Level-Set Topology Optimization with PDE Generated Conformal Meshes

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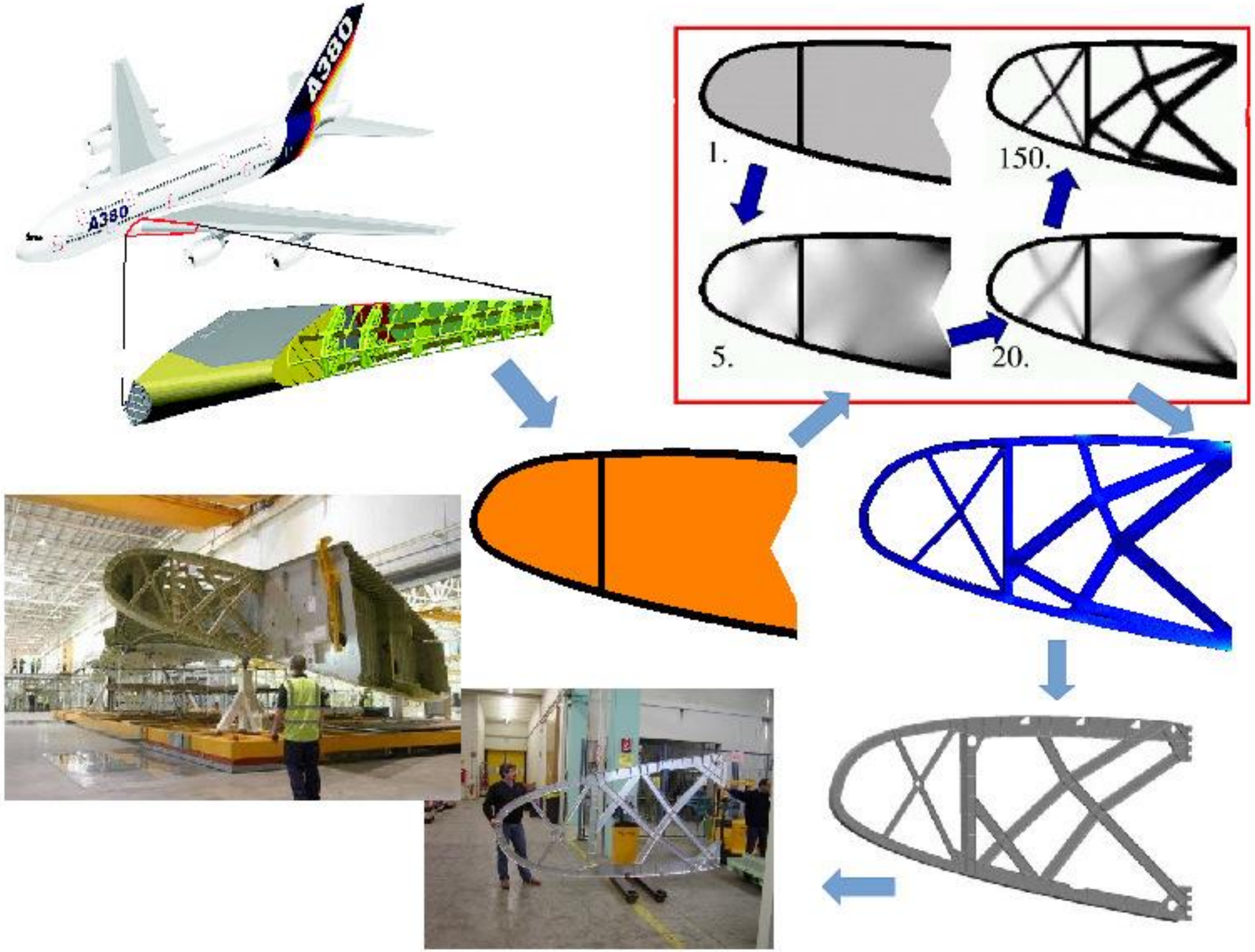
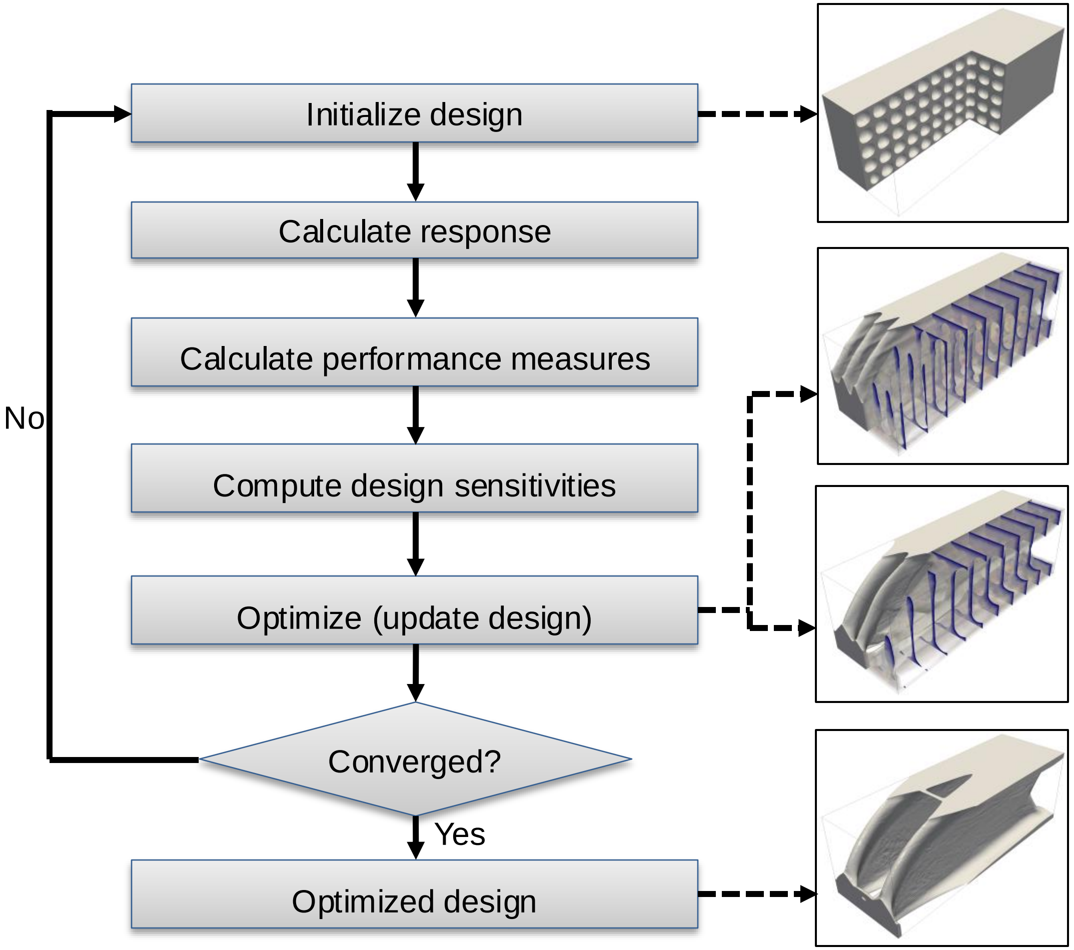
Motivation

Reduction in weight by 700kg per aircraft



Optimization Process



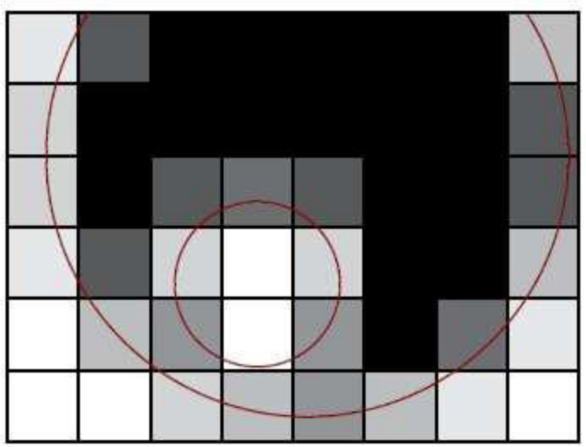
- Cost and constraint functions

$$\min_{s} z\big(s, u(s)\big) = w_1 F(s, u(s)) + w_2 P_{Reg}(s)$$

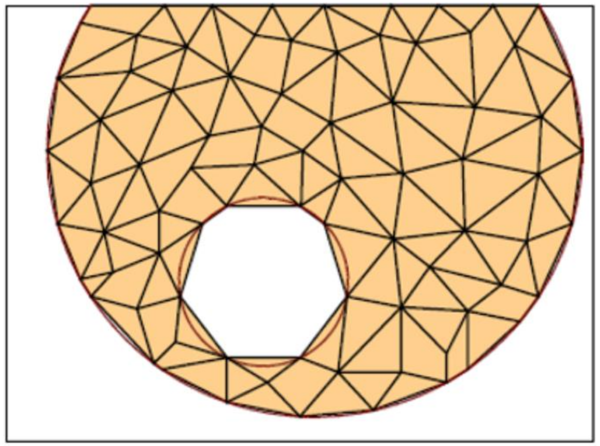
$$s.t.: \quad g_i\big(s, u(s)\big) \leq 0, i = 1, \ldots, N_g$$

