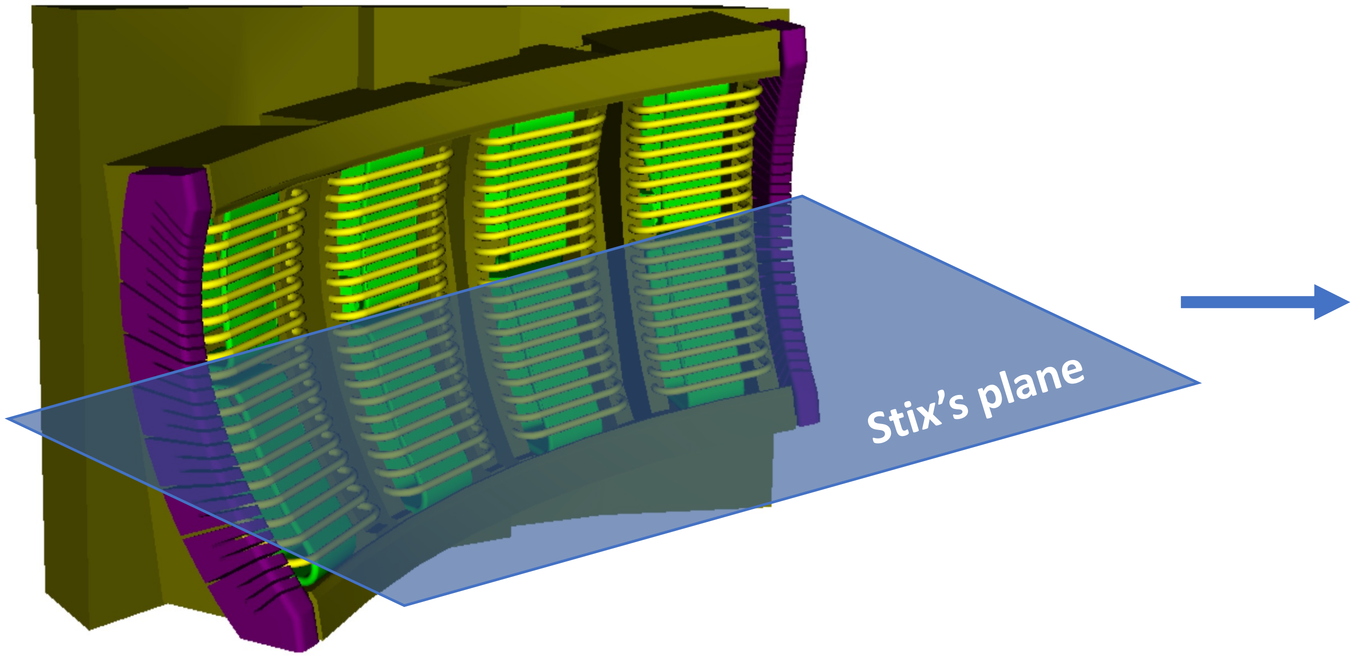

[Courtesy of R. Diab, MIT]




[S.J. Wukitch, et. al., 22nd Top. Conf. on Radio frequency Power in Plasmas (2017)]

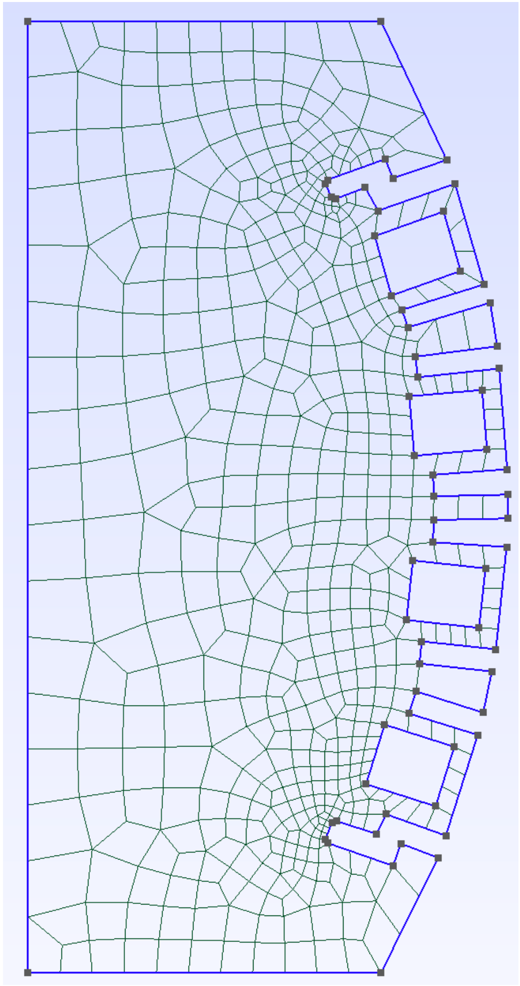
# How does this C-Mod case look like computationally in Stix?

