

# Level-Set Topology Optimization with PDE Generated Conformal Meshes

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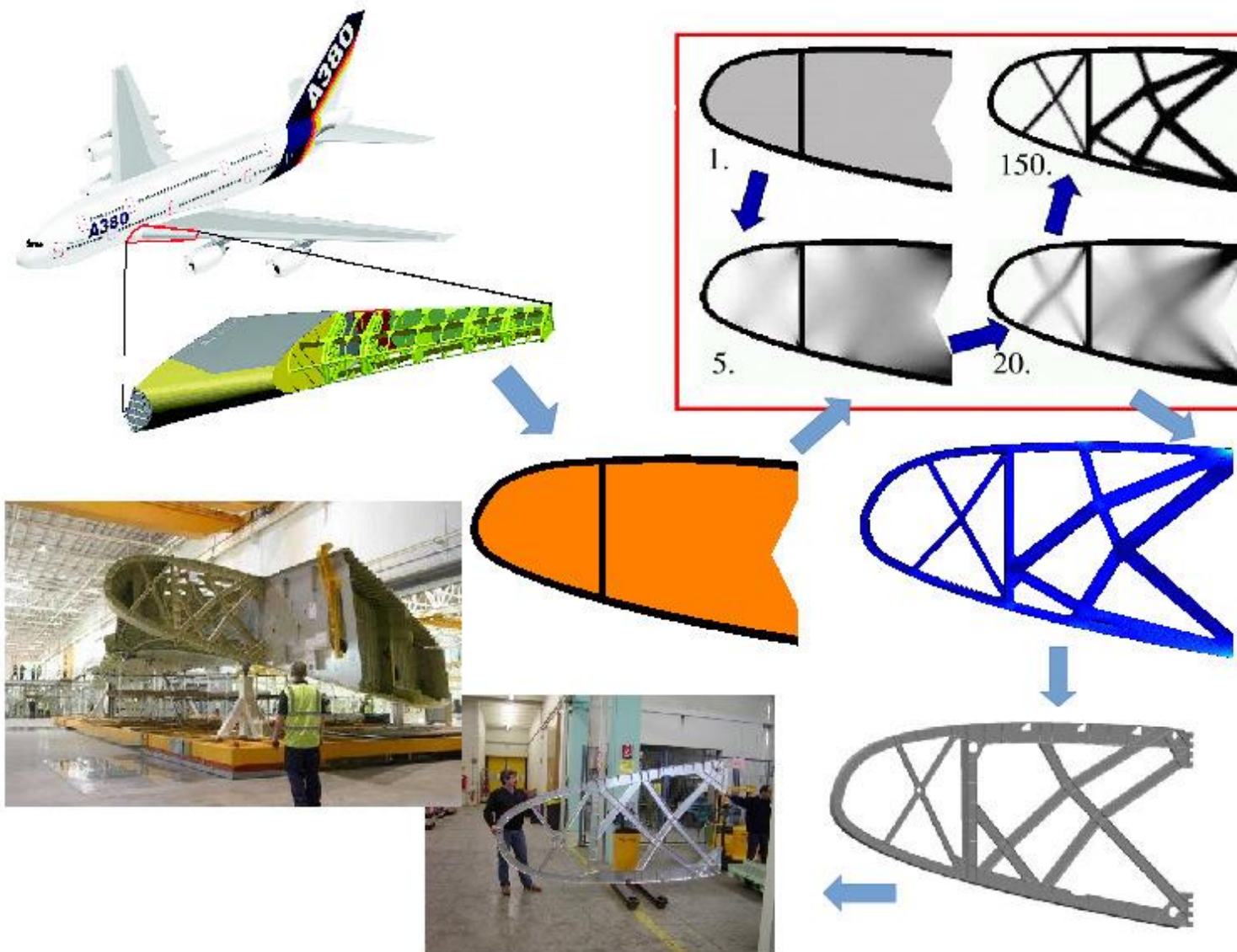
This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

# Motivation

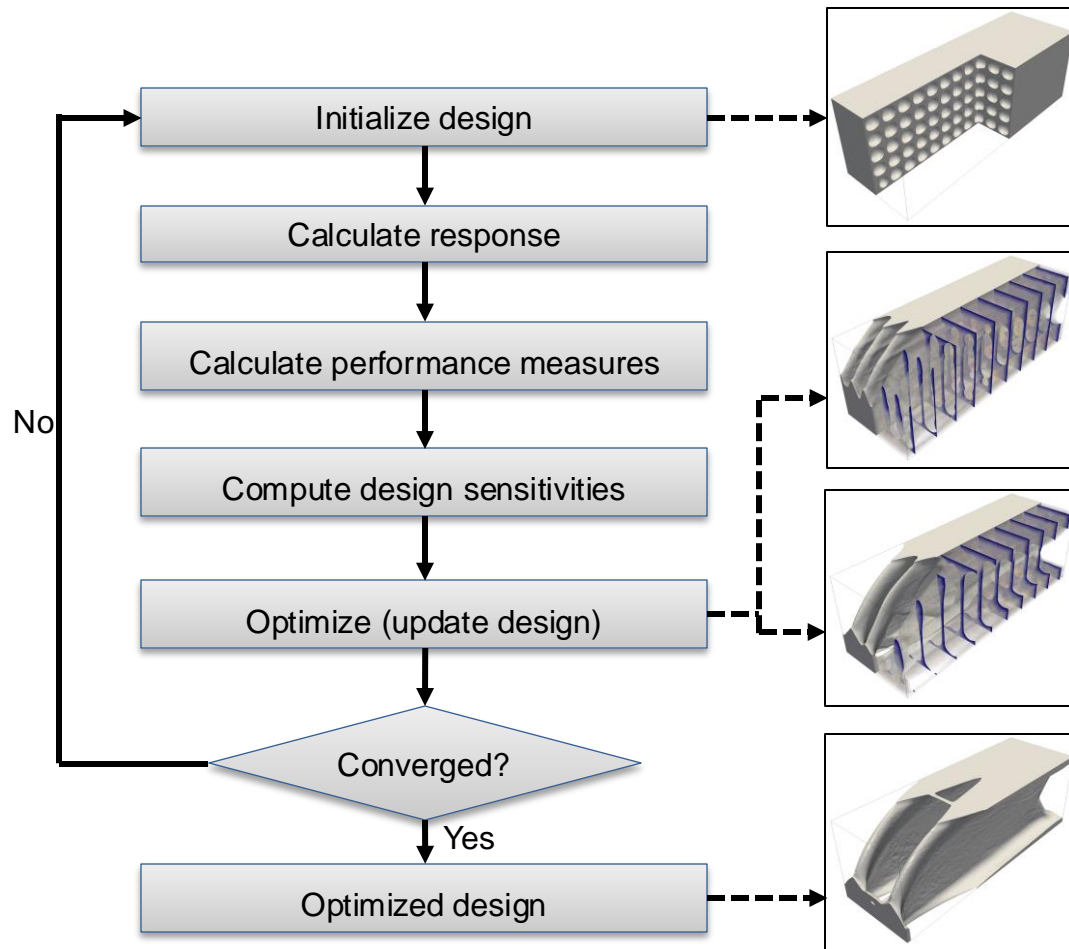




# Reduction in weight by 700kg per aircraft



## Optimization Process



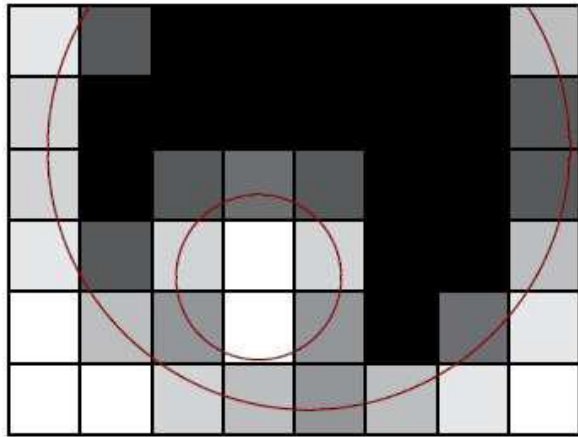
### ➤ Cost and constraint functions

$$\min_s z(s, u(s)) = w_1 F(s, u(s)) + w_2 P_{Reg}(s)$$
$$s.t.: \quad g_i(s, u(s)) \leq 0, i = 1, \dots, N_g$$