Analysis models

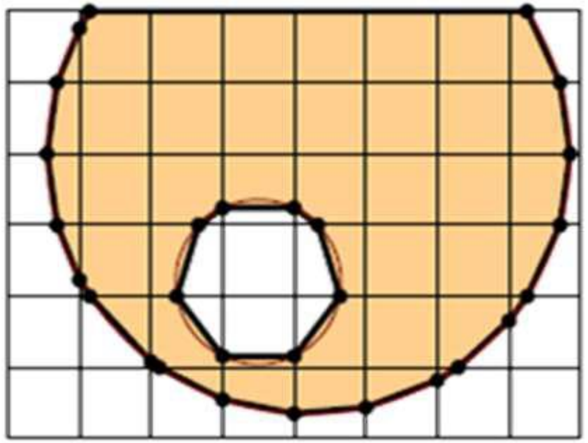
- Three types of analysis models



Density methods



Body fitted analysis mesh



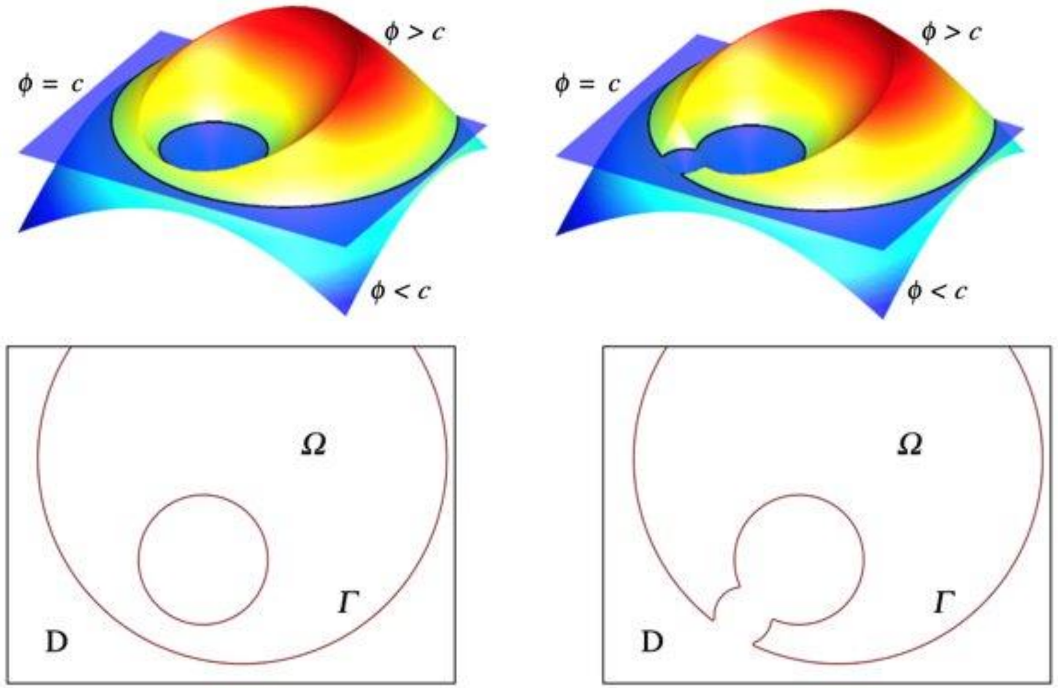
Immersed analysis methods

Level-set optimization uses isocontour to describe geometry

Geometry definition

- Design variables employed as coefficients for level-set field discretization
- Geometry implicitly defined by isocontour ϕ_t, of a level-set function ϕ

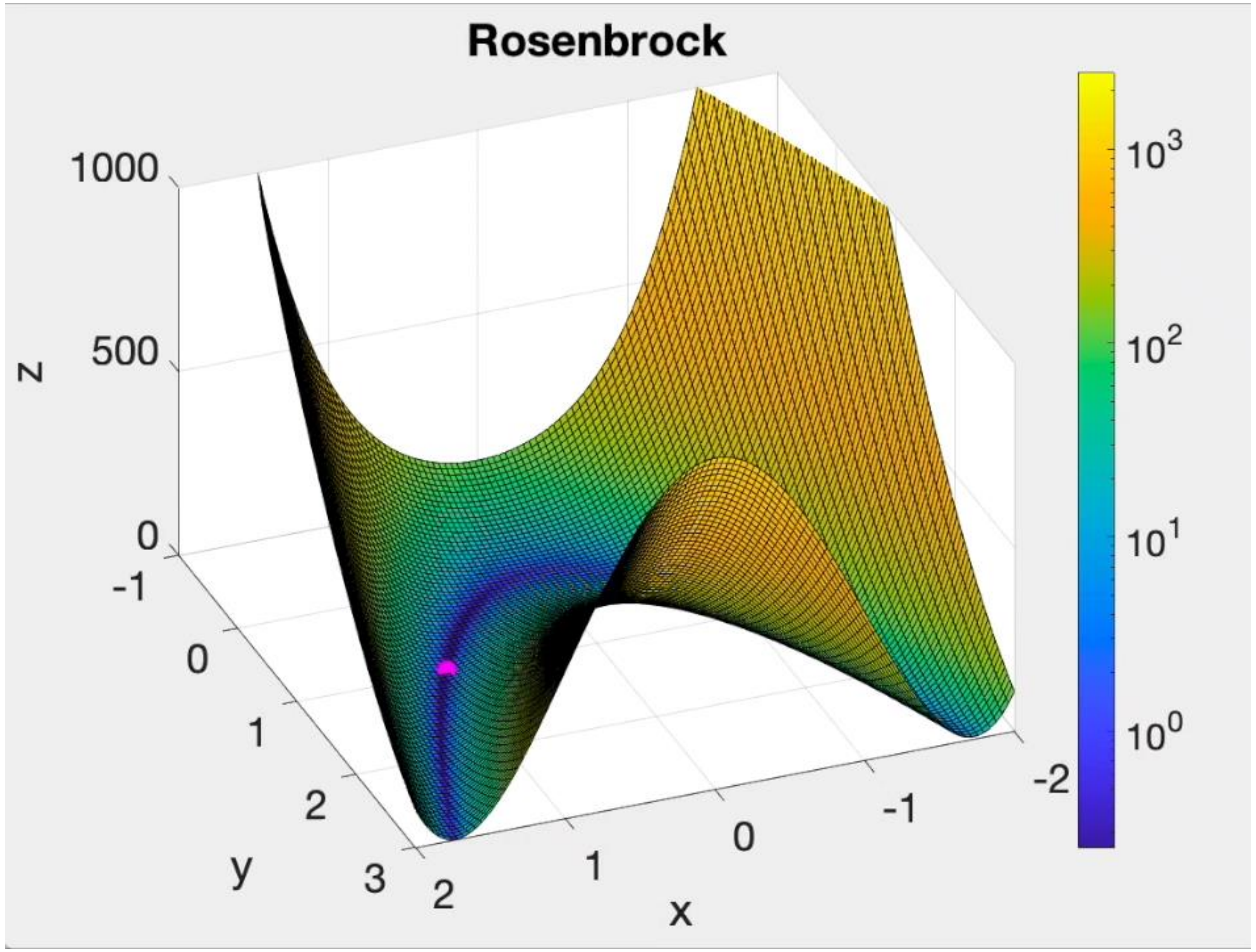
$$\phi(x) > \phi_t, \forall x \in \Omega_+$$
$$\phi(x) < \phi_t, \forall x \in \Omega_-$$
$$\phi(x) = \phi_t, \forall x \in \Gamma_\pm$$



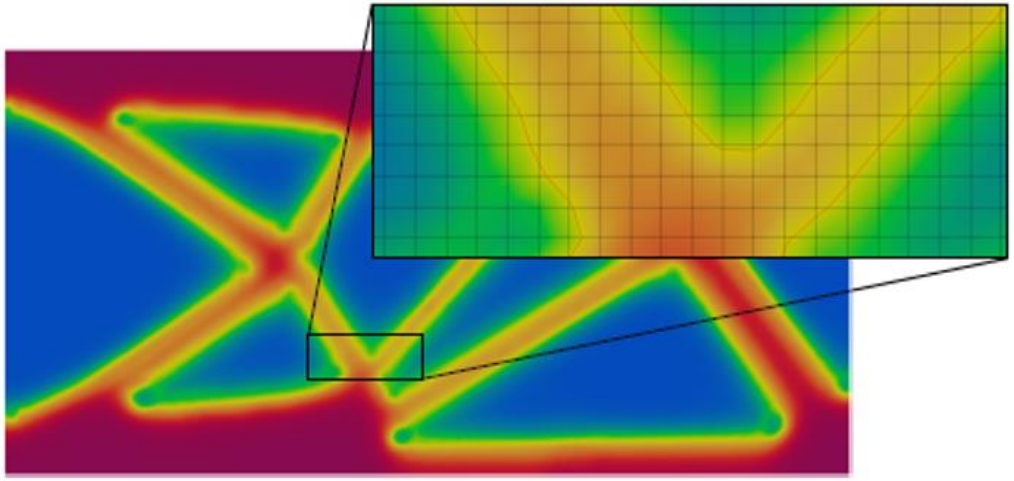
Level-set function and resulting isocontour [Van Dijk 2012]

Method of Moving Asymptotes available in MFEM

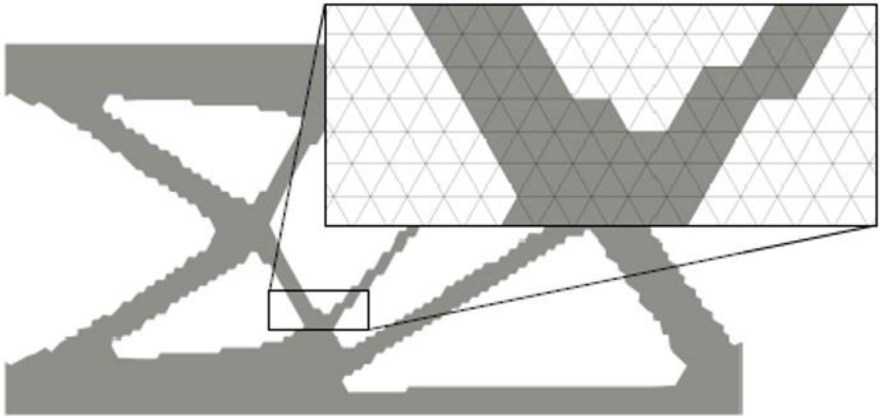
- MMA is a first order non-linear programming technique
- Used to update design variables throughout the optimization process
- MPI capable version of MMA available in MFEM