3D View of Domain:




[Courtesy of Tom Jenkins, TechX]

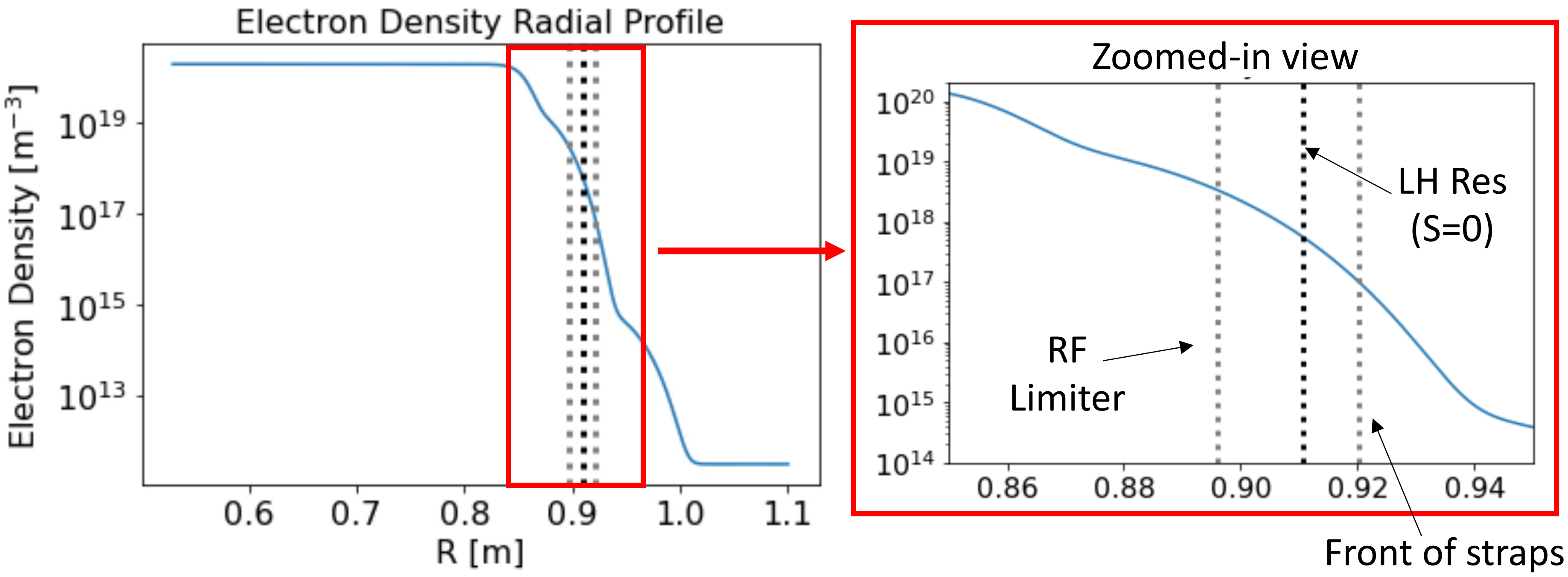
2D View of Domain: Stix's Mesh

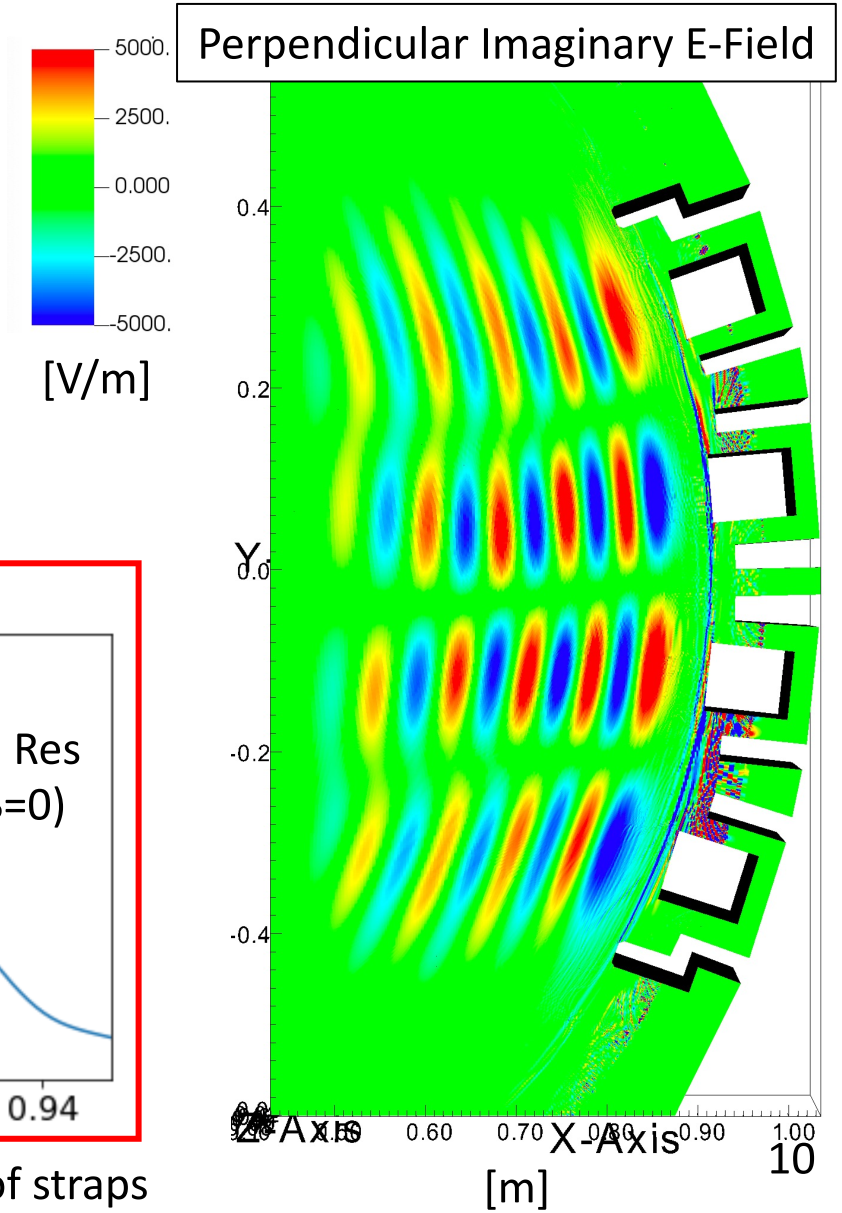


[mesh before serial + AMR refinement]

# Stix's simulation setup

- EQDSK magnetic field from experiment
- $[0\ \pi\ 0\ \pi]$ antenna phasing
- Sheath BC along RF limiter edges
- Artificial collisional profile to damp waves propagating into the core
- Pedestal-like density profile:
  - $n_{e,max} = 2\text{x}10^{20}\ m^{-3}$ and $n_{e,min} = 1\text{x}10^{11}\ m^{-3}$
  - Exciting a parasitic SW in front of the straps