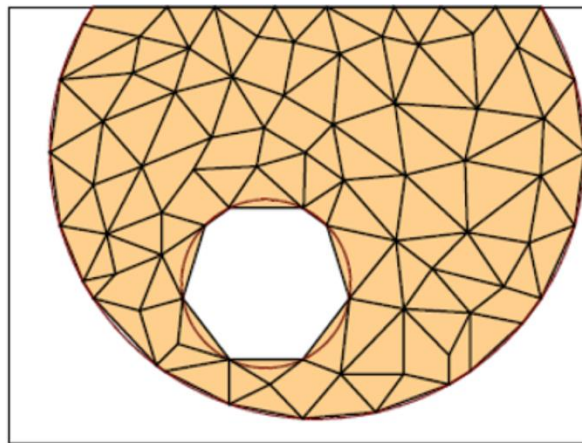
# Classic TO approaches have drawbacks

## Analysis models

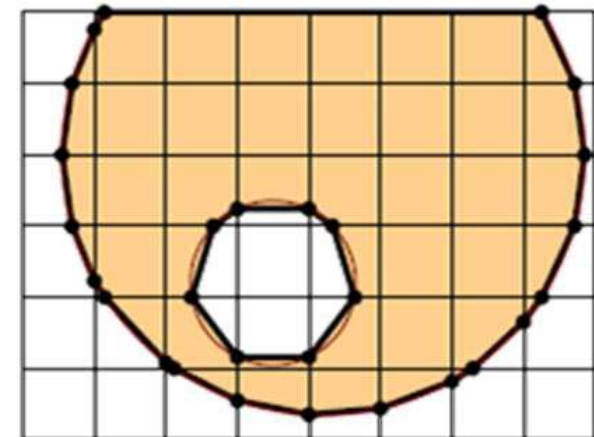
- Three types of analysis models



Density methods



Body fitted analysis mesh



Immersed analysis methods

# Level-set optimization uses isocontour to describe geometry

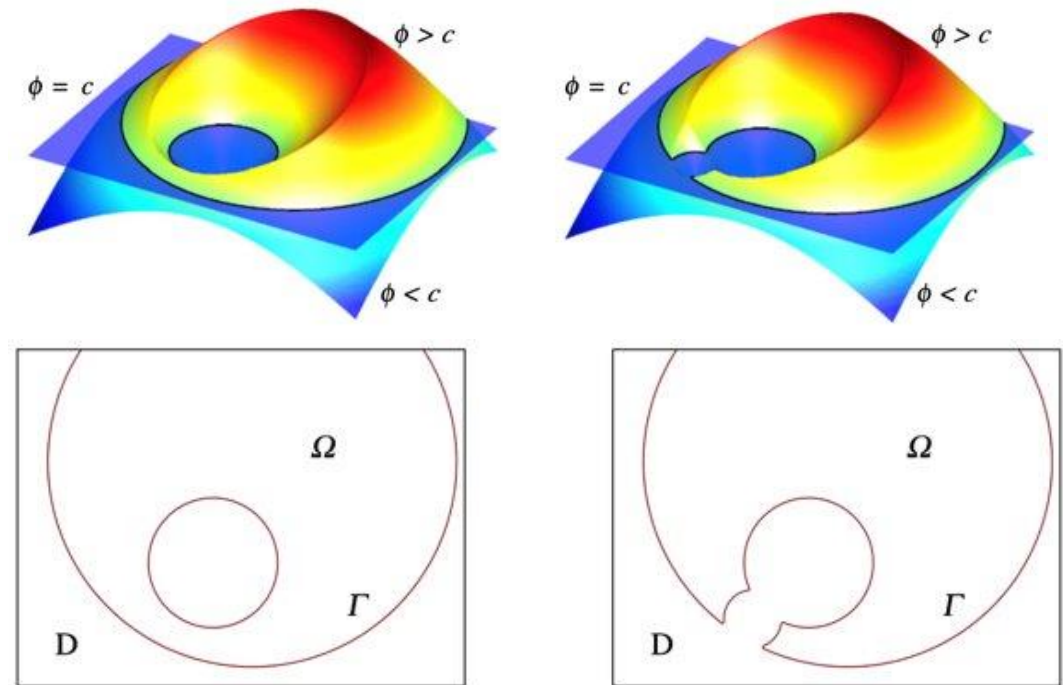
## Geometry definition

- Design variables employed as coefficients for level-set field discretization
- Geometry implicitly defined by isocontour  $\phi_t$ , of a level-set function  $\phi$

$$\phi(x) > \phi_t, \forall x \in \Omega_+$$

$$\phi(x) < \phi_t, \forall x \in \Omega_-$$

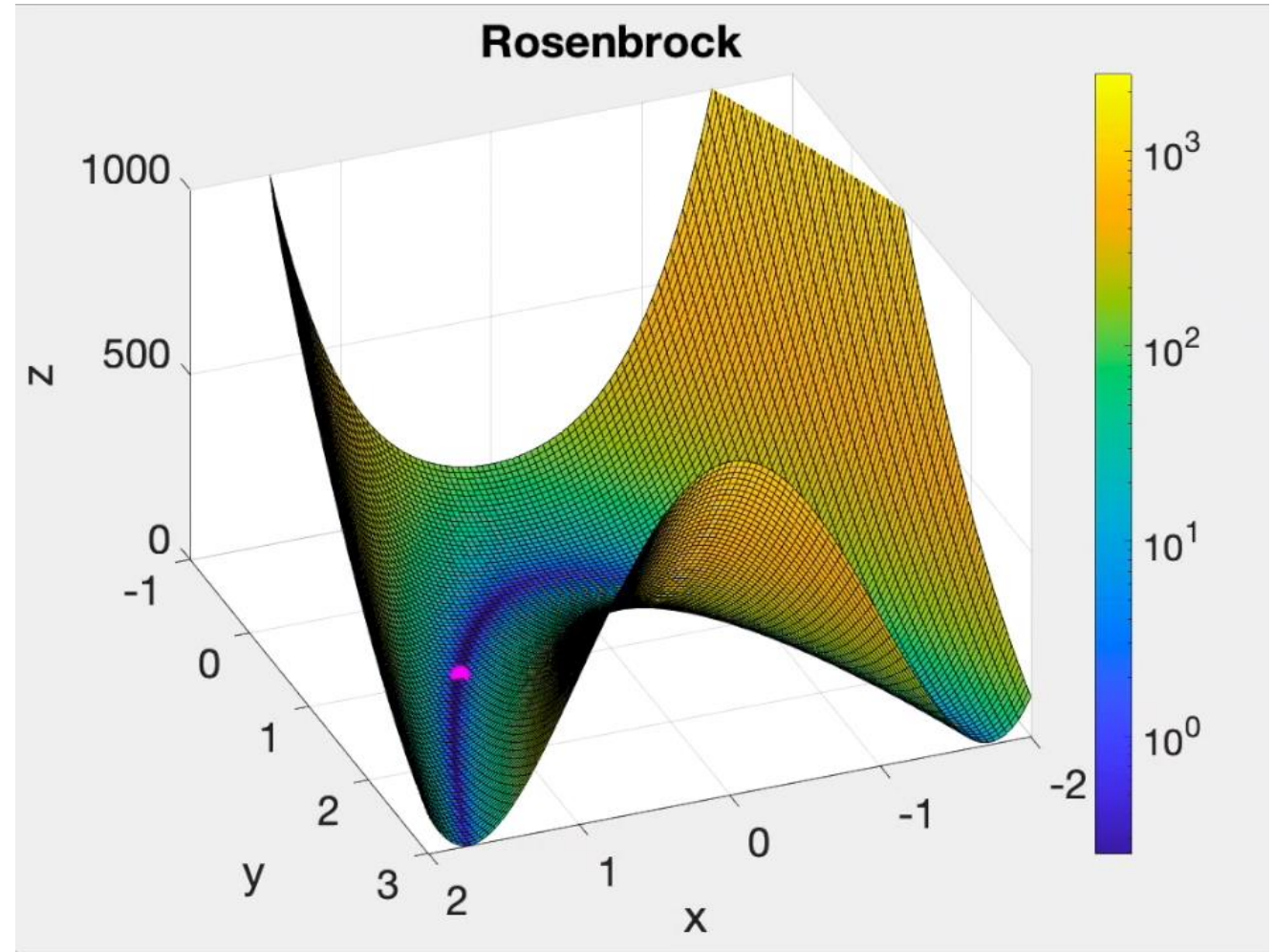
$$\phi(x) = \phi_t, \forall x \in \Gamma_{\pm}$$



Level-set function and resulting isocontour [Van Dijk 2012]

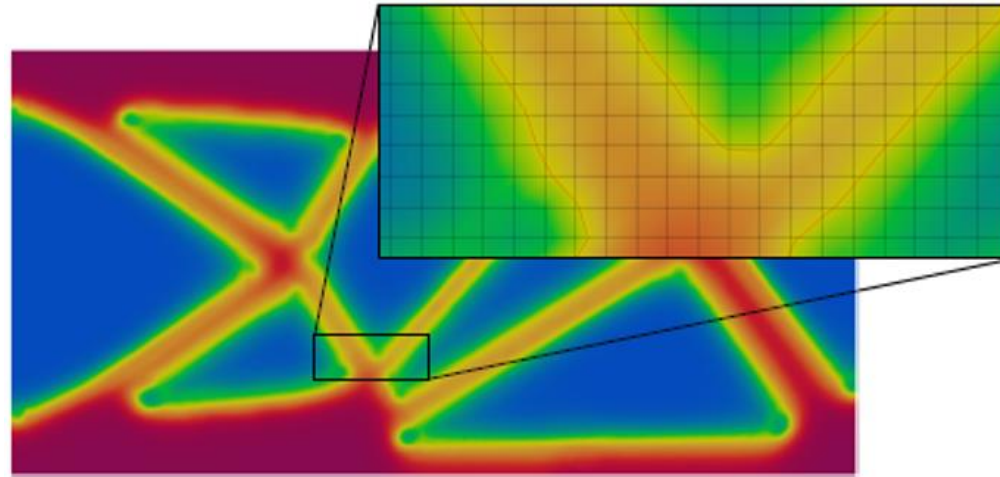
# Method of Moving Asymptotes available in MFEM

- MMA is a first order non-linear programming technique
- Used to update design variables throughout the optimization process
- MPI capable version of MMA available in MFEM