Level-Set Topology Optimization

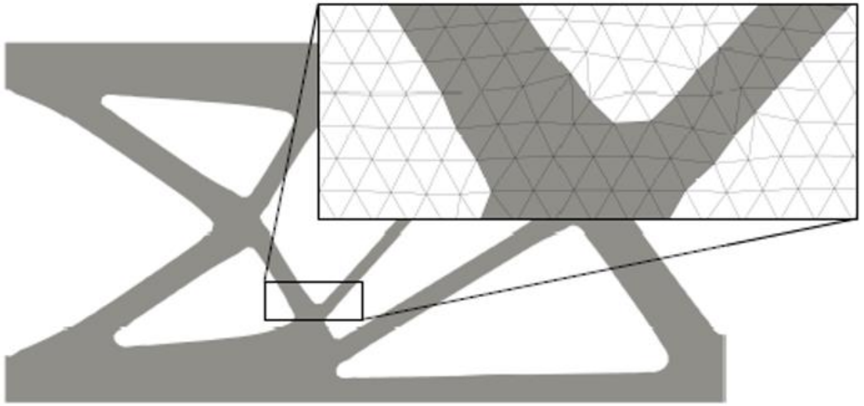


Design mesh and level-set field



Non-conformal mesh

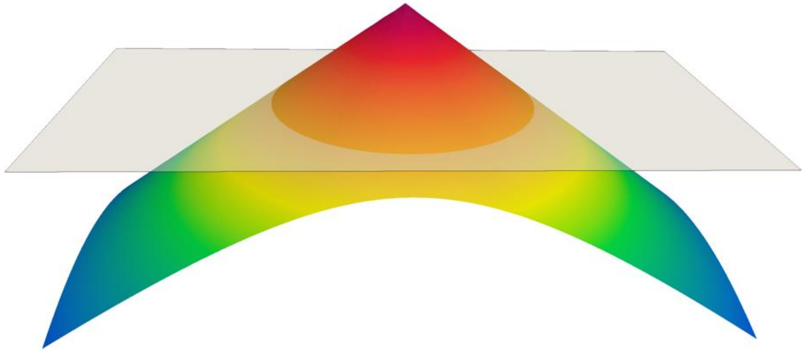
PDE based conformal
mesh morphing



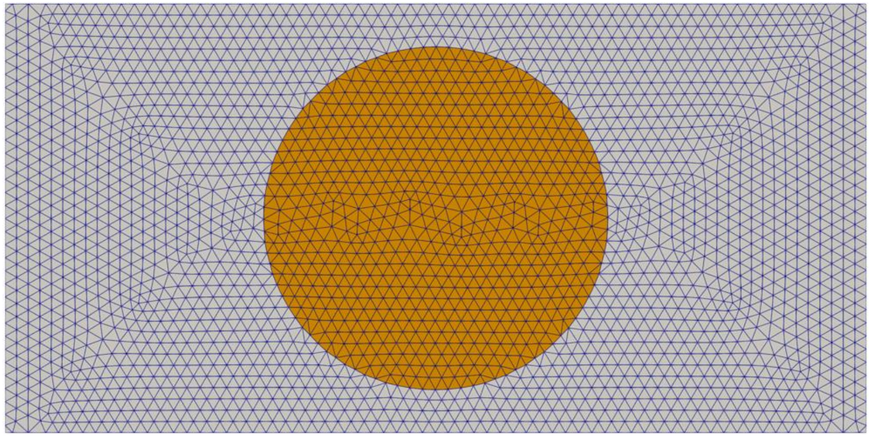
Conformal mesh

Mesh morphing through Target-Matrix Optimization Paradigm

$$F(x) = \int_{\Omega} \mu\big(T(x)\big)d\Omega + \omega_\sigma \sum_{s\in S} \phi(x_s)^2$$



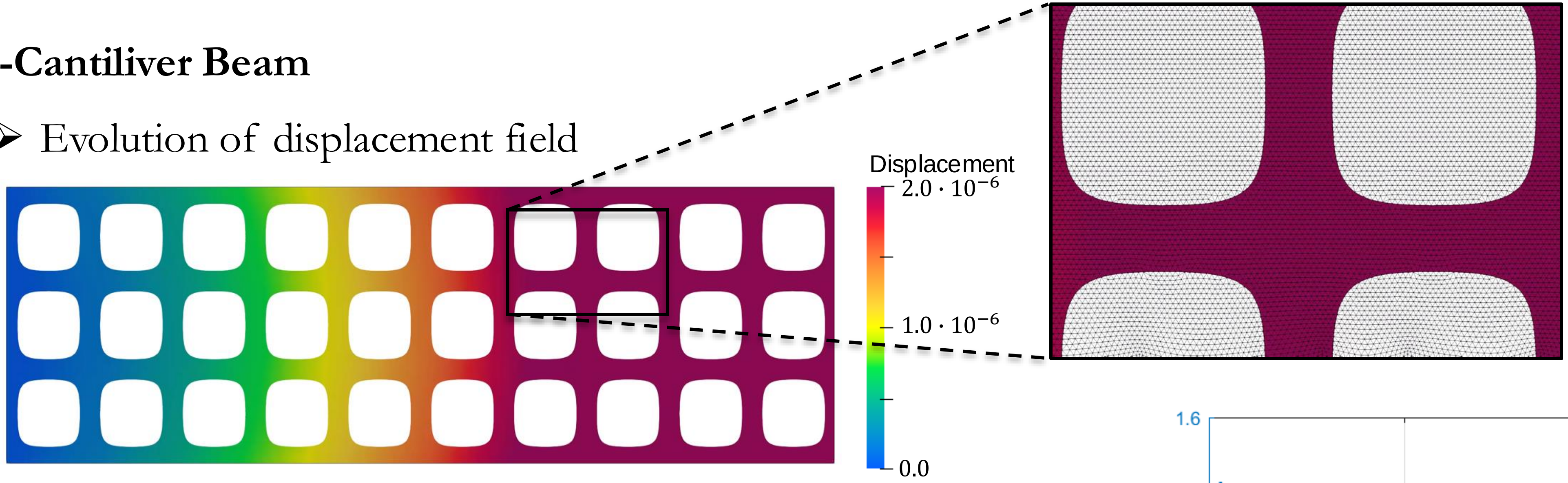
- $\mu(T)$ element quality
- ω_σ interface fitting weight



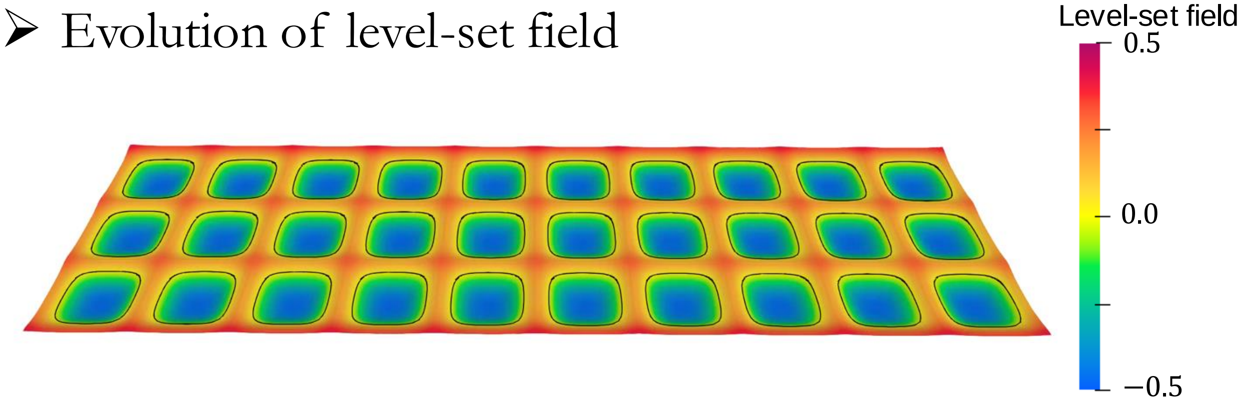
- TMOP adapts meshes to conform to an implicit geometry
- TMOP aims to preserve element quality