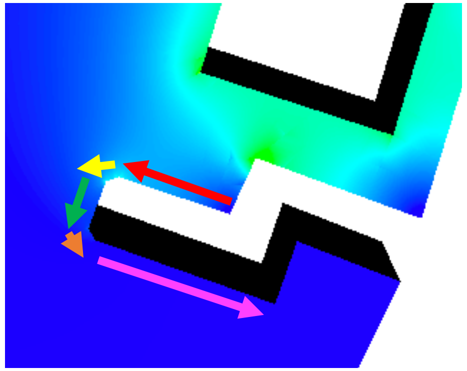


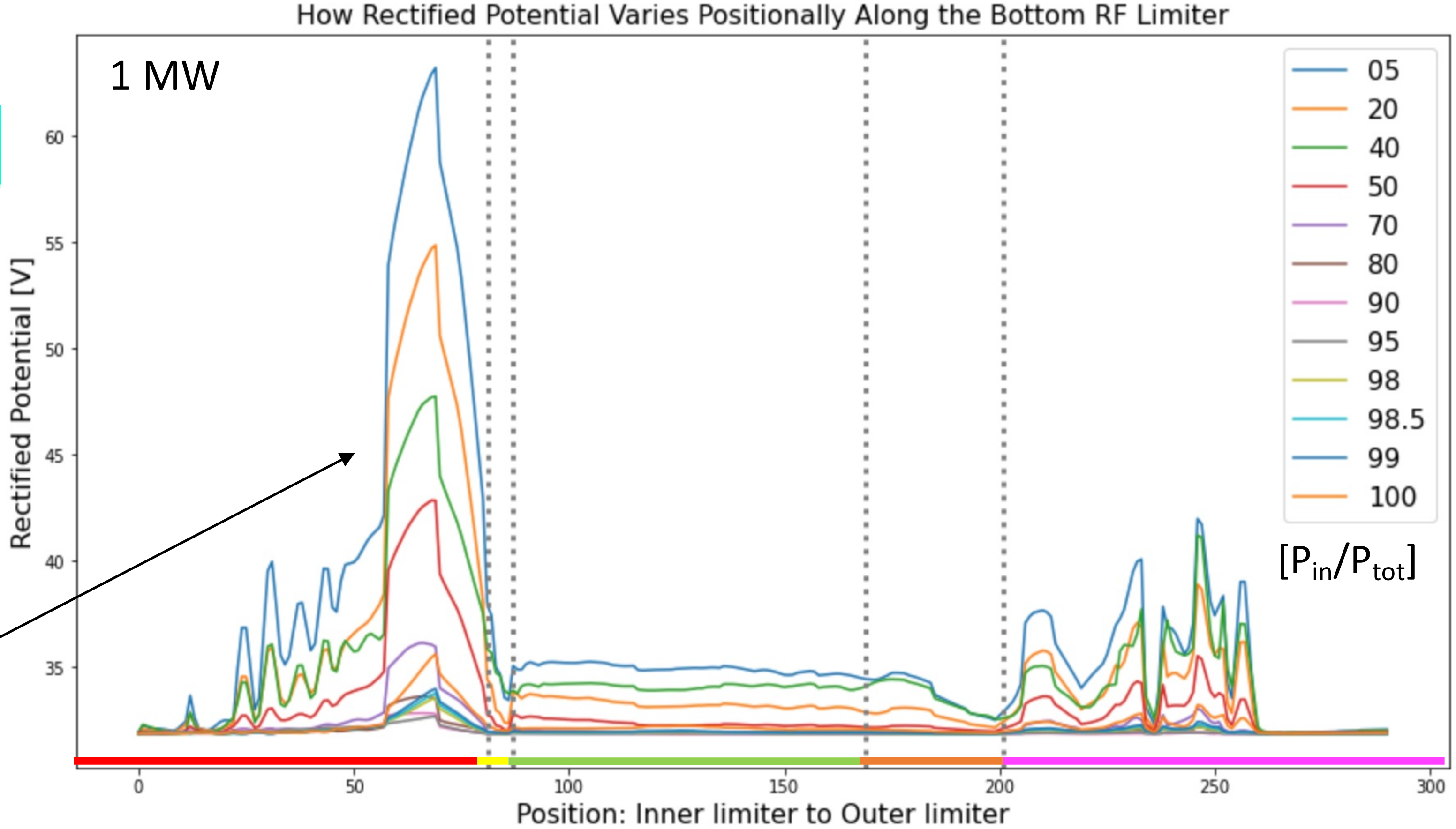
# Stix’s resulting rectified potentials

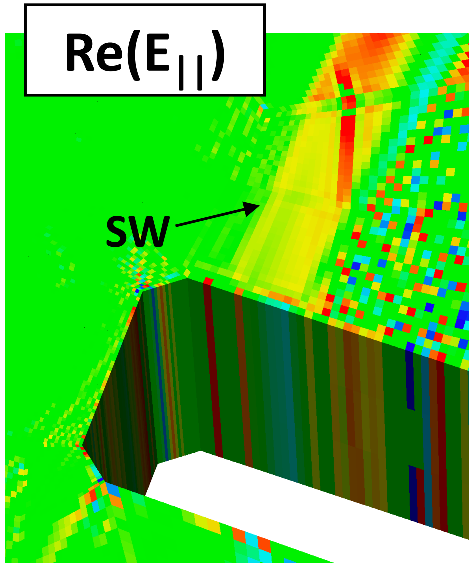
- Sheath BC was placed all along RF limiter
  - Found rectification only on inner facing limiter (red arrow)
- Rectification happened between densities of: $\sim 5.5\times10^{16} - 1.2\times10^{17}\ m^{-3}$
  - Below the lower hybrid (LH) resonance which occurs at $5.4\times10^{17}\ m^{-3}$
  - RF sheath caused by propagating slow wave (SW)