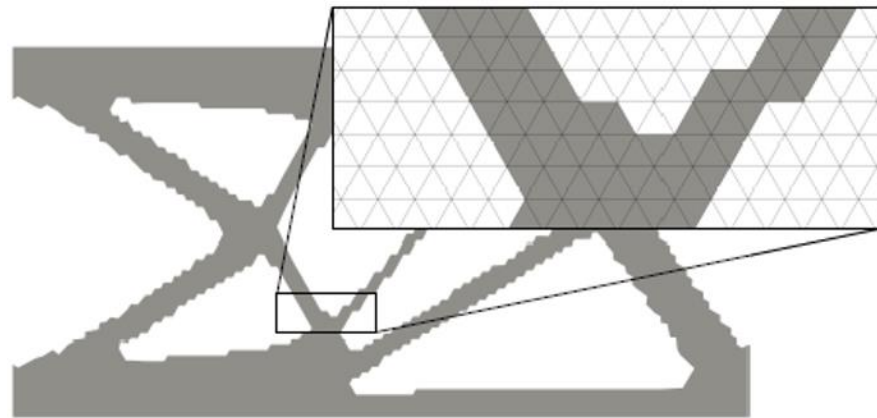




# Level-Set Topology Optimization

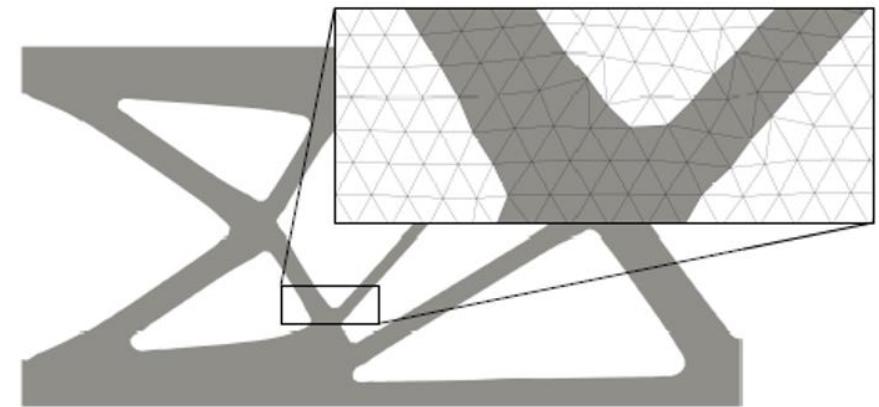


Design mesh and level-set field



Non-conformal mesh

→  
PDE based conformal  
mesh morphing



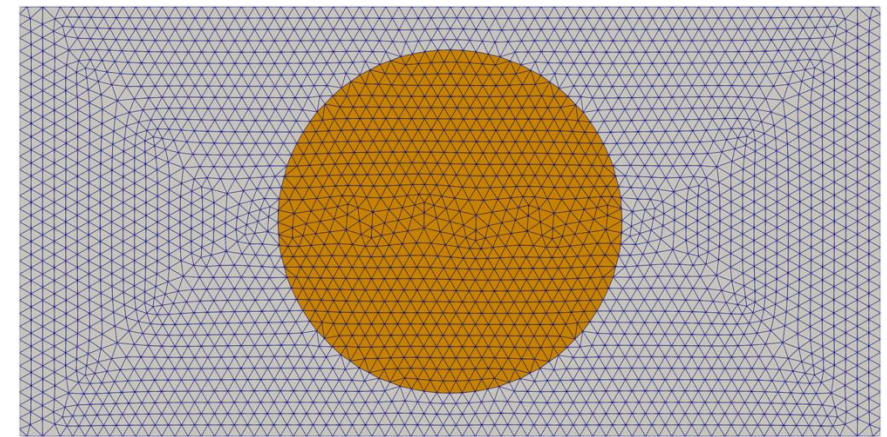
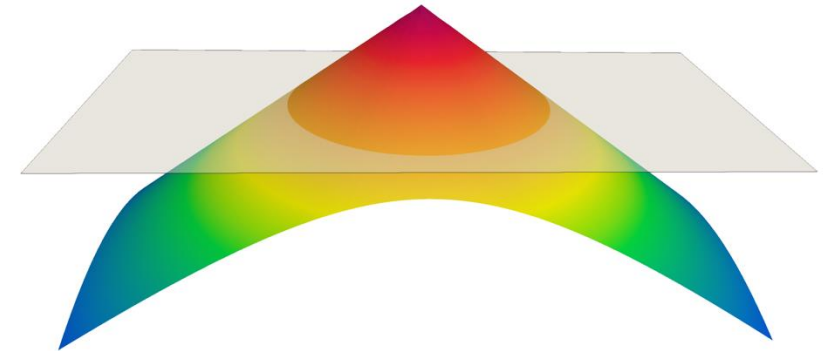
Conformal mesh



## Mesh morphing through Target-Matrix Optimization Paradigm

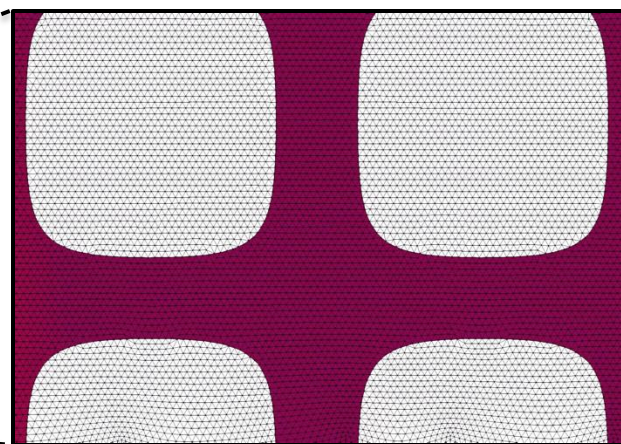
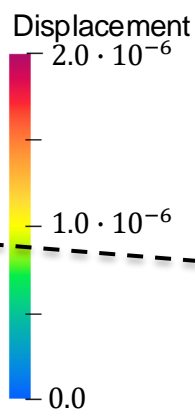
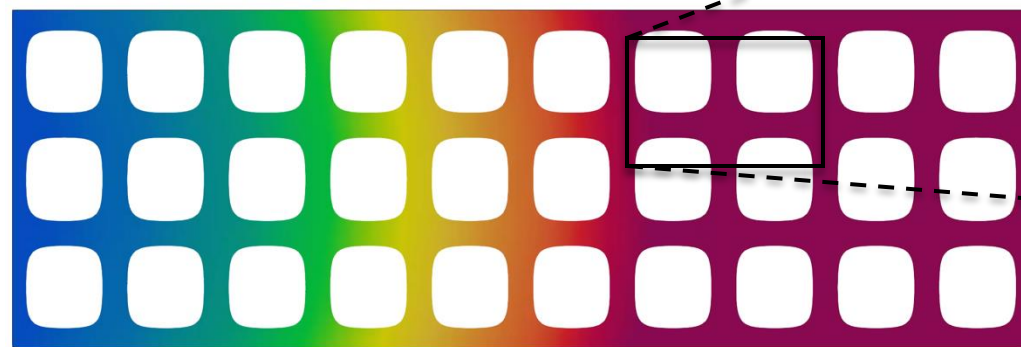
$$F(x) = \int_{\Omega} \mu(T(x)) d\Omega + \omega_{\sigma} \sum_{s \in S} \phi(x_s)^2$$

- $\mu(T)$  element quality
- $\omega_{\sigma}$  interface fitting weight
  
- TMOP adapts meshes to conform to an implicit geometry
- TMOP aims to preserve element quality

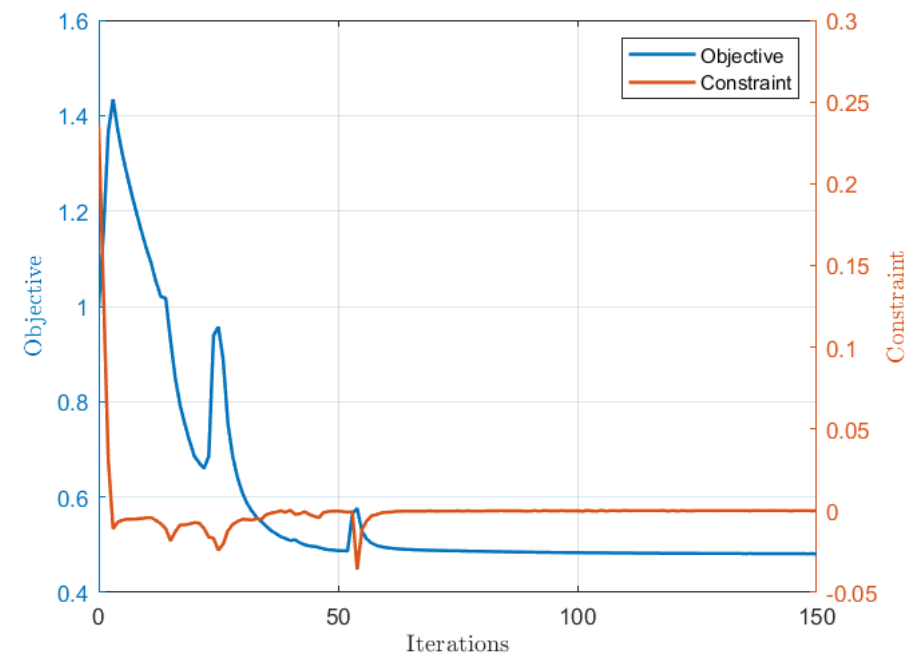
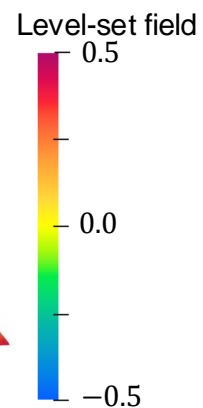
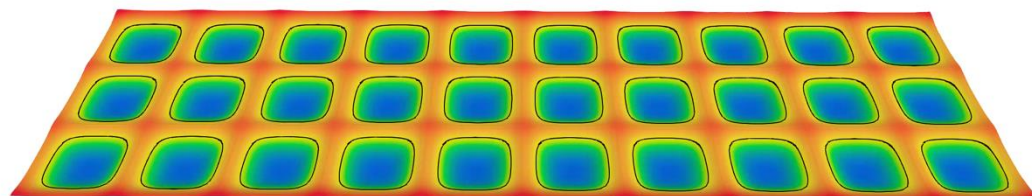


