

OPEN-SOURCE FRAMEWORK FOR IMMERSED FSI MODELING USING FEBIO AND MFEM

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- CHOP NHLBI T32 Program

- Jolley Lab

- Matthew Jolley
- Jalaj Maheshwari
- Wensi Wu (Penn)
- Jolley Lab Team

- FEBio Team

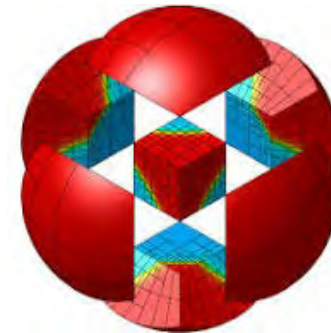
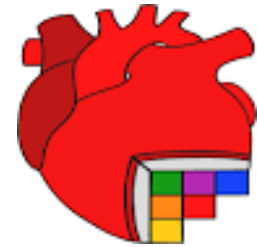
- Jeffrey Weiss
- Steve Maas

- MFEM Team

- Tzanio Kolev
- John Camier
- Veselin Dobrev



National Institutes
of Health



OUTLINE

- Clinical Problem
 - Valve Disease, Treatments, Challenges
- Fluid-Structure Interaction (FSI) modeling for heart valves
- Immersed FSI method
- Open-source computational FSI framework
- Current Efforts
- Summary and Future Work

VALVE DISEASE AND DYSFUNCTION

Types of Valve Dysfunction

Stenosis

Normal aortic valve



Open

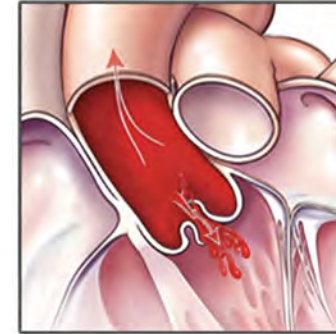
Aortic valve stenosis



Open

Restricted opening
→ accelerated jet

Regurgitation



Incomplete closure
→ backward flow

How does dysfunction arise in CHD?

Valve Morphology



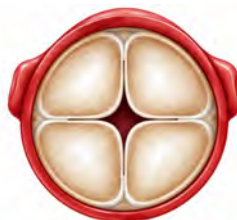
Unicuspid (single cusp)



Bicuspid (two cusps)



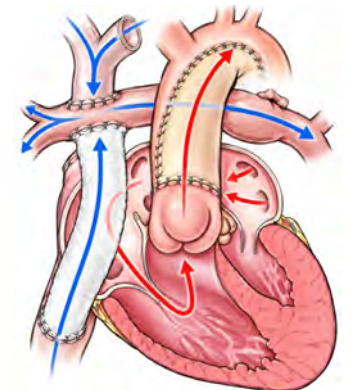
Tricuspid (3 cusps)