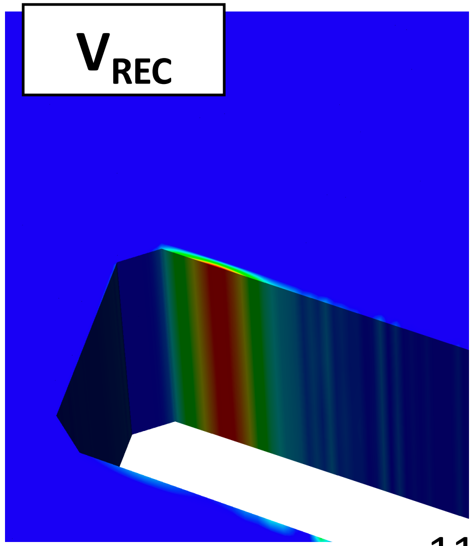
**2D View of RF limiter**



Largest rectification occurs on inner facing RF limiter closest to the plasma

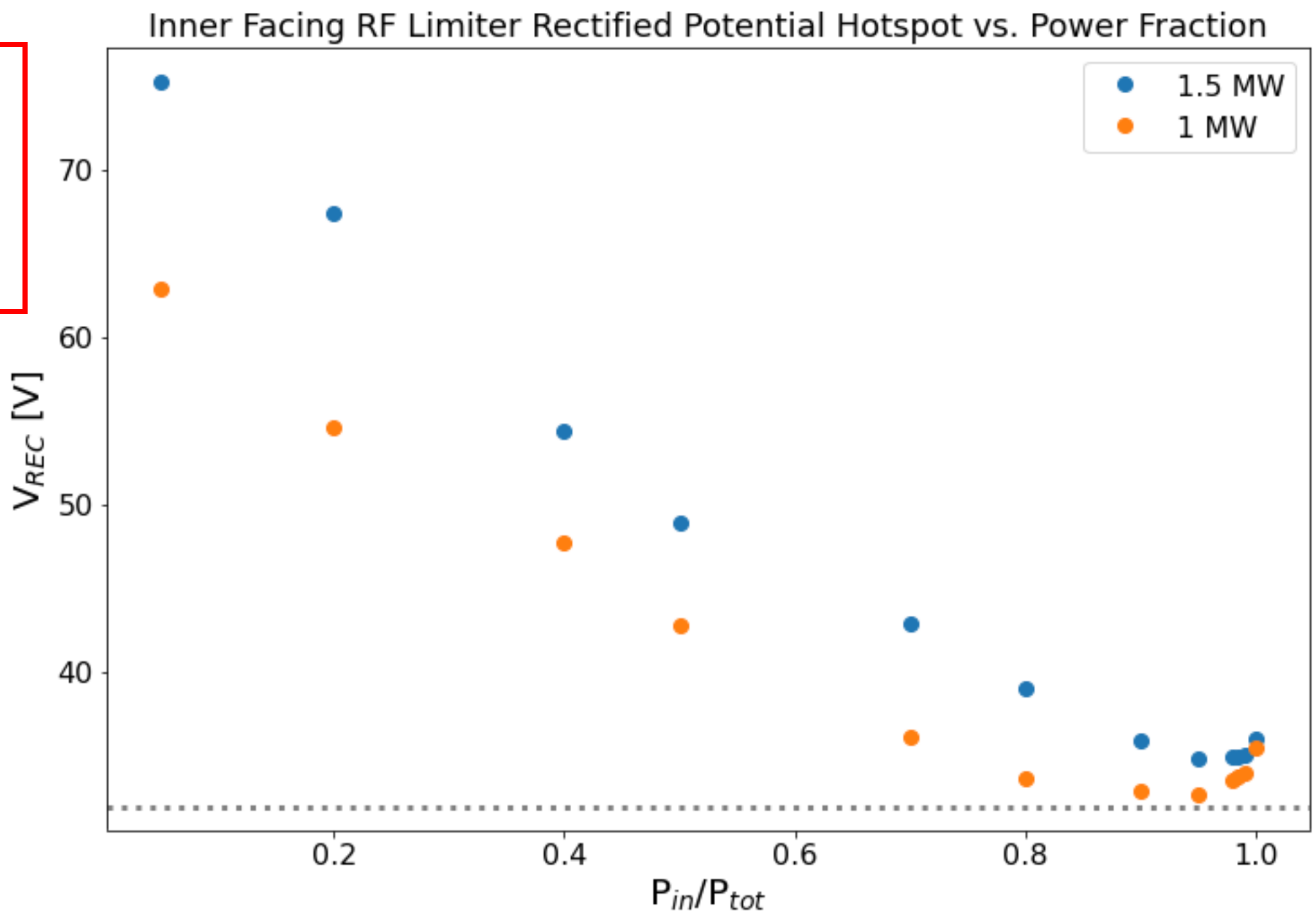


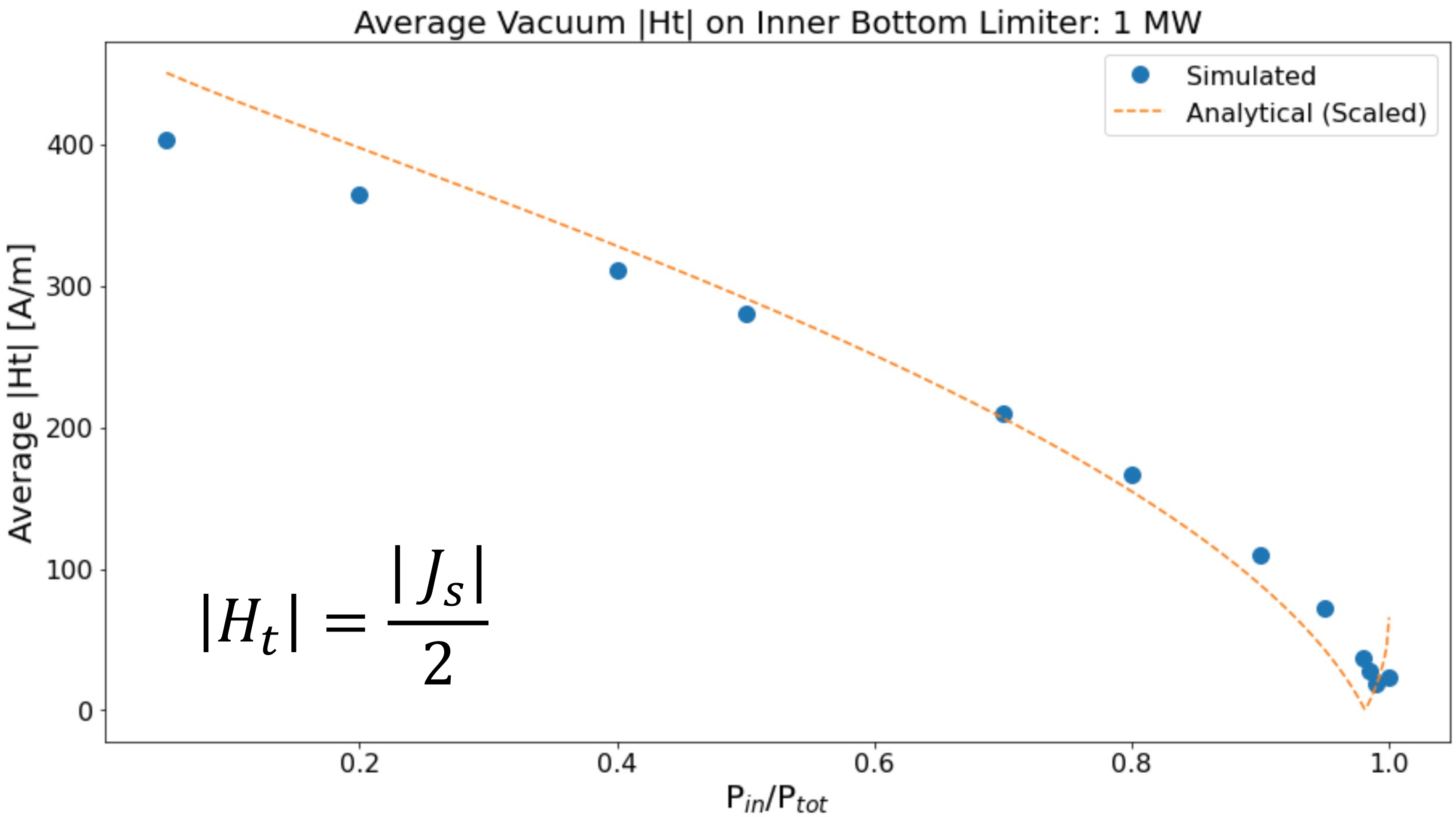



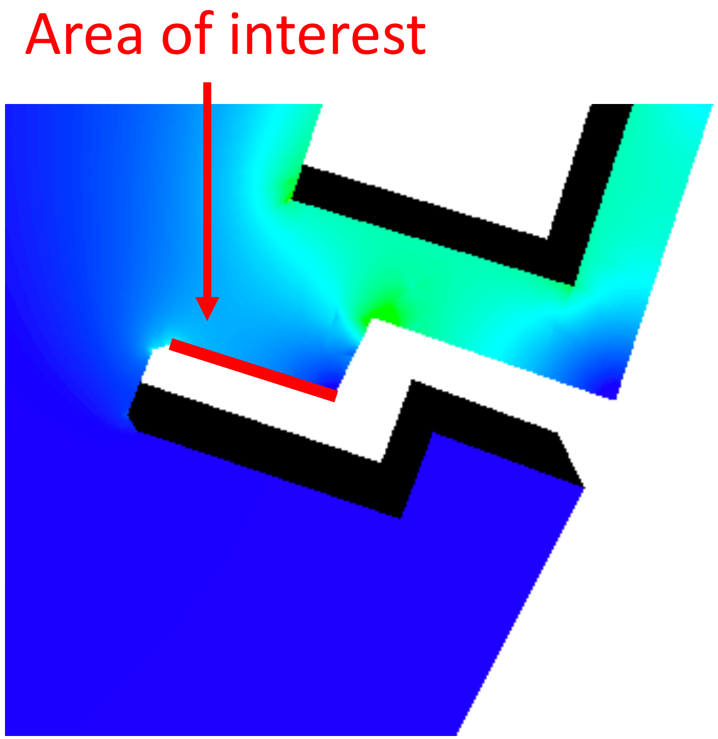
# Stix's rectified potentials vs. power phasing scan finds the same trend as in experiment

- Found $V_{REC}$ minimized between:
  - $P_{cent}/P_{tot}$ ~ 0.8 – 0.95 for 1 MW
  - $P_{cent}/P_{tot}$ ~ 0.9 – 1.0 for 1.5 MW
- Minimum is pushed to a higher power fraction for high antenna power
- 1.5 MW doesn't reach Bohm sheath at minimum like 1 MW does
- Edge profiles are kept constant for both power scans