## 2D-Cantiliver Beam

➤ Evolution of displacement field

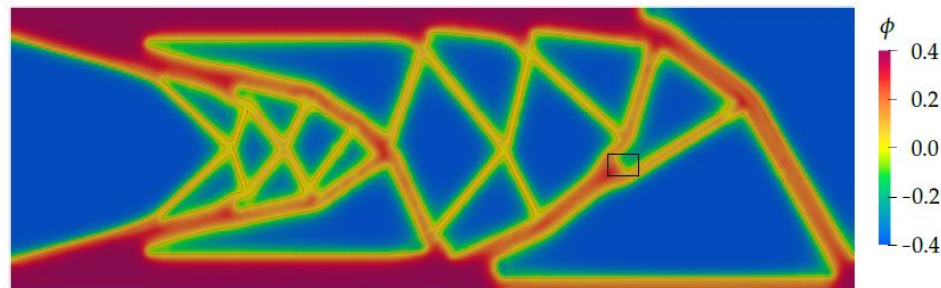


➤ Evolution of level-set field

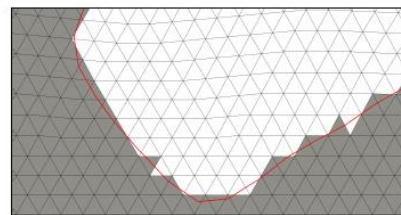


# Mesh quality

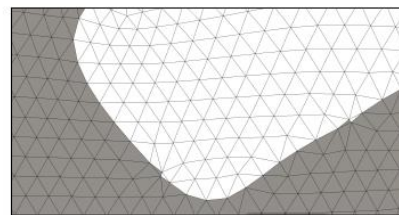
- Mesh quality varies based on element type and subphase assignment



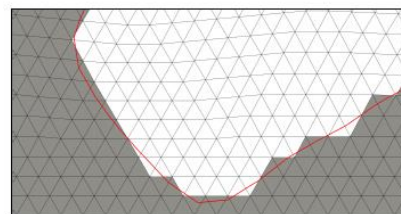
Level-set field  $\phi$



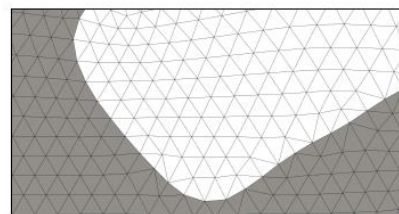
Homeomorphic Mesh a.)



Conformal Mesh a.)

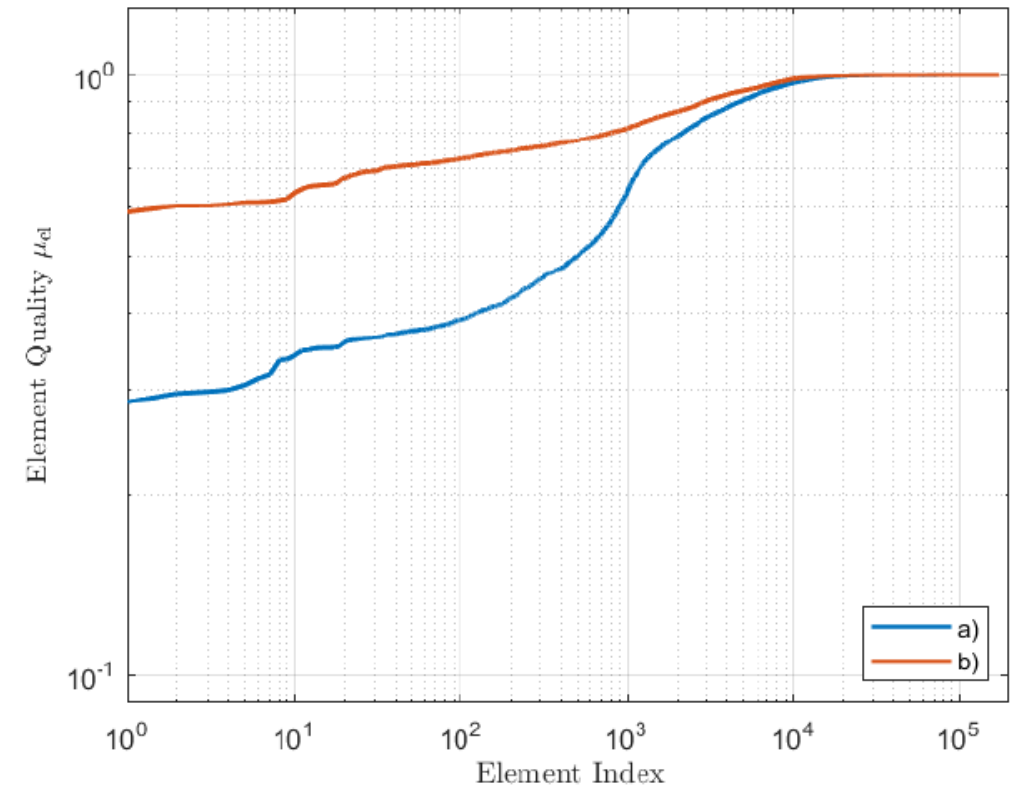


Homeomorphic Mesh b.)



Conformal Mesh b.)

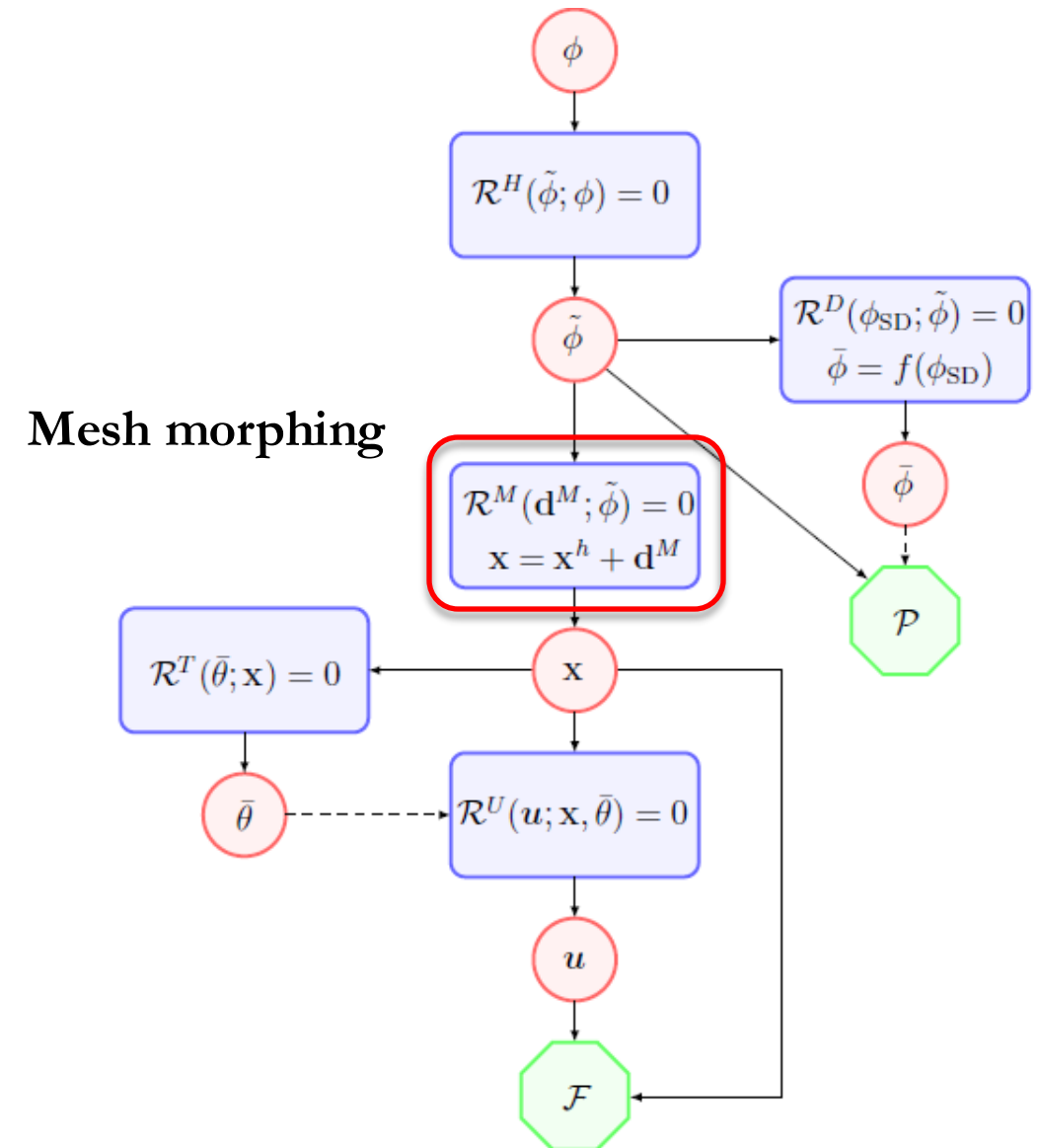
$$\mu_{el} = \frac{1}{|\Omega_e|} \int_{\Omega_e} n_{dim} \frac{(\det(\mathcal{F}^M))^{\frac{2}{n_{dim}}}}{|\mathcal{F}^M|^2} d\Omega_e.$$





## Optimization Graph

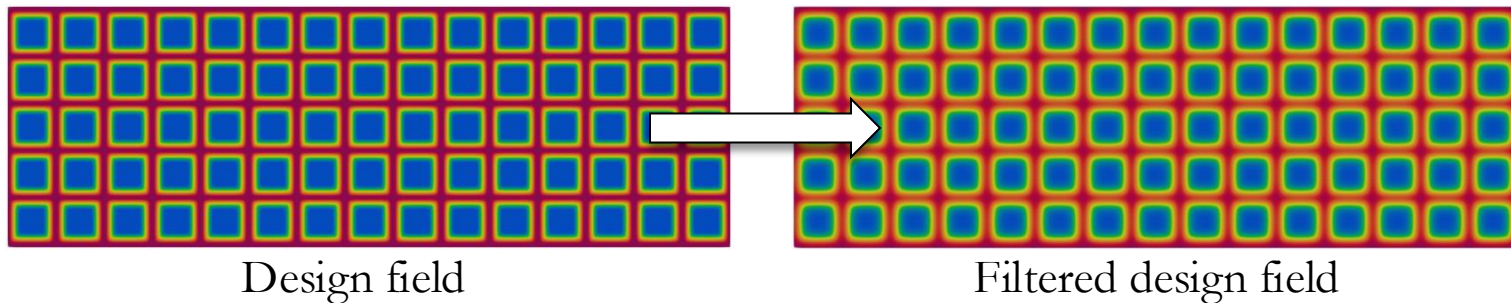
- Design variable filter
- PDE-based mesh morphing
- Island detection
- Physical analysis
- Level-set regularization