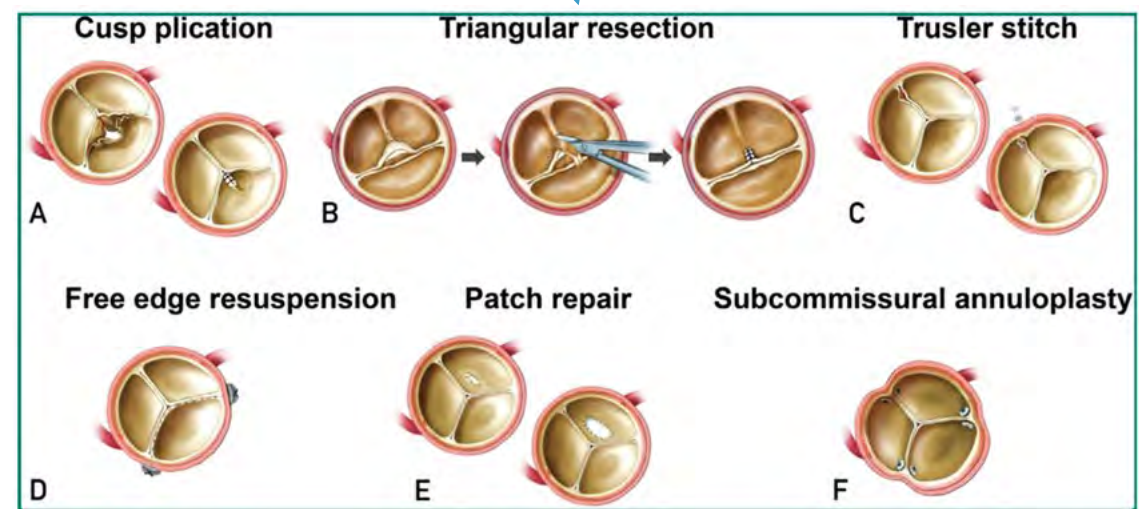
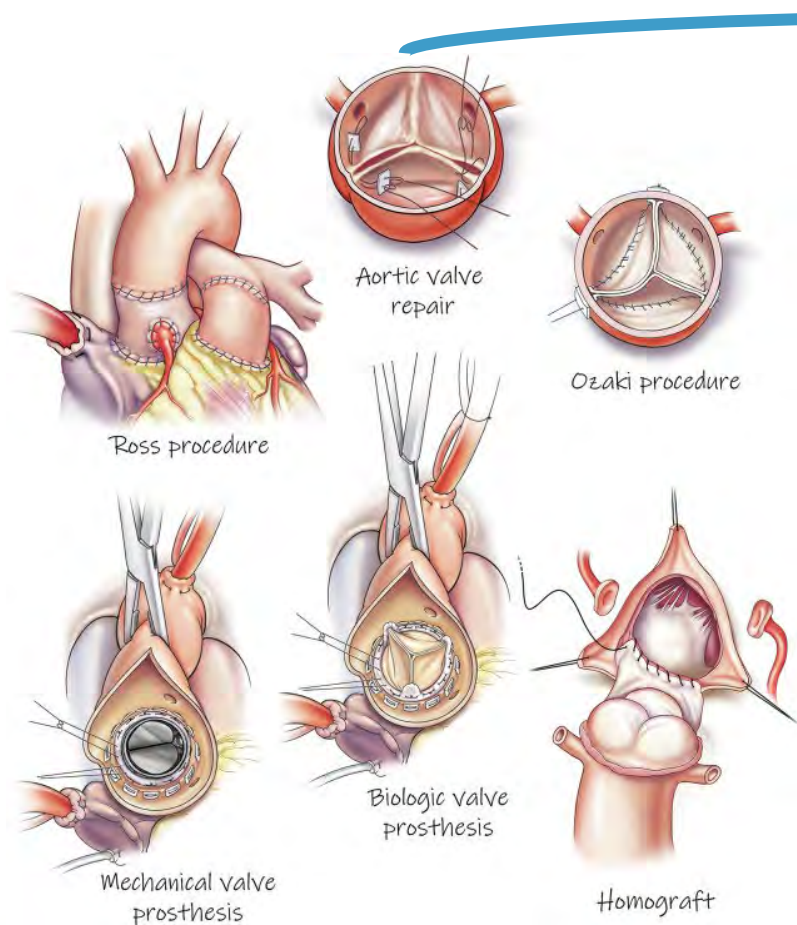
Quadricuspid (4 cusps)