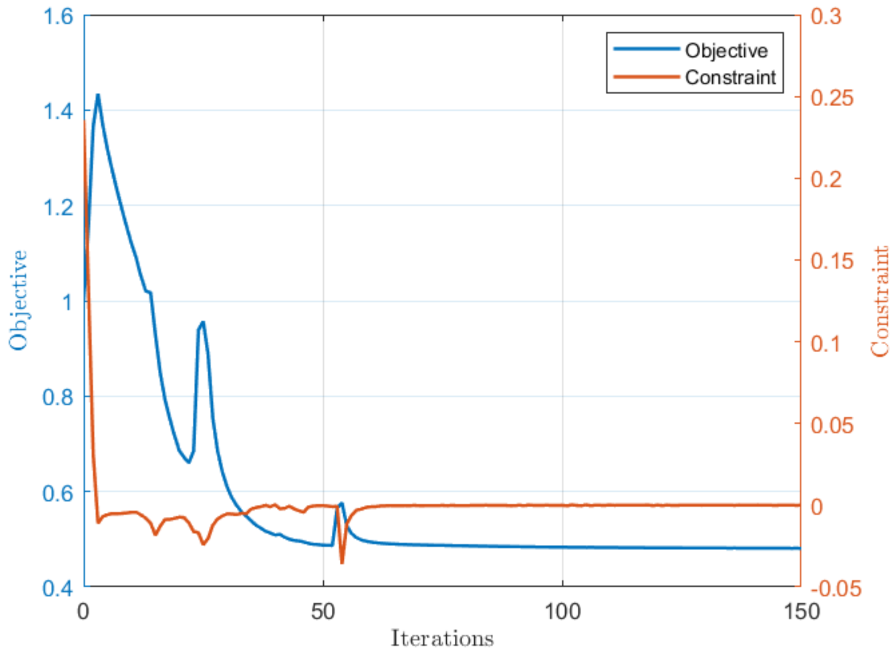
2D-Cantiliver Beam

- Evolution of displacement field



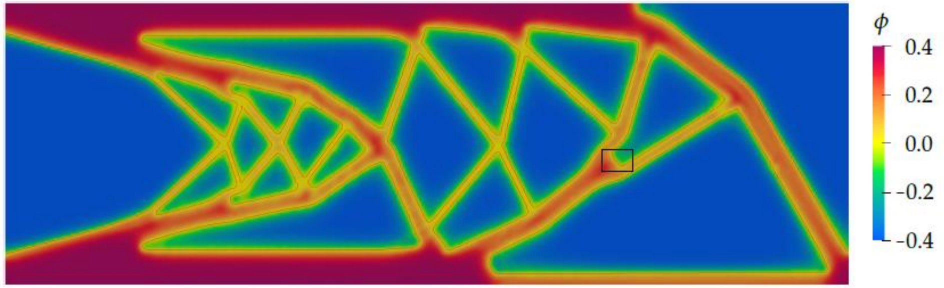
- Evolution of level-set field



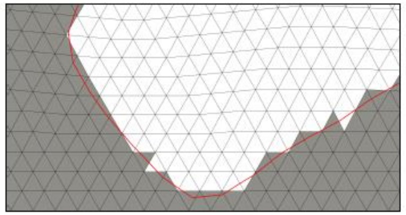


Mesh quality

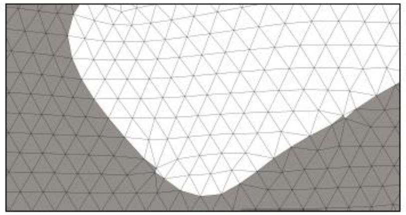
- Mesh quality varies based on element type and subphase assignment



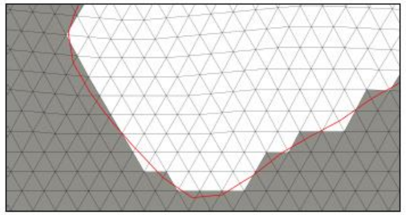
Level-set field ϕ



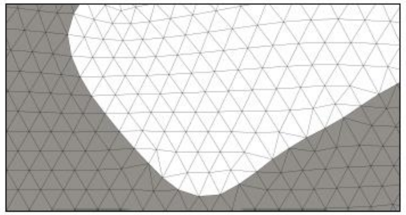
Homeomorphic Mesh a.)



Conformal Mesh a.)

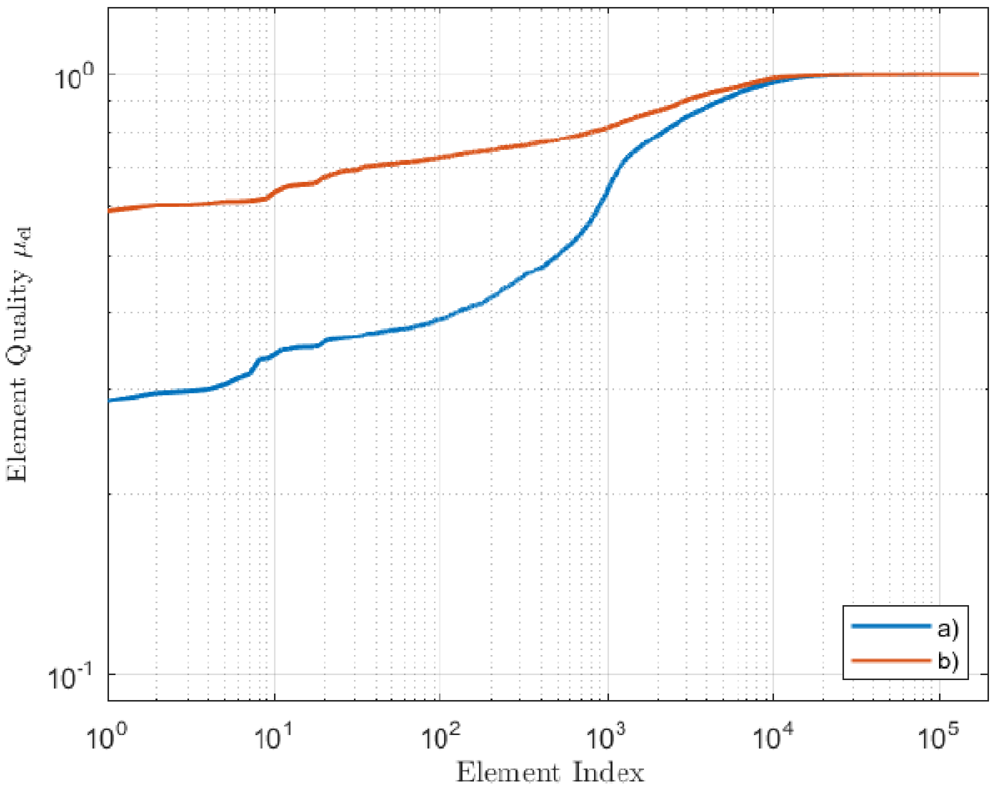


Homeomorphic Mesh b.)



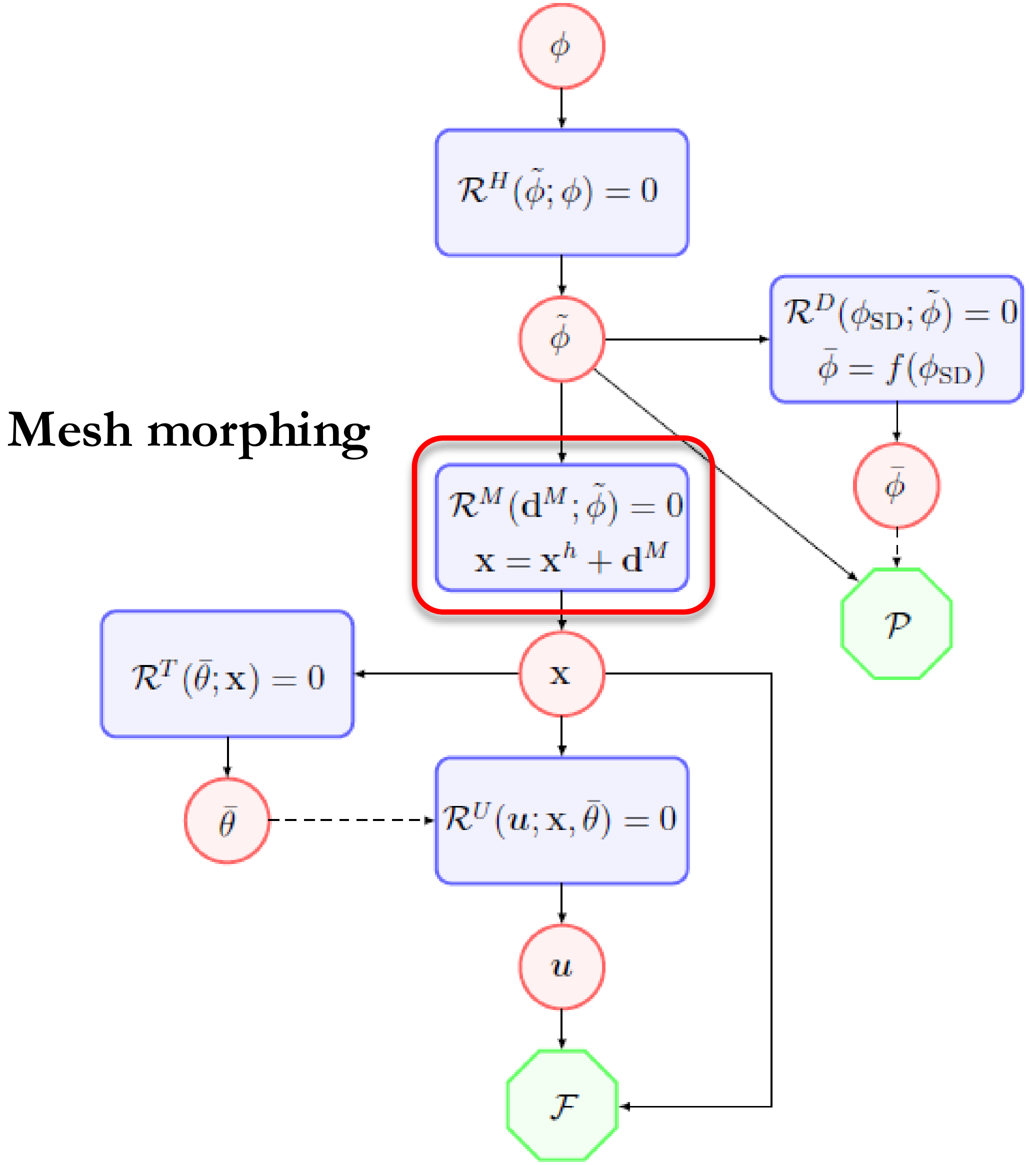
Conformal Mesh b.)

$$\mu_{el} = \frac{1}{|\Omega_e|}\int_{\Omega_e} n_{\dim} \frac{(\det(\mathcal{F}^M))^{\frac{2}{n_{\dim}}}}{|\mathcal{F}^M|^2}\, d\Omega_e$$



Optimization Graph

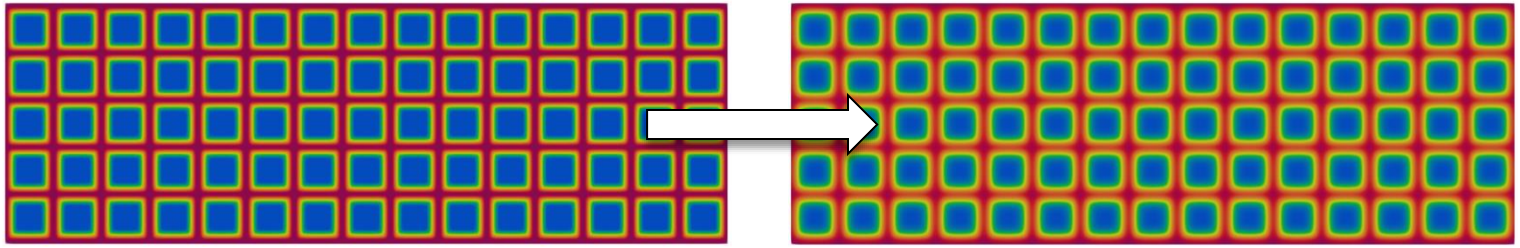
- Design variable filter
- PDE-based mesh morphing
- Island detection
- Physical analysis
- Level-set regularization