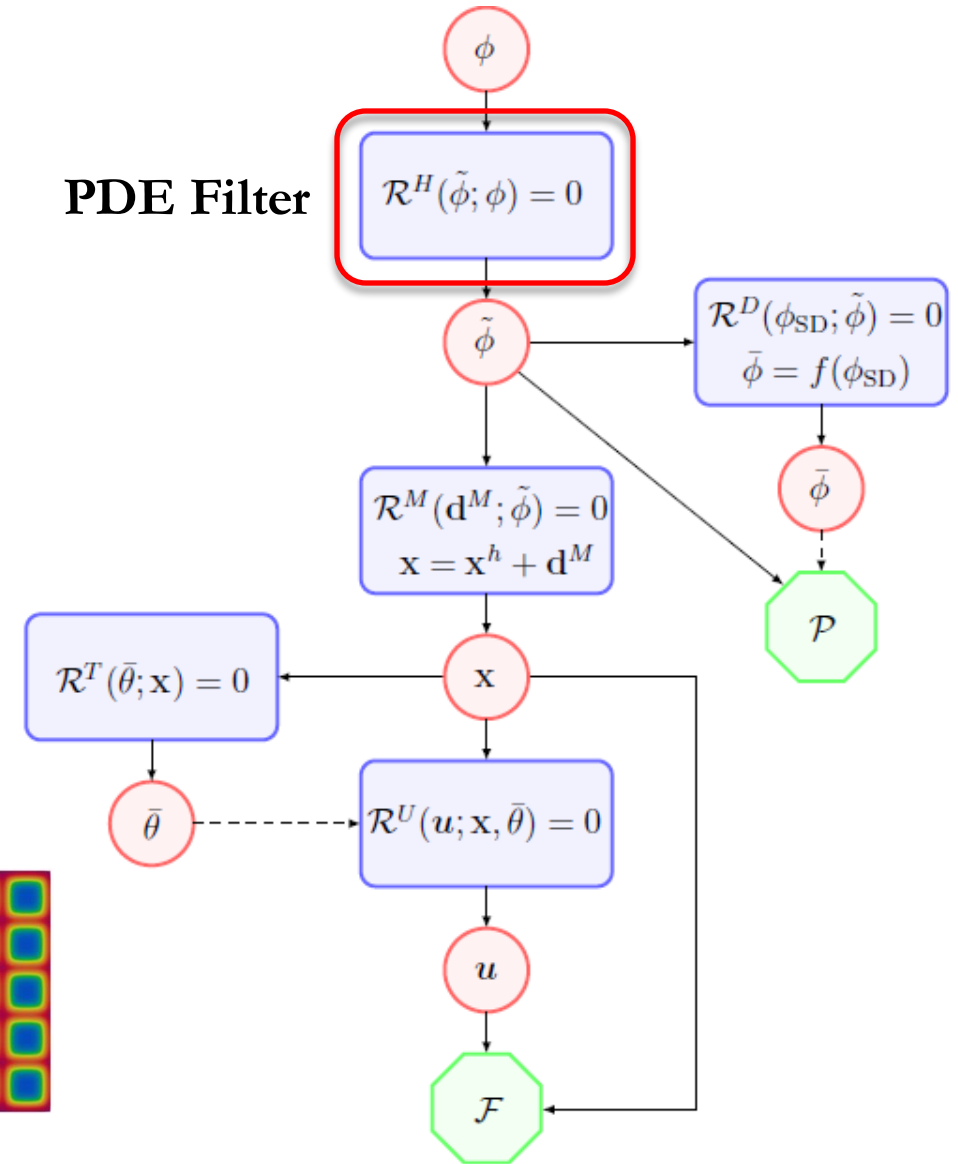
## Optimization Graph

- Increased convergence of TO problem
- Mesh independence
- Promotes smooth filtered LS field with smooth gradients
- *common* miniapp

$$\mathcal{R}^H(\tilde{\phi}; \phi) = \int_{\Omega^d} \left( \nabla \delta w r^2 \nabla \tilde{\phi} + \delta w (\tilde{\phi} - \phi) \right) d\Omega$$



## PDE Filter

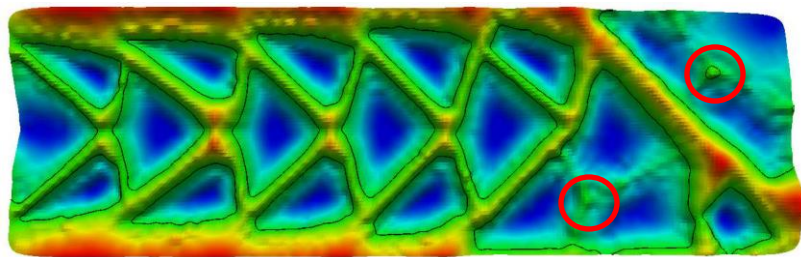


## Optimization Graph

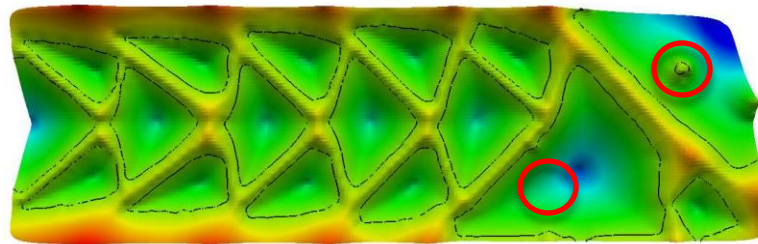
- Distance field computation: *common* miniapp
- Promotes consistent fitting

$$R^D(\phi_{SD}; \phi) = \int_{\Omega^d} (\nabla \delta w | \nabla \phi_{SD}|^{p-2} \nabla \phi_{SD} - \delta w) d\Omega$$

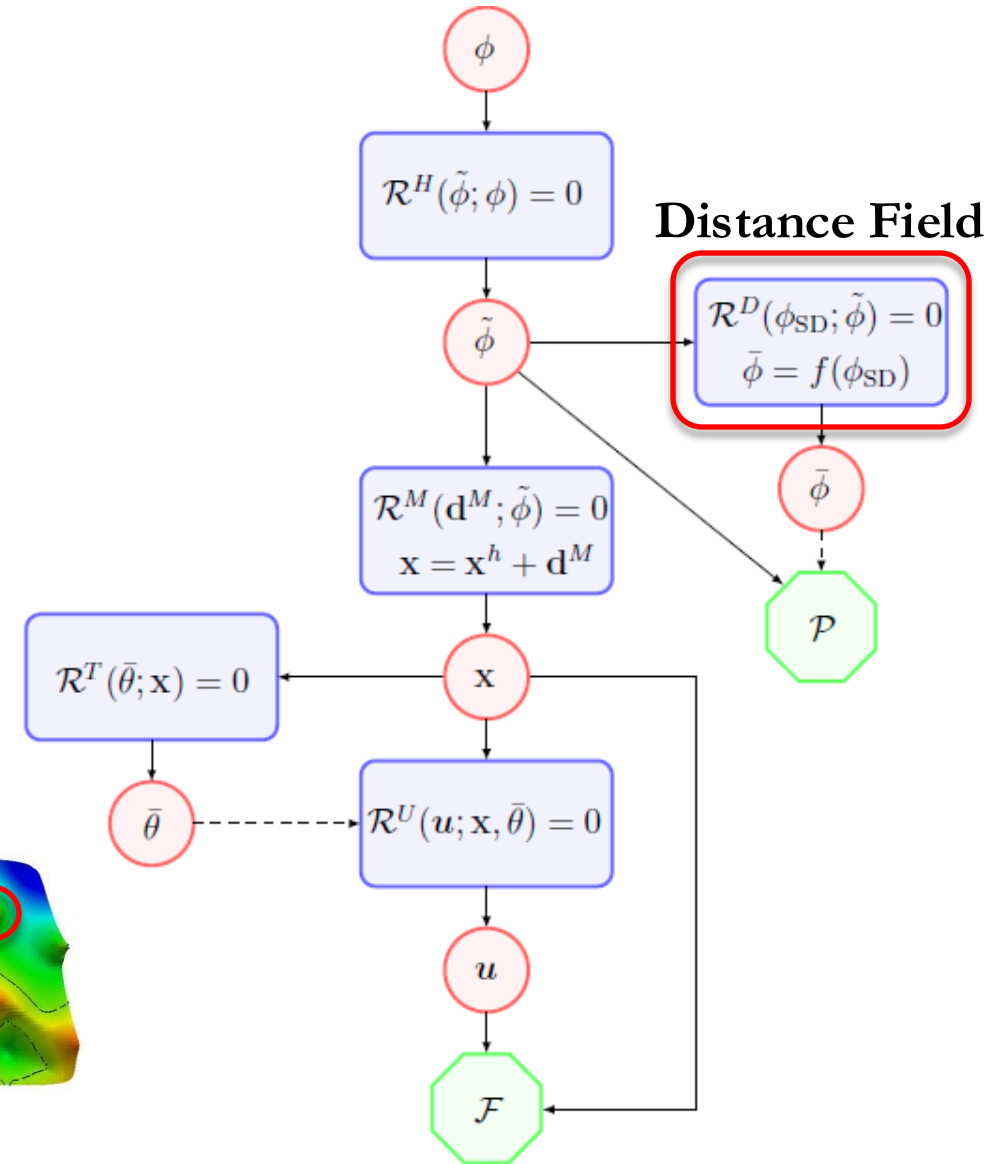
$$P = \frac{\int_{\Omega^d} w_\phi (\tilde{\phi} - \bar{\phi})^2 d\Omega^d}{\int_{\Omega^d} \phi_{Bnd}^2 d\Omega^d} + \frac{\int_{\Omega^d} w_{\nabla\phi} |\nabla \tilde{\phi} - \nabla \bar{\phi}|^2 d\Omega^d}{\int_{\Omega^d} d\Omega^d}$$