Mechanical Loading

Surgically altered geometry and circulation



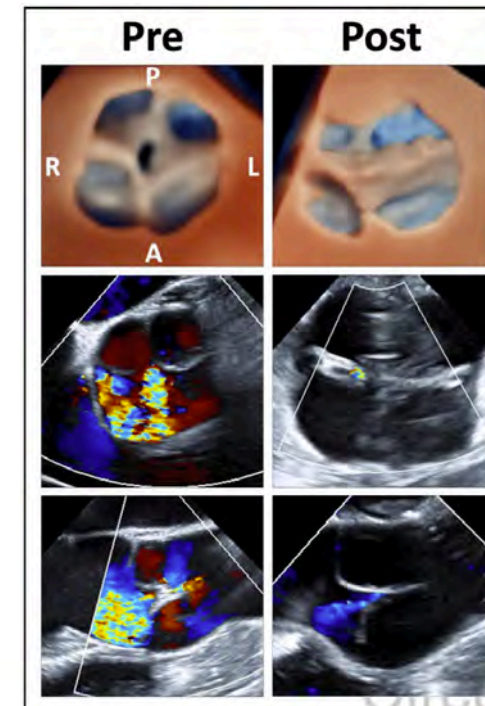
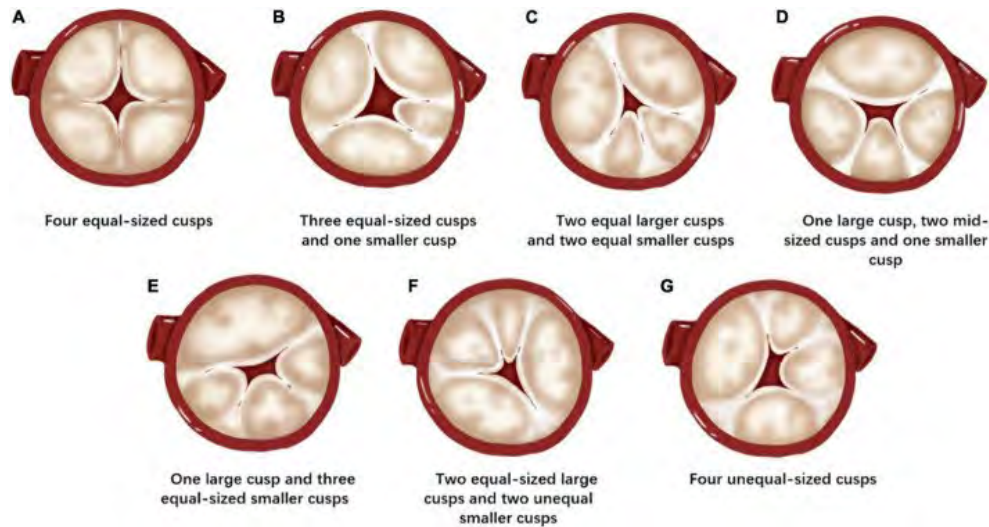
HEART VALVE TREATMENTS