Optimization Graph

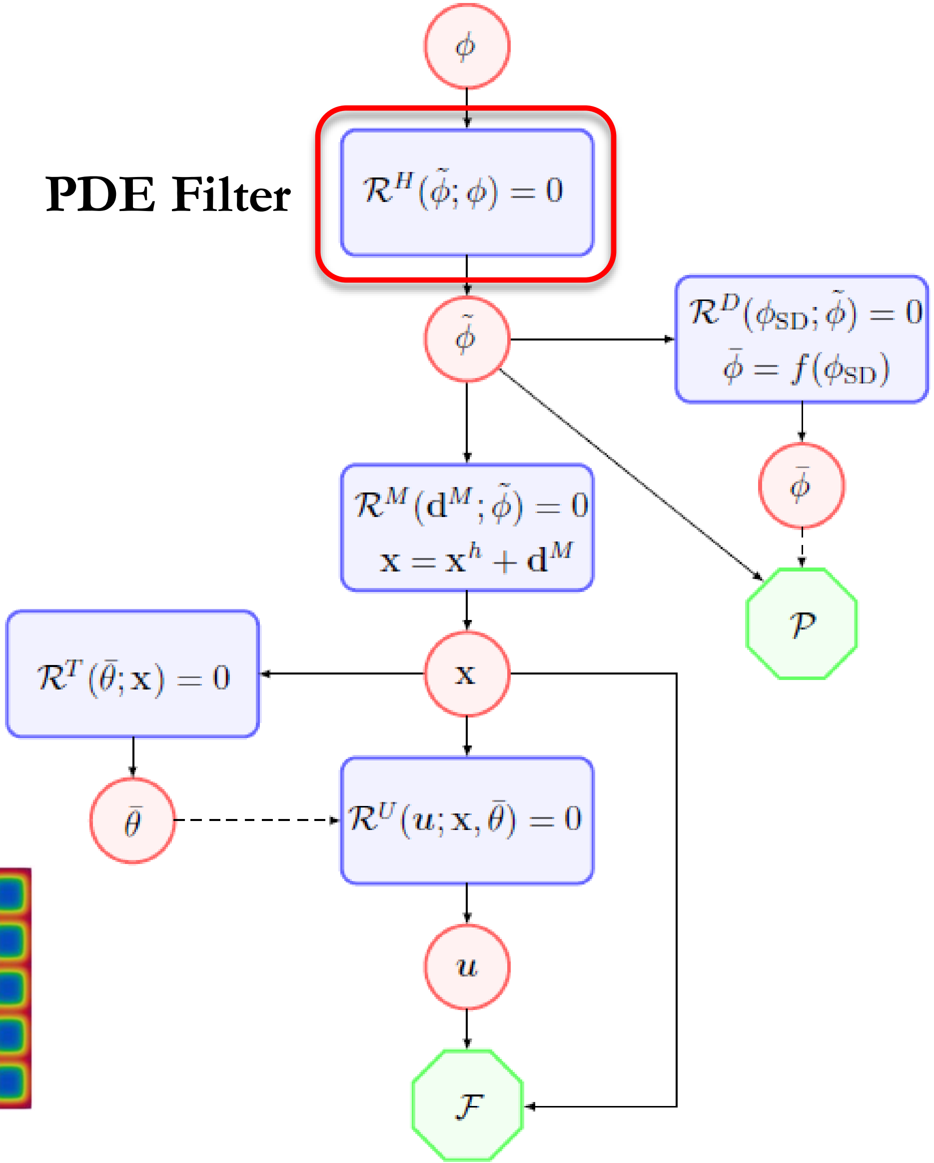
- Increased convergence of TO problem
- Mesh independence
- Promotes smooth filtered LS field with smooth gradients
- *common* miniapp

$$R^H(\tilde{\phi};\phi) = \int_{\Omega^d} \left(\nabla\delta w\, r^2\, \nabla\tilde{\phi} + \delta w(\tilde{\phi} - \phi)\right) d\Omega$$



Design field

Filtered design field

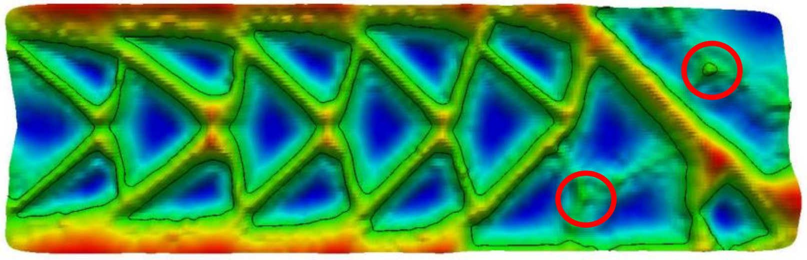


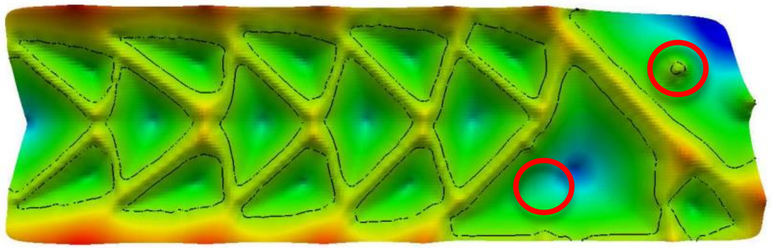
Optimization Graph

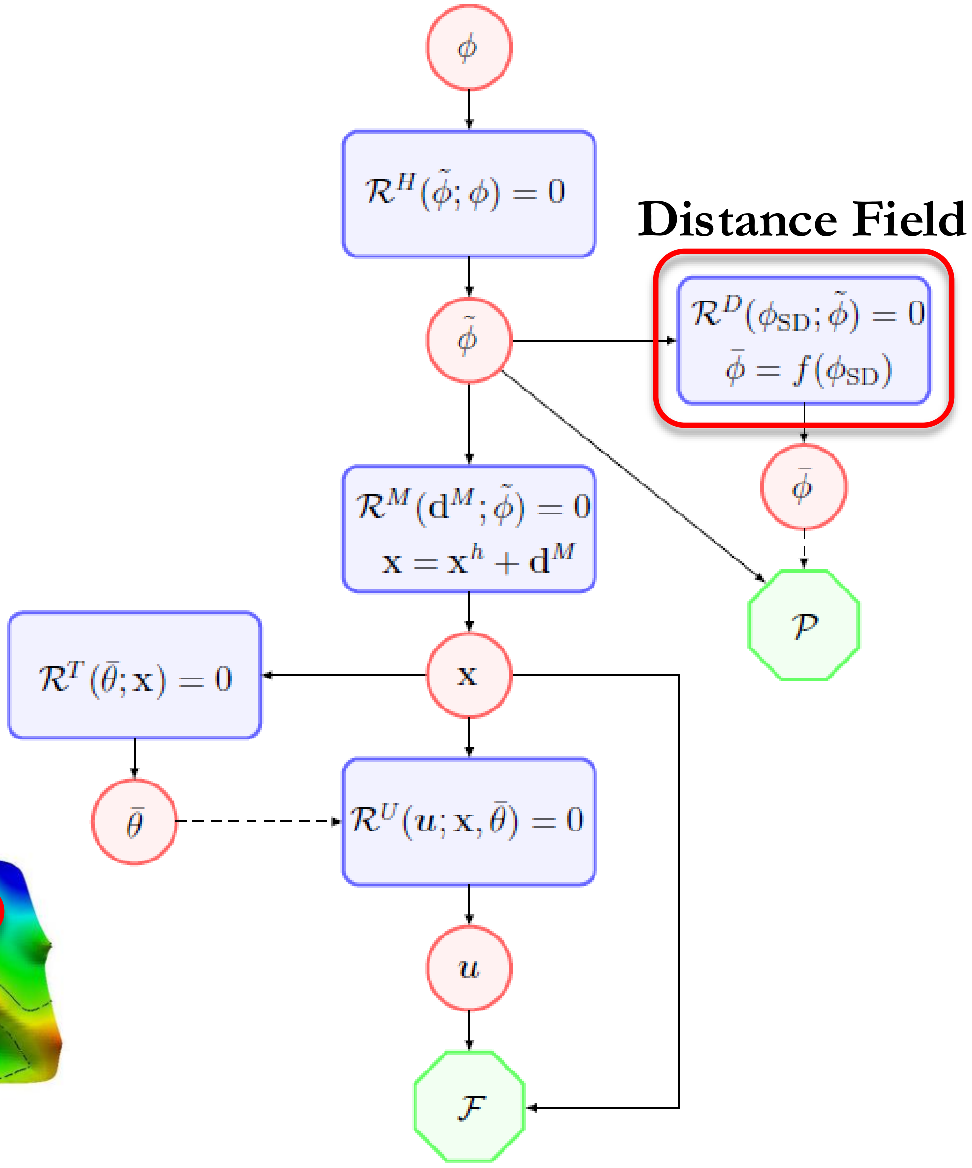
- Distance field computation: *common* miniapp
- Promotes consistent fitting

$$R^D(\phi_{SD};\phi) = \int_{\Omega^d} (\nabla\delta w \mid \nabla\phi_{SD}|^{p-2}\nabla\phi_{SD} - \delta w)\, d\Omega$$

$$\mathcal{P} = \frac{\int_{\Omega^d} w_\phi \left(\tilde{\phi} - \bar{\phi}\right)^2 d\Omega^d}{\int_{\Omega^d} \phi^2_{\mathrm{Bnd}} d\Omega^d} + \frac{\int_{\Omega^d} w_{\nabla\phi}|\nabla\tilde{\phi} - \nabla\bar{\phi}|^2 d\Omega^d}{\int_{\Omega^d} d\Omega^d}$$