Filtered design field



Signed distance field

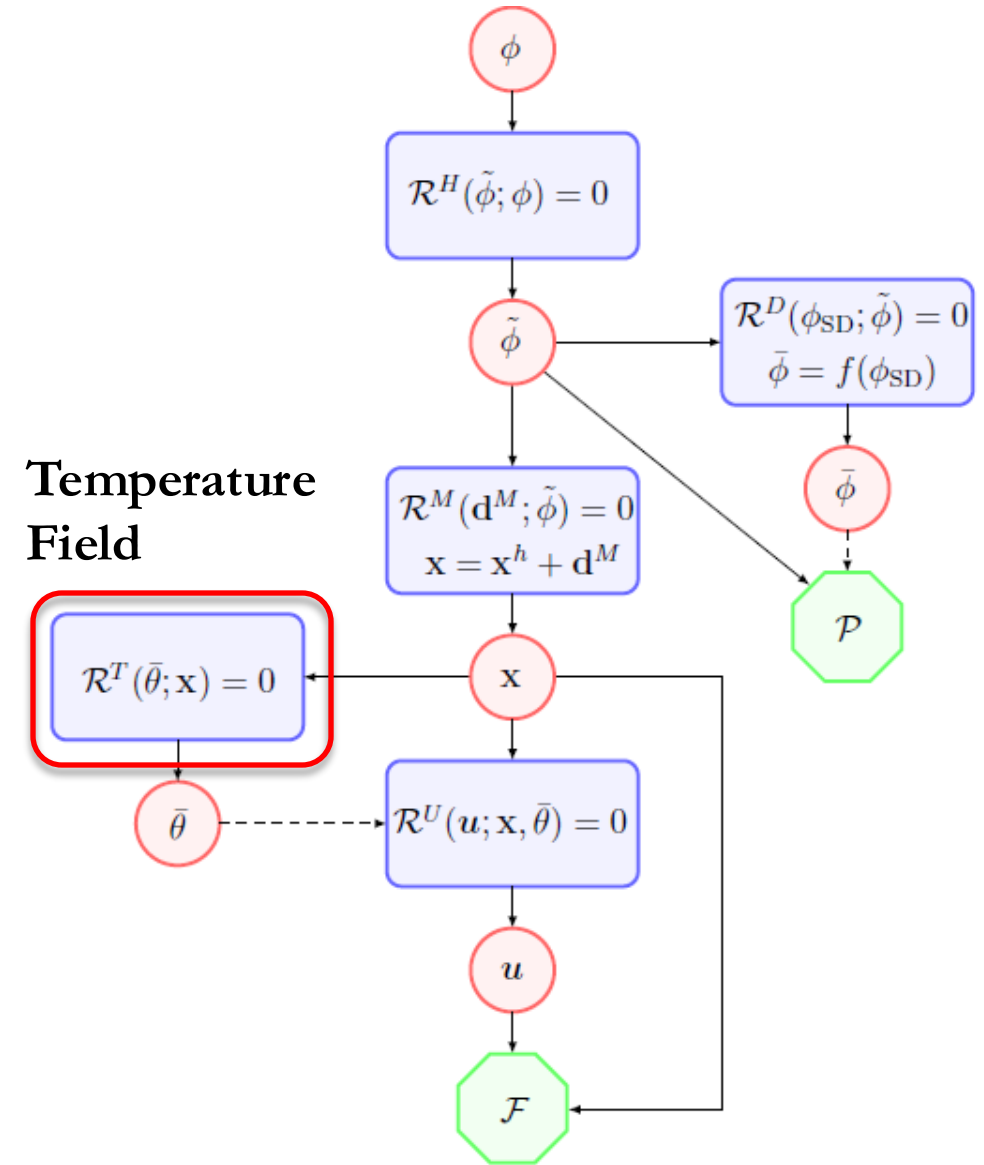
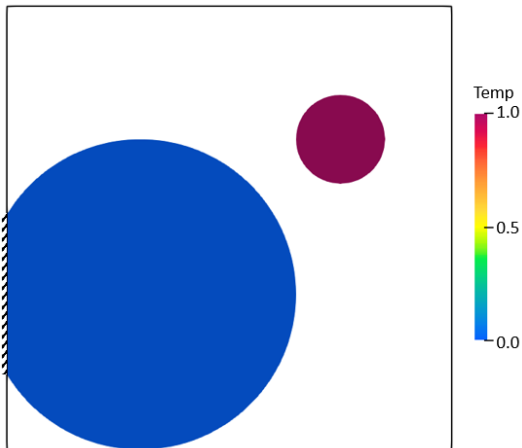




## Optimization Graph

- Instabilities through disconnected subregions
- Island detection

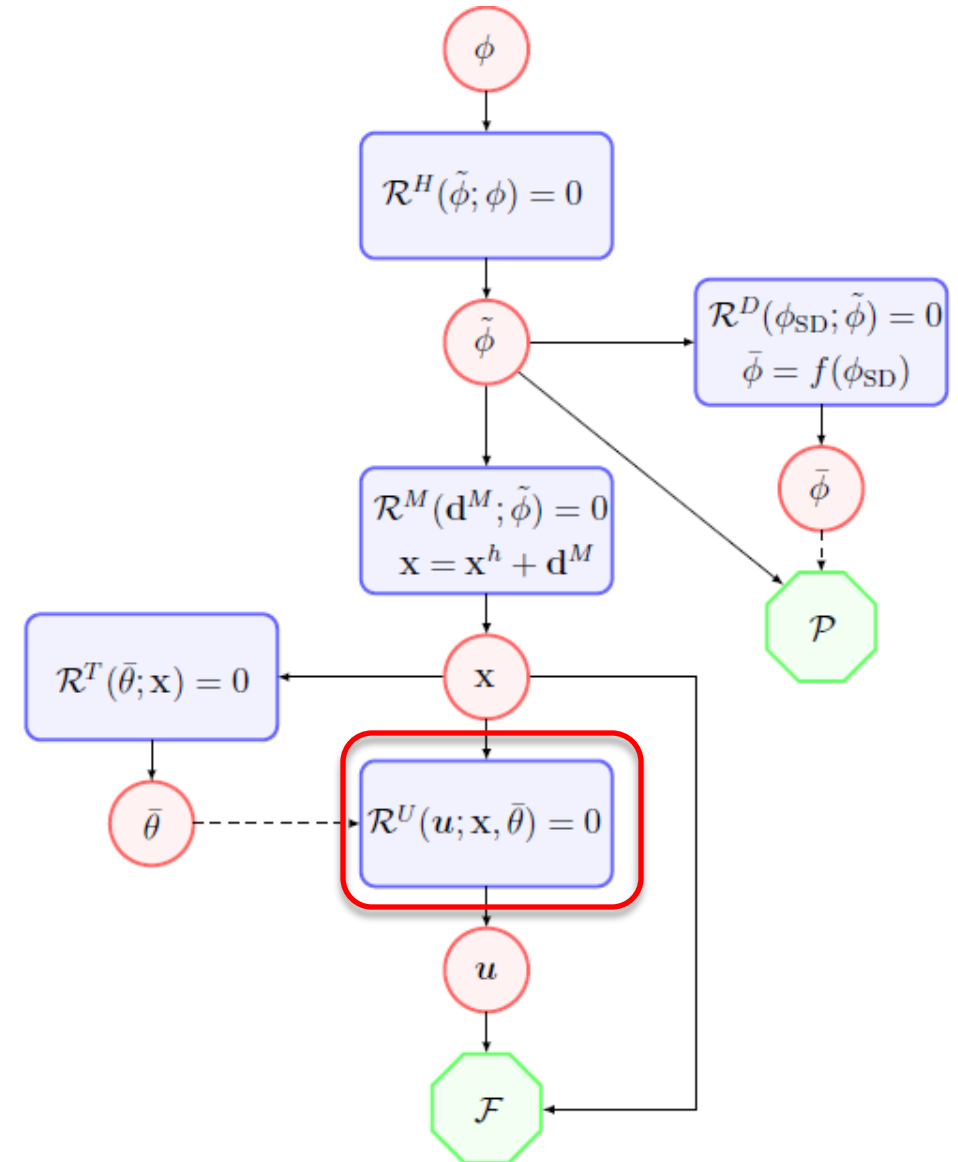
$$R^T(\theta; x) = \int_{\Omega} (\nabla \delta w \kappa \nabla \theta + \delta w (\theta - 1)) d\Omega$$



## Optimization Graph

- Structural islands are stabilized

$$\mathcal{R}^U(u; x, \theta) = \int_{\Omega_I} \delta \boldsymbol{\varepsilon}(w) : \boldsymbol{\sigma}(u) d\Omega - \int_{\Gamma_N^{\Omega_I}} \delta w \mathbf{t}_N d\Gamma + \int_{\Omega_I} \delta w u r_s \frac{E}{h^2} \theta d\Omega$$



