MFEM Workshop 2023

Discontinuous Galerkin in the Time Domain for Maxwell's Equations

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What do we want to solve?

Hesthaven, J.S. and Warburton, T. (2008) Nodal Discontinuous Galerkin Methods/Algorithms, Analysis, and Applications

Maxwell's Equations

$$\mu\frac{\partial \boldsymbol{H}}{\partial t} = -\nabla\times\boldsymbol{E},\quad \varepsilon\frac{\partial \boldsymbol{E}}{\partial t} = \nabla\times\boldsymbol{H} \xRightarrow{\text{IPP x2}}{\text{Strong Form}} \begin{cases} \hat{\boldsymbol{n}}\cdot(\boldsymbol{F}_H - \boldsymbol{F}_H^*) = \dfrac{1}{2\{\{Y\}\}}\hat{\boldsymbol{n}}\times\left(Y^+[\boldsymbol{E}] + \alpha\hat{\boldsymbol{n}}\times[\boldsymbol{H}]\right) \\ \hat{\boldsymbol{n}}\cdot(\boldsymbol{F}_E - \boldsymbol{F}_E^*) = -\dfrac{1}{2\{\{Z\}\}}\hat{\boldsymbol{n}}\times\left(Z^+[\boldsymbol{H}] - \alpha\hat{\boldsymbol{n}}\times[\boldsymbol{E}]\right) \end{cases}$$

Jump terms

$$[q] = q^- - q^+$$

What do we need for our Multidimensional Maxwell explicit scheme solver in DGTD?

- ❏ Bilinear Integrators with customised jump terms for our Evolution TDO
- ❏ Total field & scattered field capabilities
- ❏ Designation of boundary conditions on true and interior(!) faces/edges
- ❏ Data extraction at specific faces/edges for post-processing purposes (RCS, Far-Field, ...)
- ❏ Spectral analysis for time step stability with high confidence
- ❏ And more...!

Briefly on Maxwell's Bilinear Integrators

We define our main equation operators

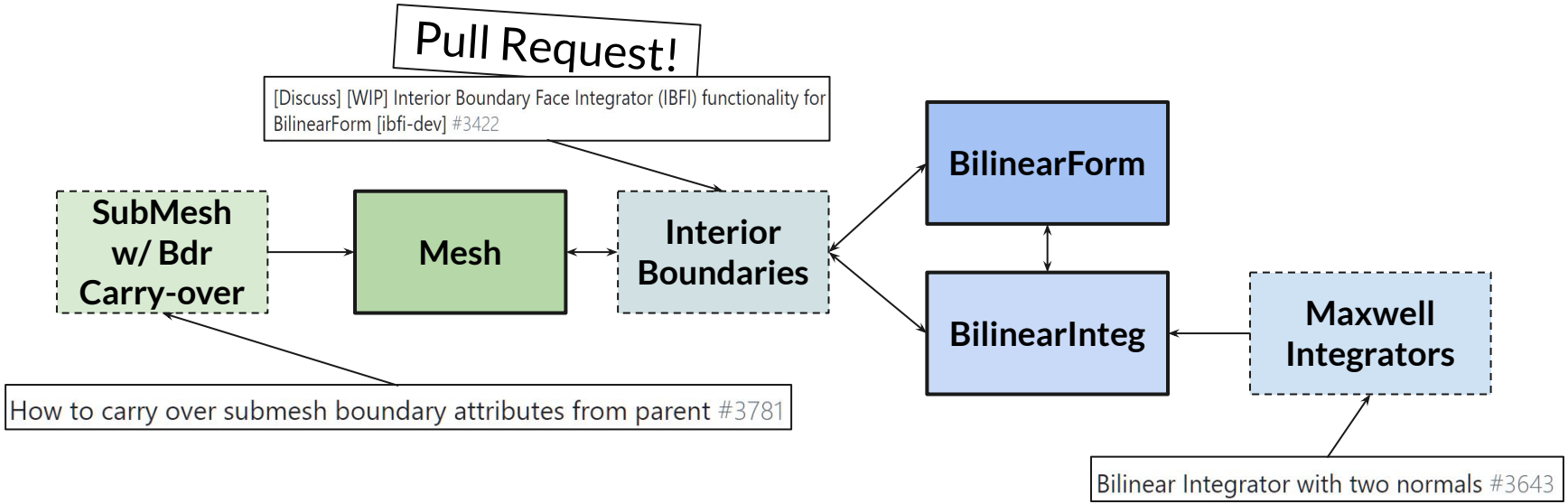
Discretized equation example

$$\frac{dE_h^x}{dt} = \mathcal{D}_y H_h^z - \mathcal{D}_z H_h^y + \frac{\mathcal{M}^{-1}}{2J} \int_{\partial \mathrm{D}^k} \left(\hat{n}_y [H_h^z] - \hat{n}_z [H_h^y] + \alpha \left([E_h^x] - (\hat{\boldsymbol{n}} \cdot [\boldsymbol{E}_h]) \hat{n}_x \right) \right) \ell(\boldsymbol{x})\, d\boldsymbol{x}$$

- Stiffness
- OneNormal: $\alpha \langle (\vec{u} \cdot n) \cdot [v], [w] \rangle$
- ZeroNormal: $\alpha \langle \vec{u} \cdot [v], [w] \rangle$
- TwoNormal: $\alpha \langle n_i (\vec{u} \cdot n) \cdot [v], [w] \rangle$