Filtered design field

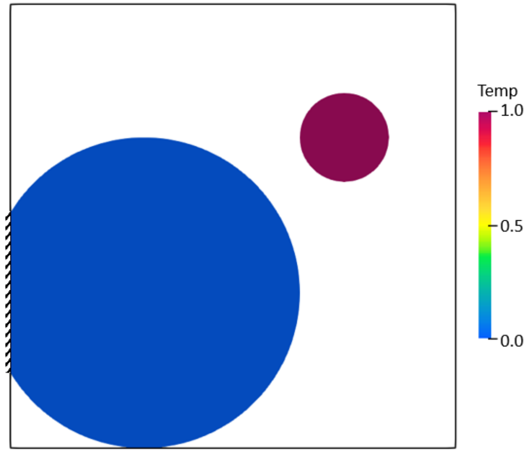

Signed distance field

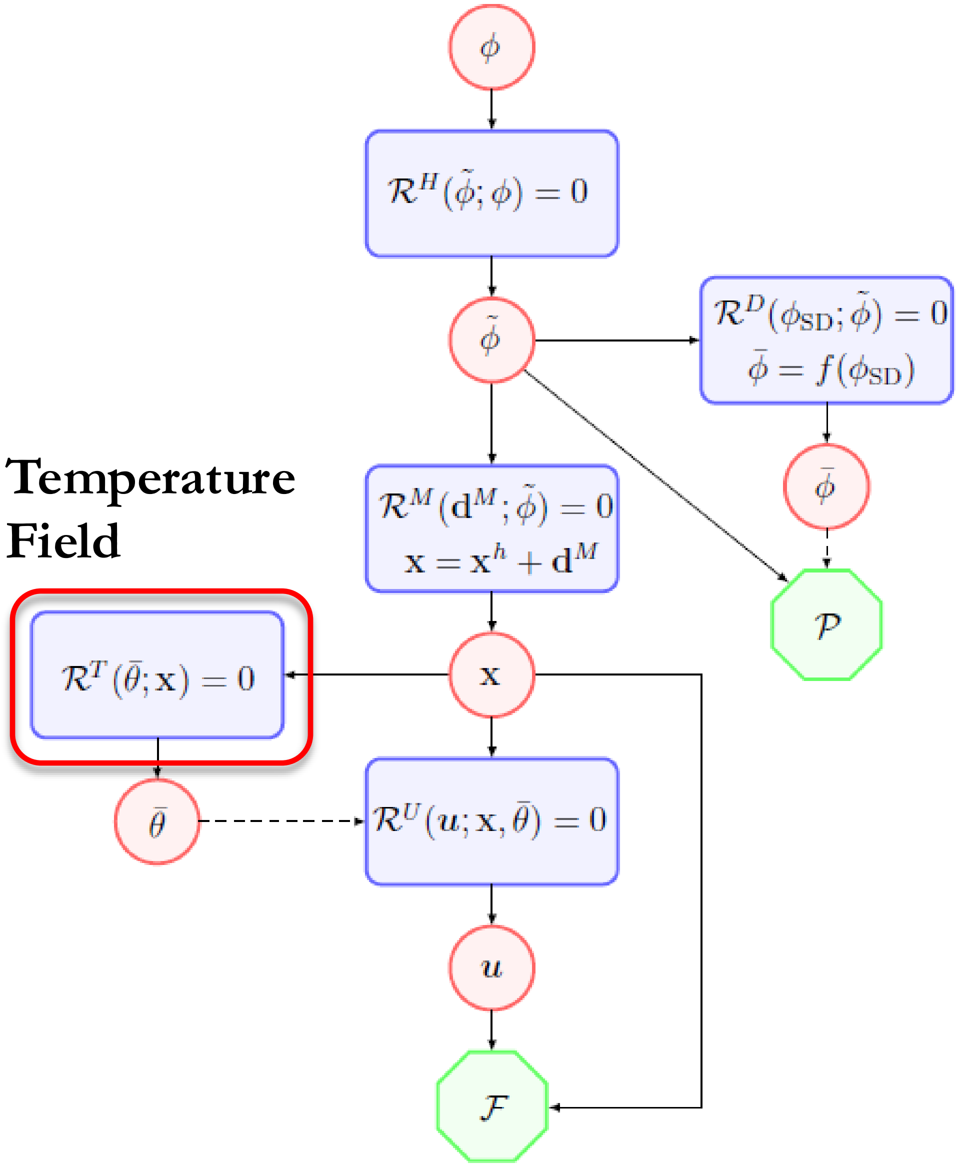


Optimization Graph

- Instabilities through disconnected subregions
- Island detection

$$R^T(\theta; x) = \int_\Omega \left(\nabla\delta w\, \kappa\nabla\theta + \delta w(\theta - 1)\right) d\Omega$$

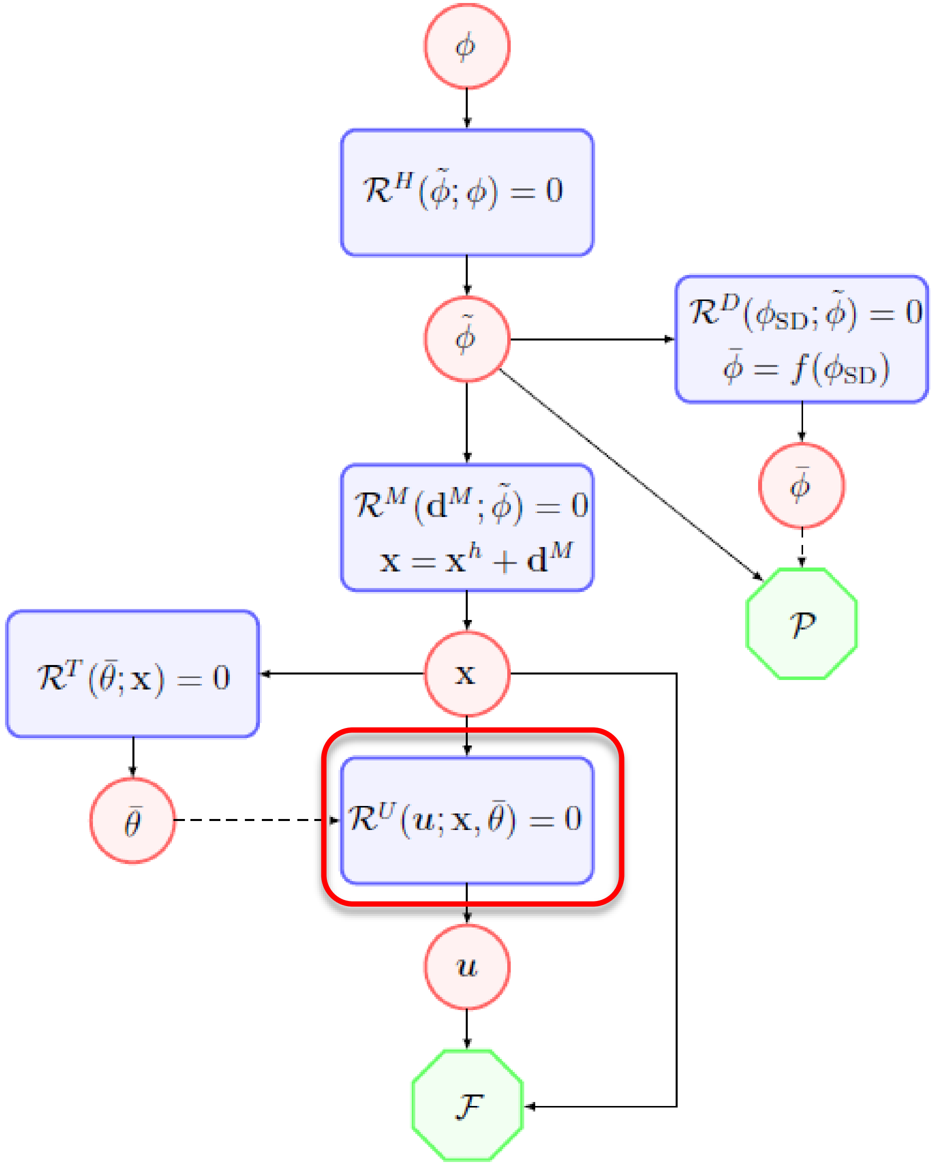




Optimization Graph

- Structural islands are stabilized

$$\boldsymbol{R}^U(u; x, \theta) = \int_{\Omega_I} \delta\boldsymbol{\varepsilon}(w) : \boldsymbol{\sigma}(u)\, d\Omega - \int_{\Gamma_N^{\Omega_I}} \delta w\, \boldsymbol{t}_N\, d\Gamma + \int_{\Omega_I} \delta w u r_s \frac{E}{h^2} \theta\, d\Omega$$



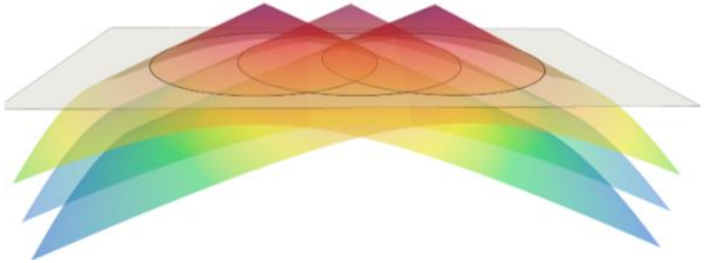
Multi-material mesh morphing through TMOP