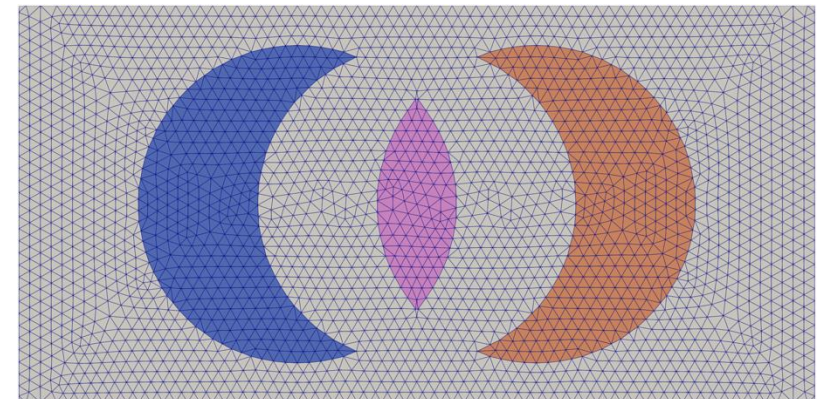
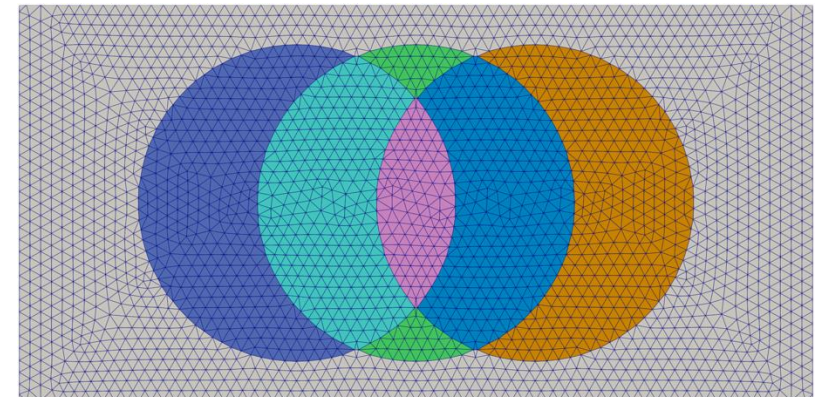
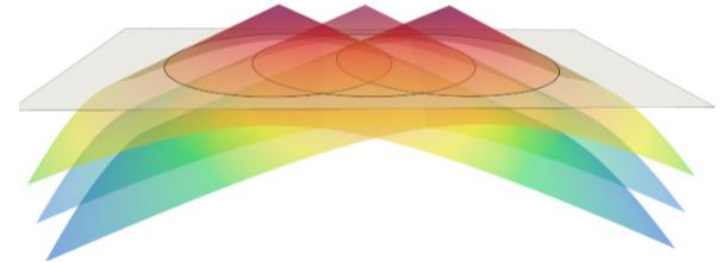
## Multi-material mesh morphing through TMOP

- $2^N$  number of subphases:
- Subphase index

$$I = \sum_{i=0} 2^i H_i$$

- User defined subphase index:
  - Table:  $I \rightarrow I_{custom}$

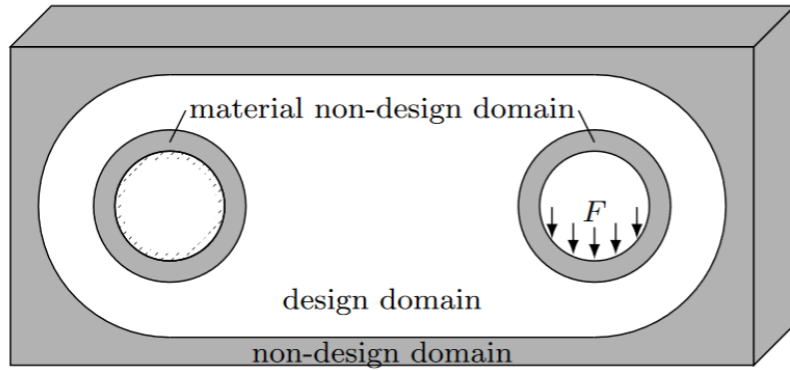
$$F(x) = \int_{\Omega} \mu(T(x)) d\Omega + \omega_{\sigma} \sum_{i \in N_{\phi}} \sum_{s \in S_{\phi_i}} \phi_i^2(x_s)$$



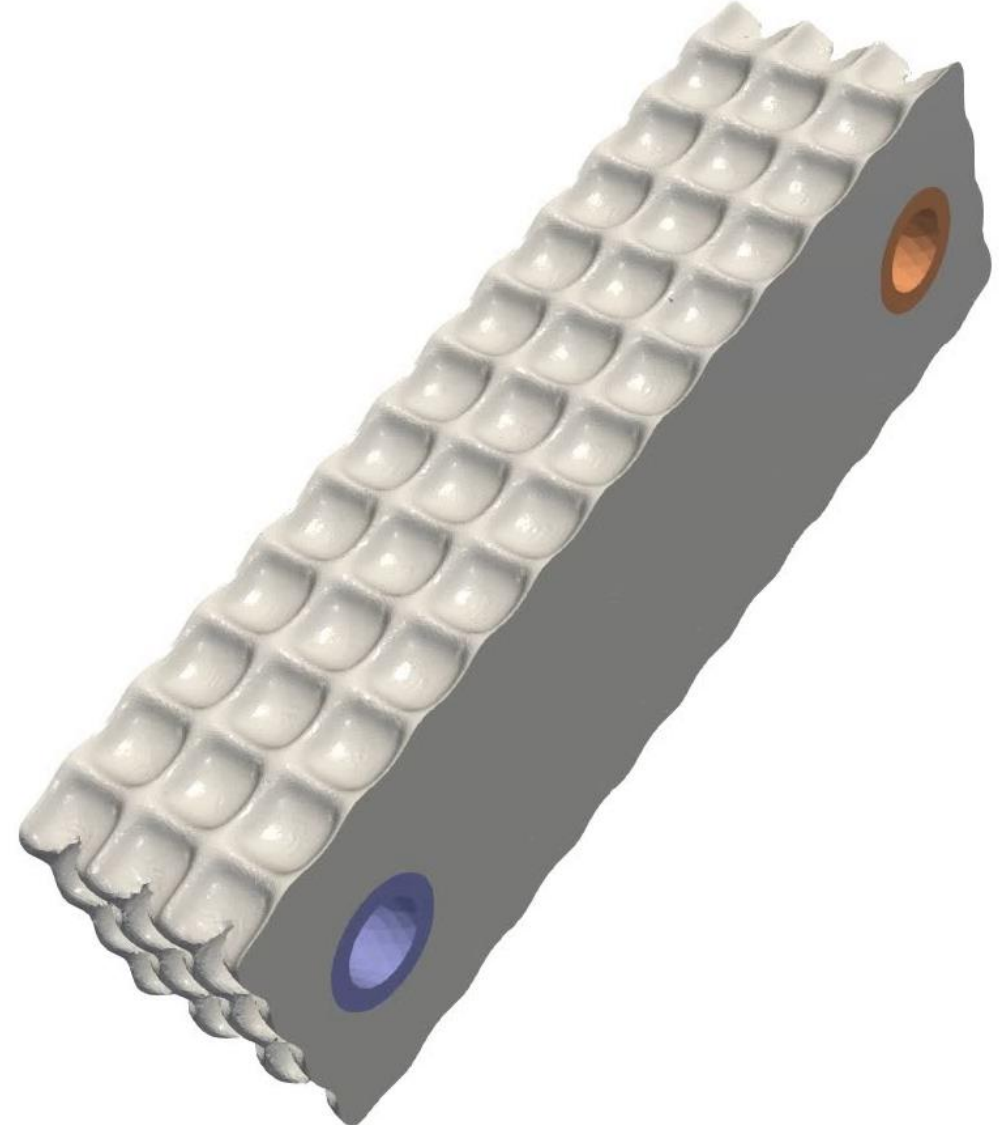
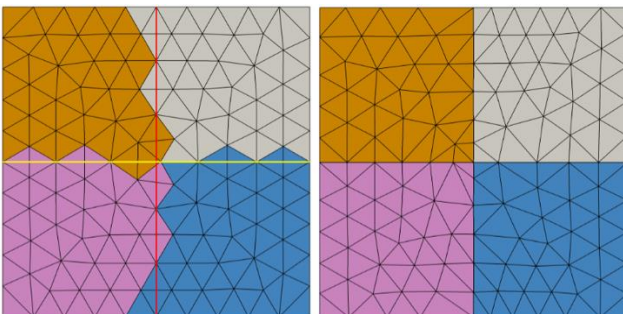


# 3D Bracket with assembly requirements

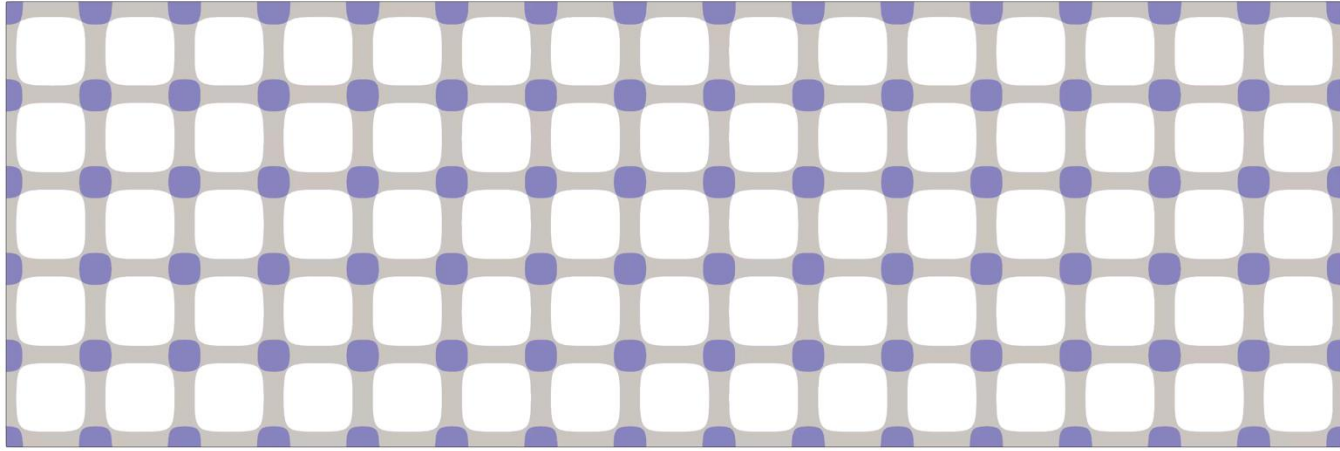
## 3D Bracket



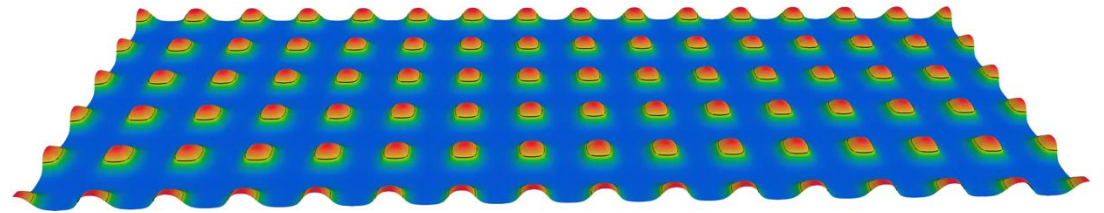
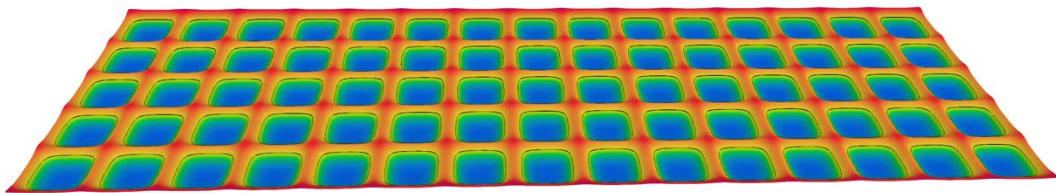
- Enables modeling of fixed components
- Enables enforcement of assembly requirements