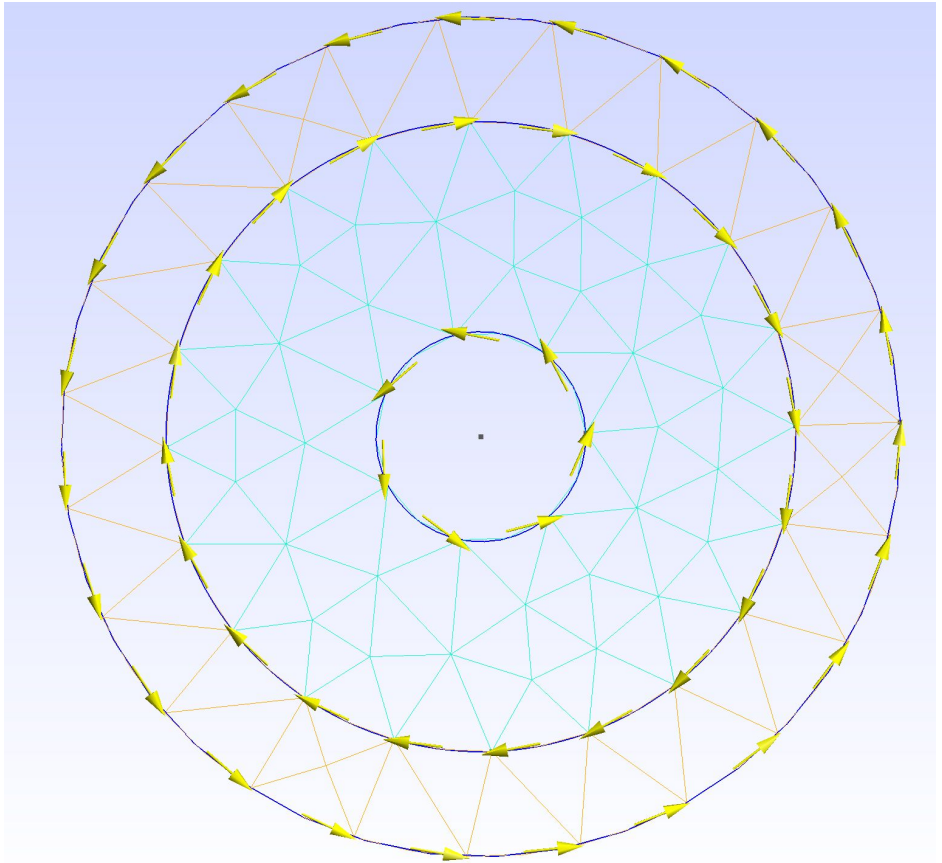
Upwind

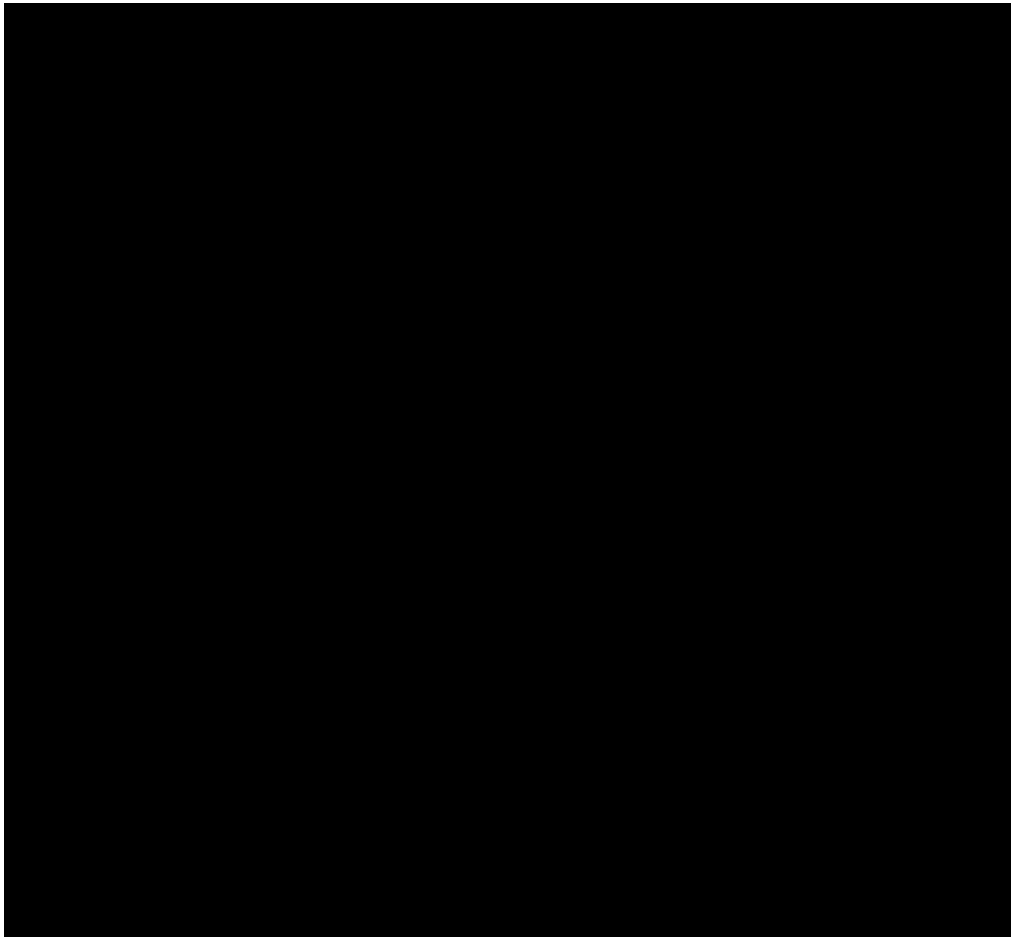
Interacting with you! Expanding MFEM's capabilities



Prepping the simulations

* -	Type	Number	Name
	Surface	1	Vacuum
	Curve	2	SMA
	Curve	3	PEC
	Curve	301	TFSF





Thank you very much!

https://github.com/OpenSEMBA/dgtd