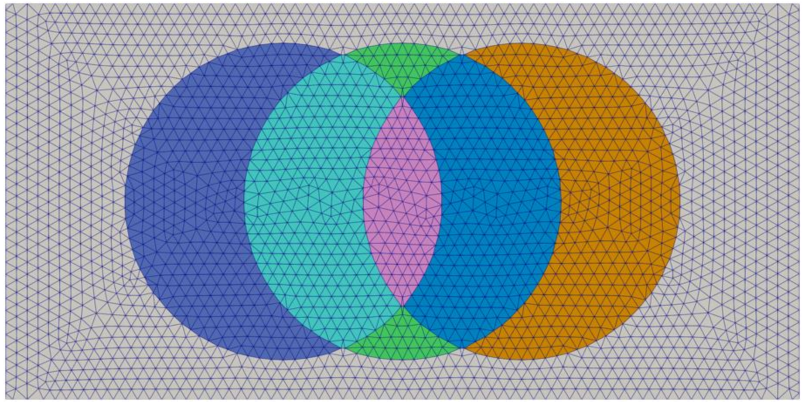
- 2^N number of subphases:
- Subphase index

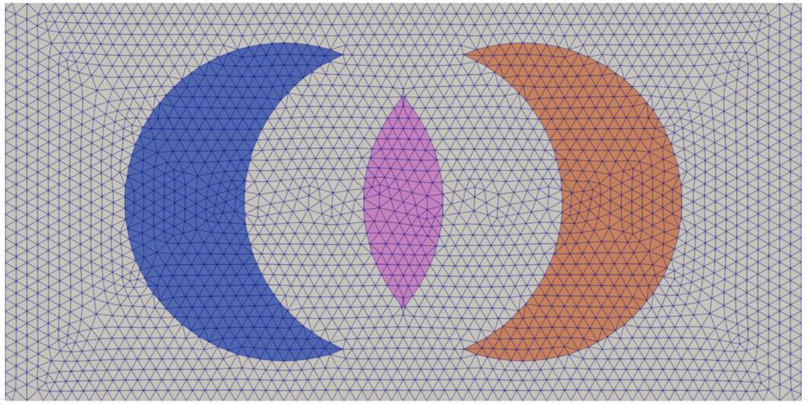
$$I = \sum_{i=0} 2^i H_i$$

- User defined subphase index:
  - Table: I → I_{custom}

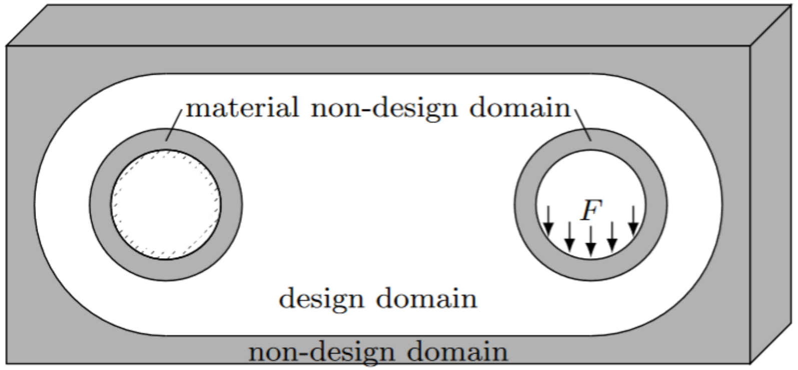
$$F(x) = \int_{\Omega} \mu(T(x))d\Omega + \omega_\sigma \sum_{i \in N_\phi} \sum_{s \in S_{\phi_i}} \phi_i^2(x_s)$$



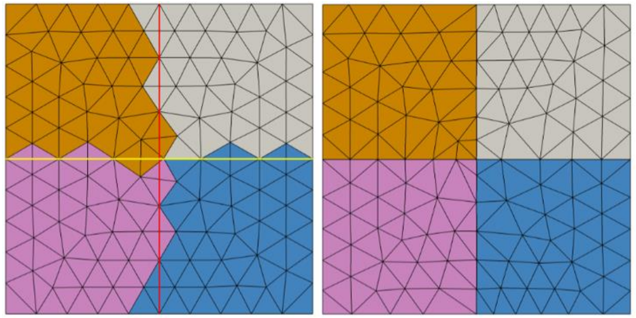


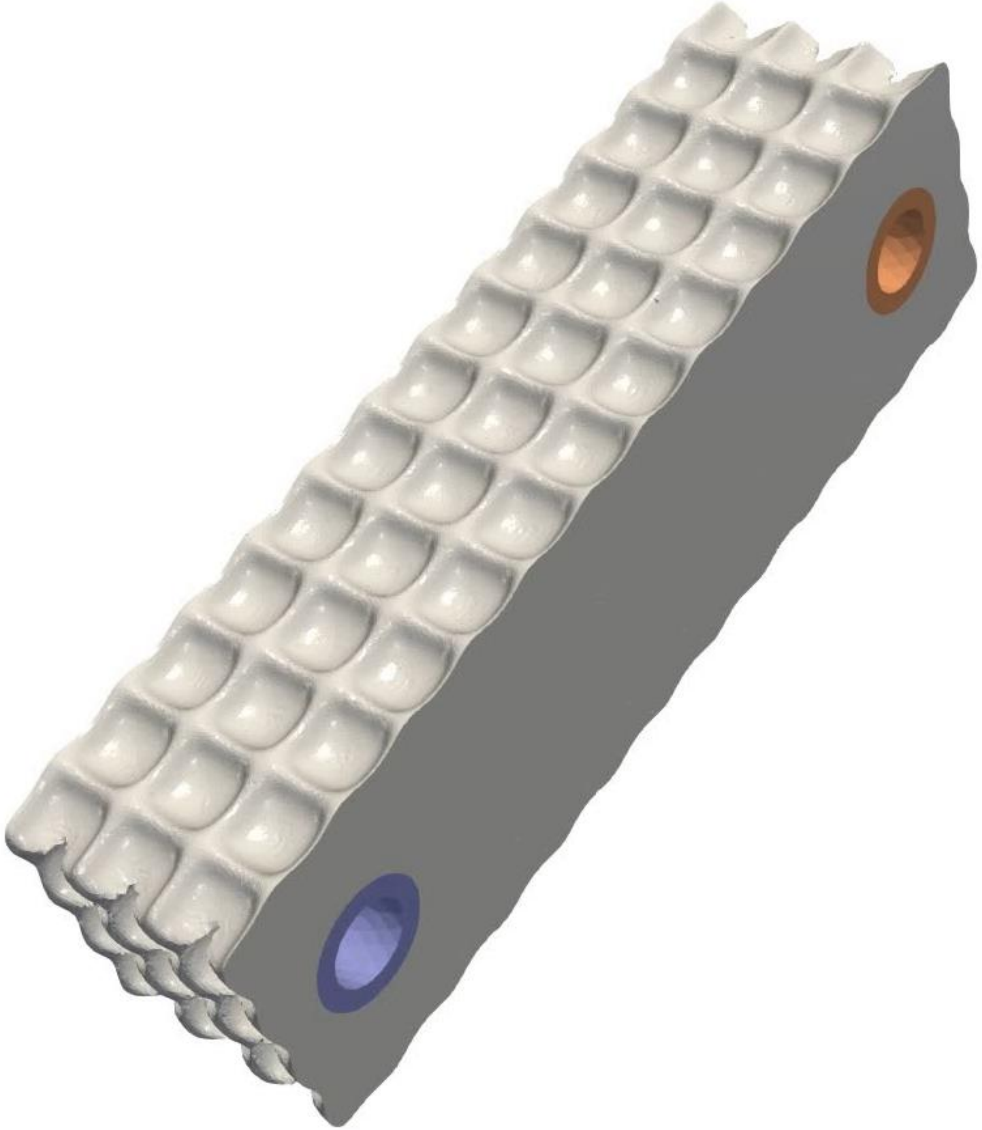


3D Bracket

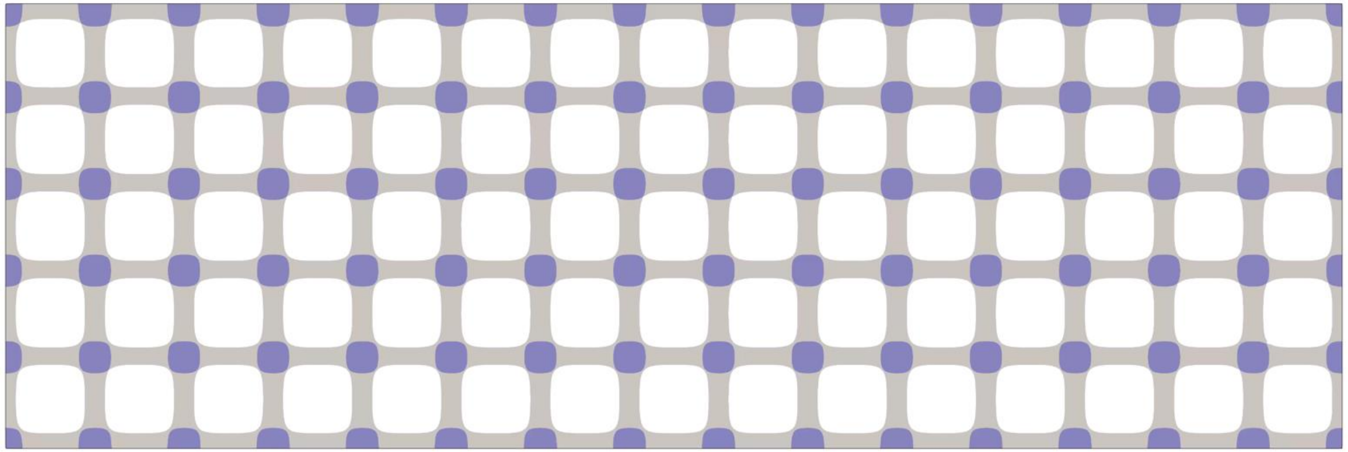


- Enables modeling of fixed components
- Enables enforcement of assembly requirements