➤ Evolution of the structure

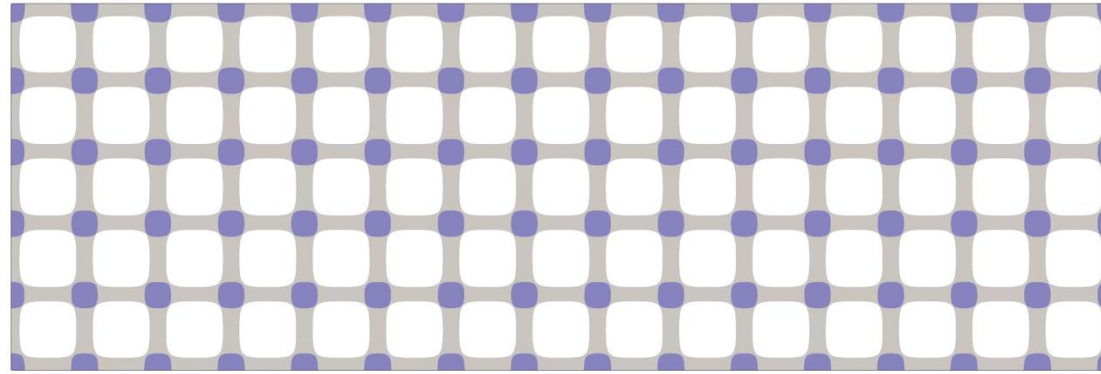


➤ Evolution of level-set fields

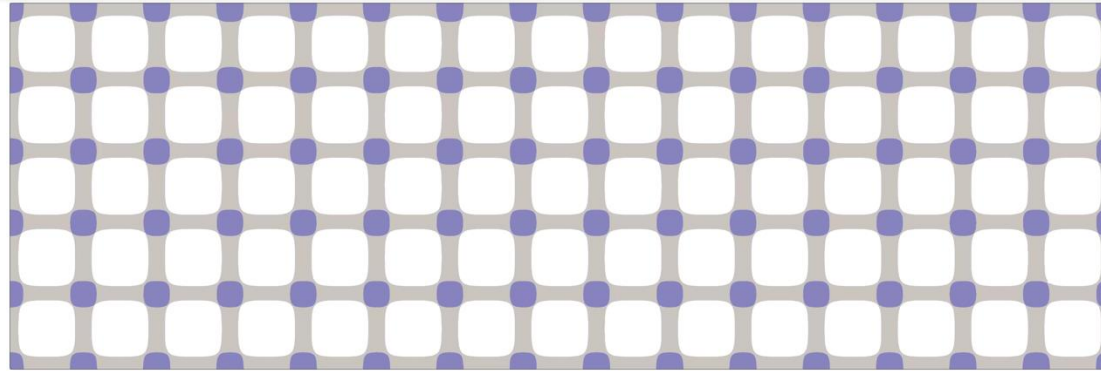


# 2D Cantilever comparison

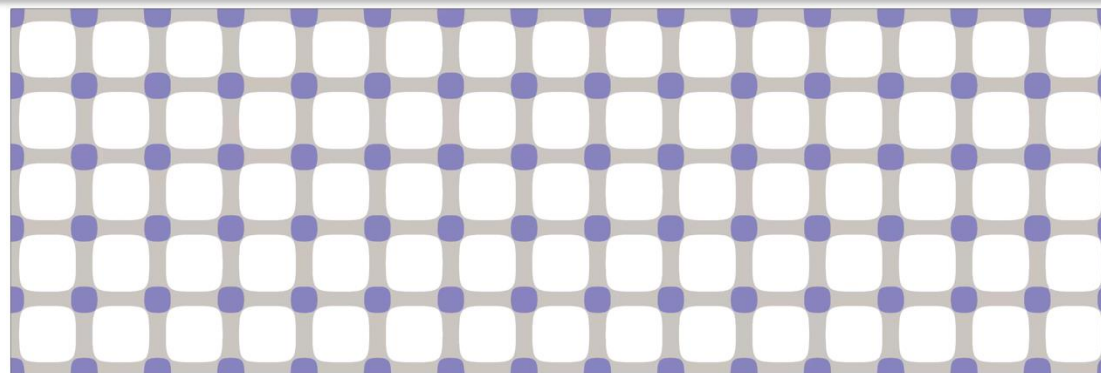
$$\frac{E_2}{E_1} = \frac{1}{2}$$



$$\frac{E_2}{E_1} = \frac{1}{10}$$



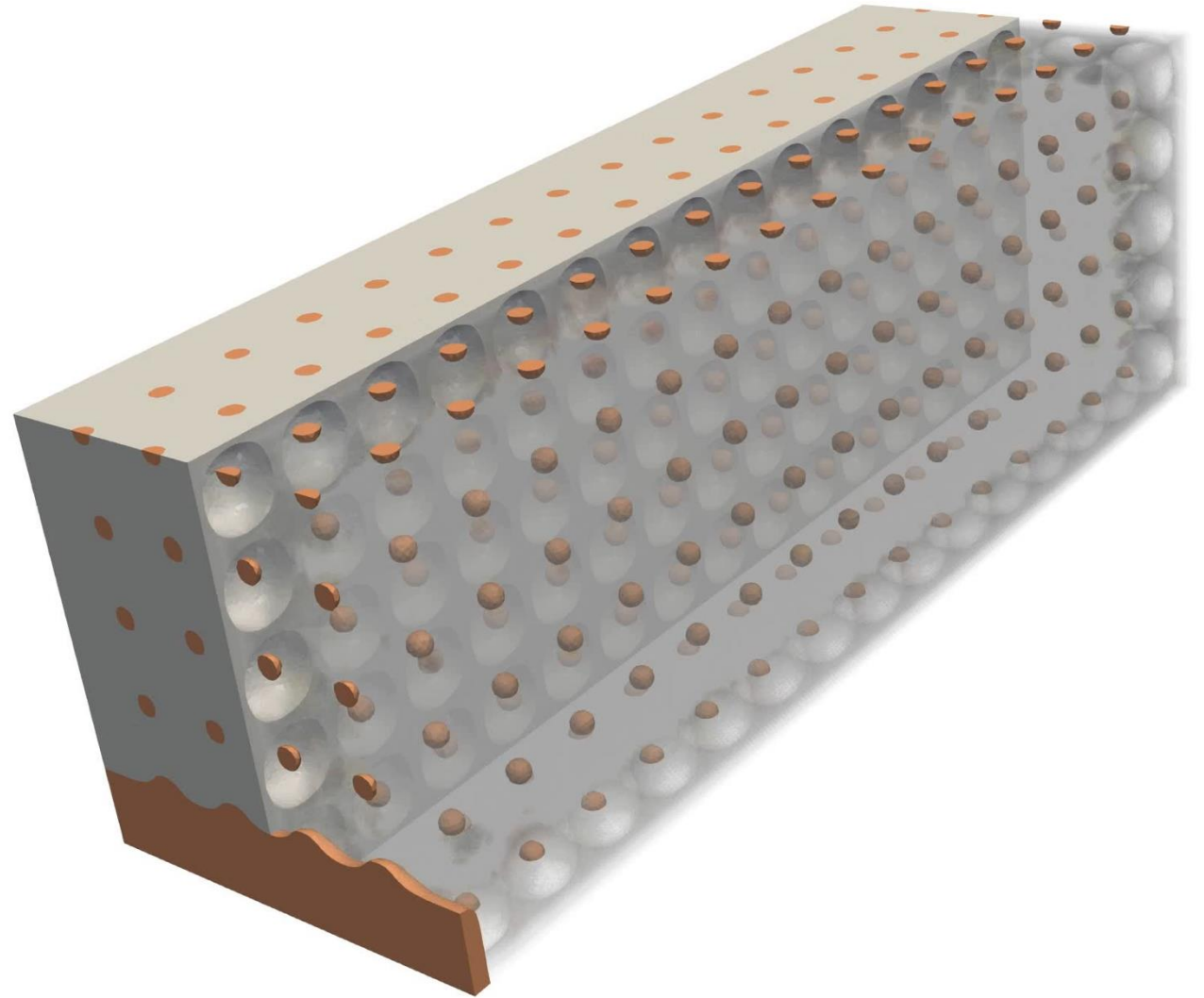
$$\frac{E_2}{E_1} = \frac{1}{20}$$



## 3D-Cantiliver Beam



- Topology optimization for external structure and internal material layout





## Summary

- Presented solely PDE-based level-set TO approach
- Utilized PDE generated conformal analysis meshes
- Highlighted selected material-void applications
- Discussed multi-LS formulation
- Presented multi-LS TO with single design field results

# Thank you!

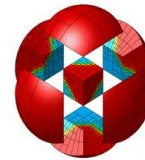
Any questions?  
schmidt43@llnl.gov



LiDO



Serac



MFEM





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