# Vacuum power scanning shows minimum surface current at higher fraction $P_{cent}/P_{tot}$ at 0.99 on inner RF limiter




This suggests that the slow wave (SW) shifts the image current cancelation to a lower power fraction from 0.99 to ~ 0.8 – 0.95 for 1 MW

# Summary

- A new cold-plasma finite element RF solver called "Stix" has been developed which couples a RF sheath BC to a global wave solve

- An experimental power-phasing study done with the 4-strap C-Mod antenna showed minimization of the rectified potentials at $P_{cent}/P_{tot} \sim 0.8 - 0.9$ for 1 MW
  - Antenna impurities were similarly minimized but for a broader range of $P_{cent}/P_{tot} \sim 0.5 - 0.9$

- 2D slice of the 4-strap C-Mod antenna was simulated using Stix to find the rectified potentials on the nearby RF limiters scanning $P_{cent}/P_{tot}$ from 0.05 to 1.0

# The key takeaways:

- Stix showed:
  - A propagating SW was the source of the rectification in the simulation
  - **The same trend of minimization of the rectified potentials at $P_{cent}/P_{tot} \sim 0.8 - 0.95$ for 1 MW**
- Vacuum scans of the same domain show a minimization of the image currents at $P_{cent}/P_{tot} \sim 0.99$
  - **Suggests that the SW pushes the image current cancelation to a lower power fraction**

## Future work:

- Integrated model:
  - Use Stix's $V_{REC}$ as an input to an impurity flux code like RustBCA
- 3D simulation of antenna:
  - Preliminarily 3D simulations of ICRF multi-strap antennas suggest that there is a lot of variation of the rectified potential poloidally along the RF limiter
    - Strongest near the corners of the antenna box: shown in vacuum scans in COMSOL via image current cancelation