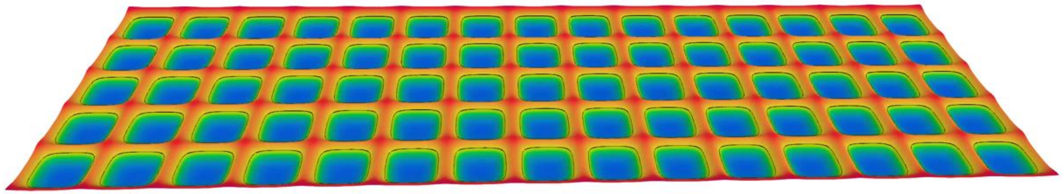


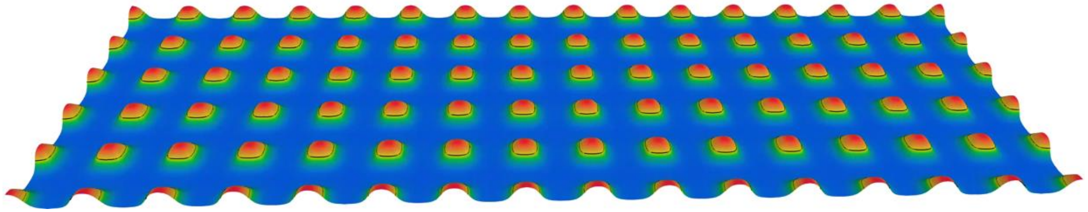


- Evolution of the structure



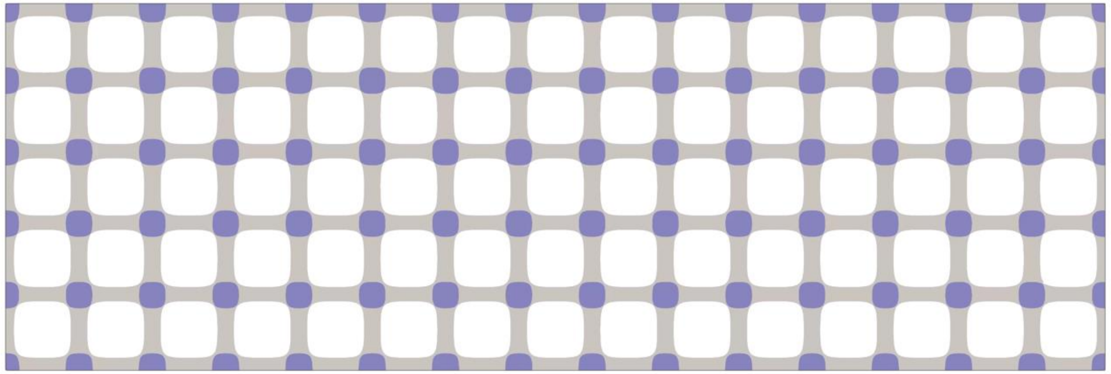
- Evolution of level-set fields



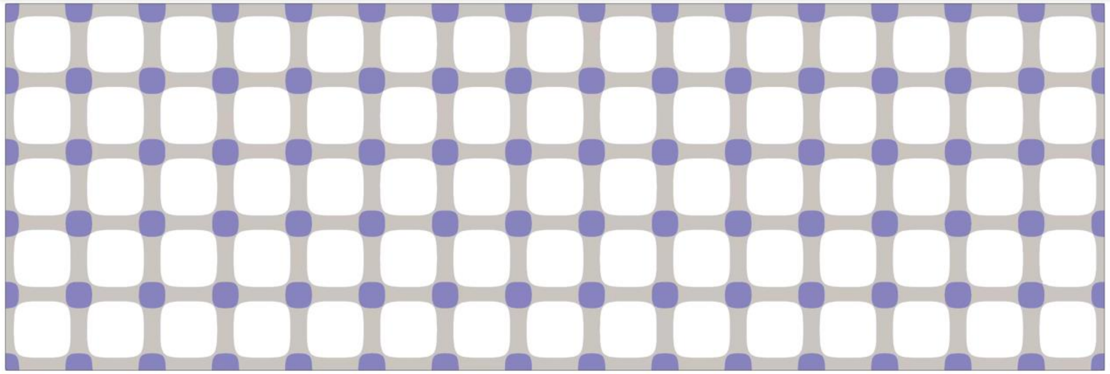


2D Cantilever comparison

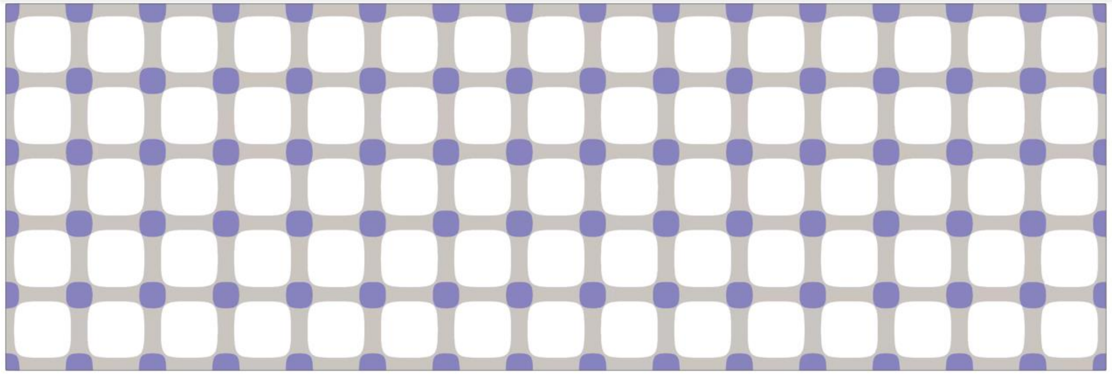
$$\frac{E_2}{E_1} = \frac{1}{2}$$



$$\frac{E_2}{E_1} = \frac{1}{10}$$



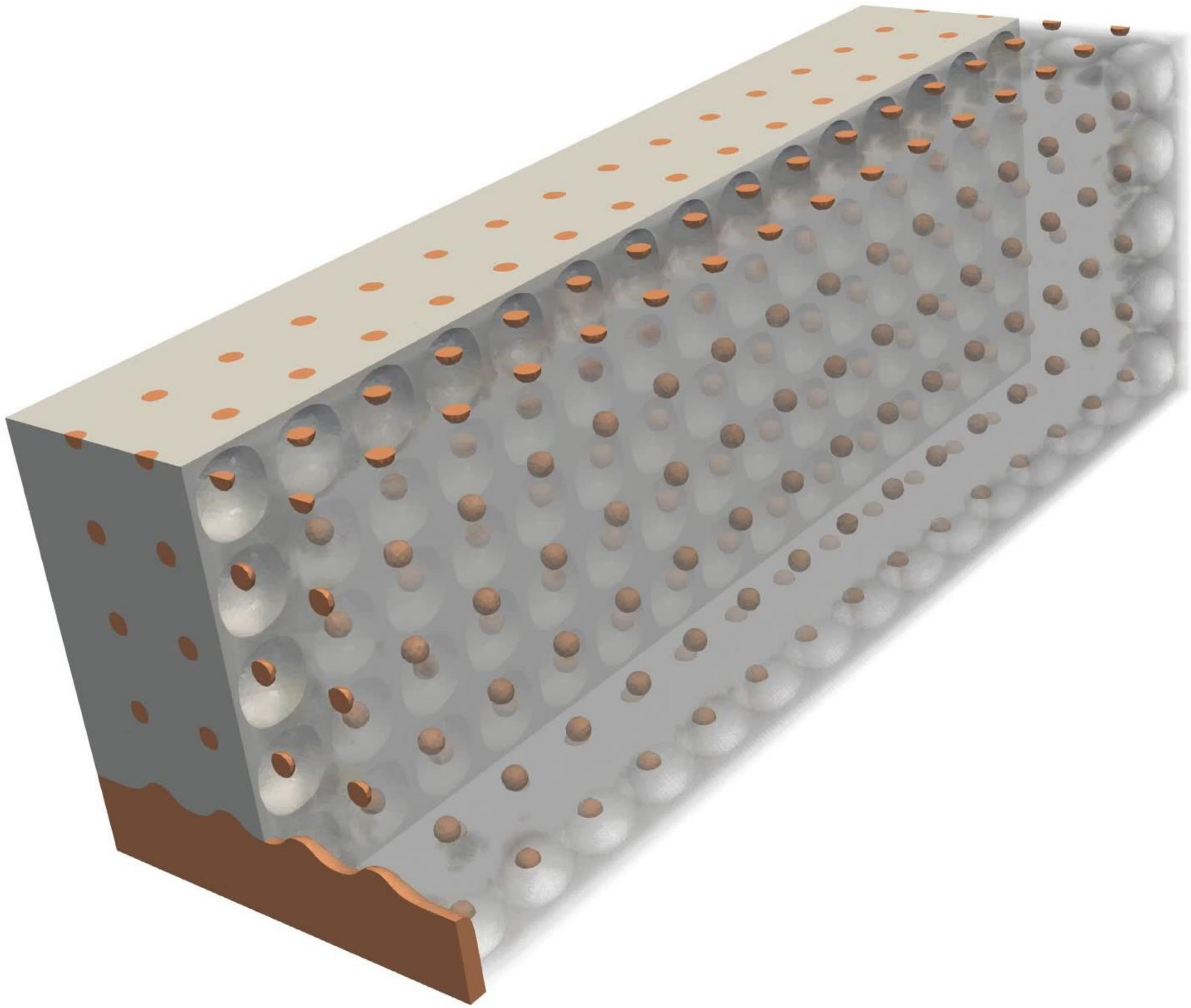
$$\frac{E_2}{E_1} = \frac{1}{20}$$



3D Multi-material cantilever

3D-Cantiliver Beam

- Topology optimization for external structure and internal material layout



Summary and outlook

Summary

- Presented solely **PDE-based** level-set TO approach
- Utilized PDE generated conformal analysis meshes
- Highlighted selected material-void applications
- Discussed multi-LS formulation
- Presented multi-LS TO with single design field results

Thank you!

Any questions?

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