HEART VALVE TREATMENTS: CHALLENGES

- Heterogeneous population → Highly **individualized** treatment

- Initially successful repair can fail a few months later

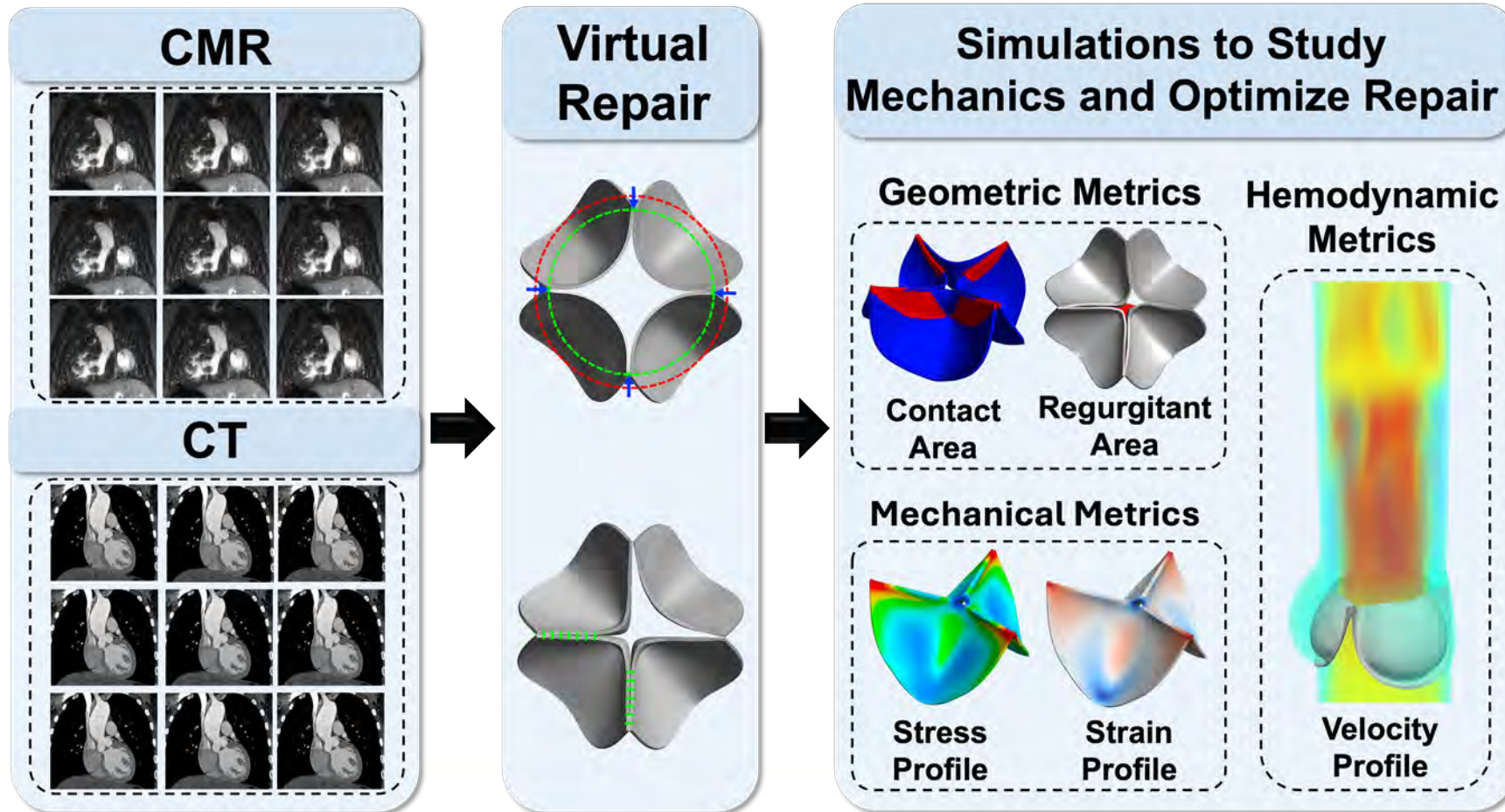


CARDIOVASCULAR IMAGES

Truncal Valve Repair: 3-Dimensional Imaging and Modeling to Enhance Preoperative Surgical Planning

Trevor R. Williams, MD¹; Alana R. Cianciulli, BS¹; Yin Wang, RDCS²; Andras Lasso, PhD³; Csaba Pinter, PhD⁴; Alison M. Pouch, PhD⁵; David M. Biko, MD⁶; Muhammad Nuri, MD⁷; Michael D. Quartermain, MD⁸; Lindsay S. Rogers, MD⁹; Jonathan M. Chen, MD¹⁰; Matthew A. Jolley, MD¹¹

COMPUTATIONAL MODELING TO INFORM PATIENT-SPECIFIC REPAIR



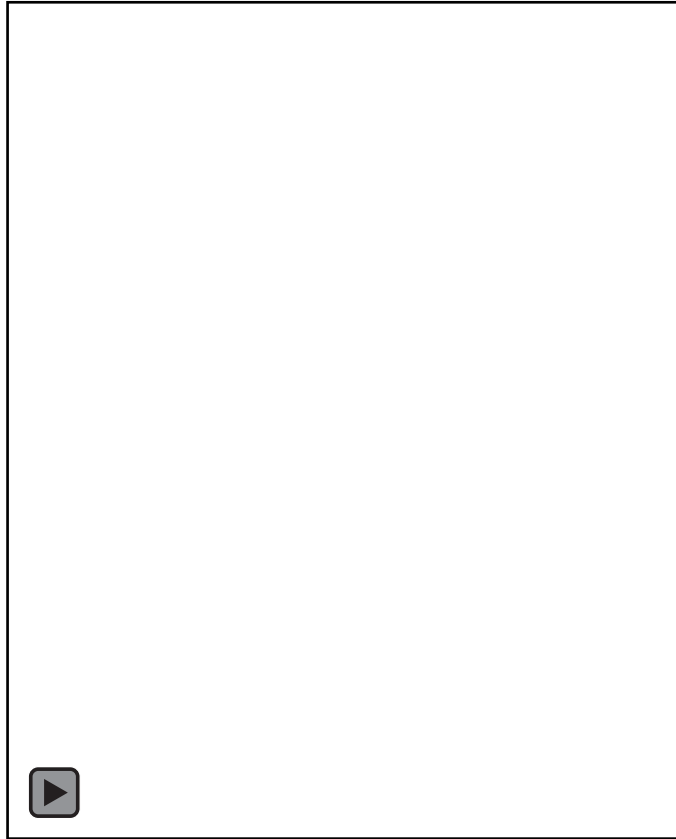
SlicerHeart

FEBio MFEM iFSI Solver

FSI MODELING FOR HEART VALVES

Flow Features:

- Nearly-transitional flow regime
- Pulsatile flow
 - Jet, Recirculation
- Sharp gradients in velocity and pressure

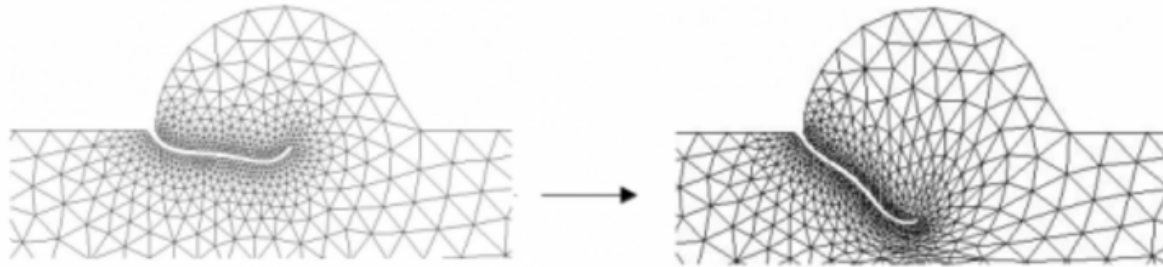


Solid Mechanics Modeling:

- Nonlinear solid material
 - Incompressible
 - Anisotropic
- Thin structure
 - Large deformations
 - Contact

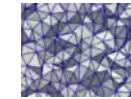
Kamensky et al. *CMAME*. 2015

FSI METHODS: VALVE FSI



Bavo et al. PLOS ONE. 2016

ALE-FSI



Fluid mesh

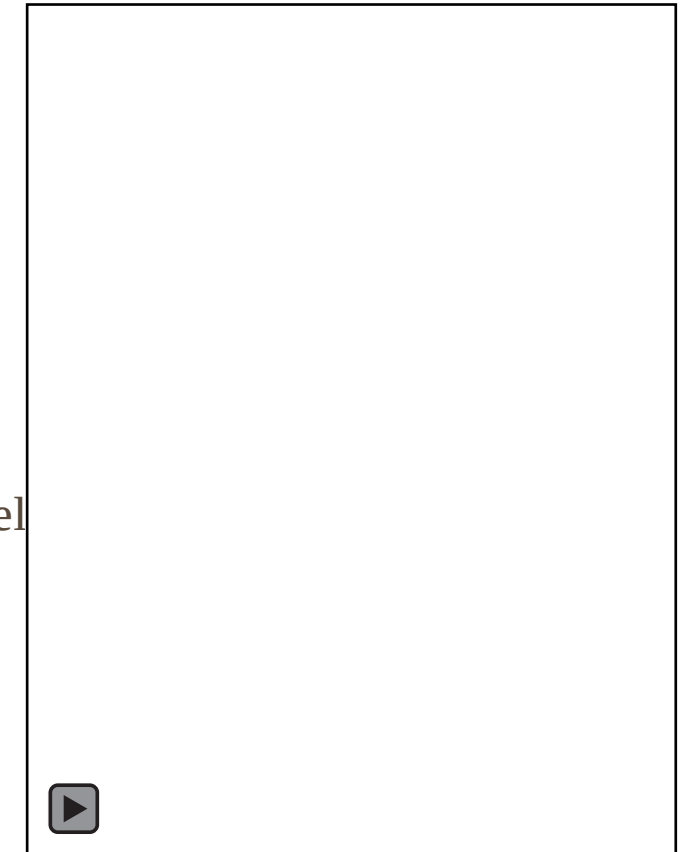


Leaflets model

Table 2. Computational time required by the two techniques. In the 3D case, for the ALE the time of the partial simulation is reported, while for the IB-FSI the time for the complete simulation is listed.

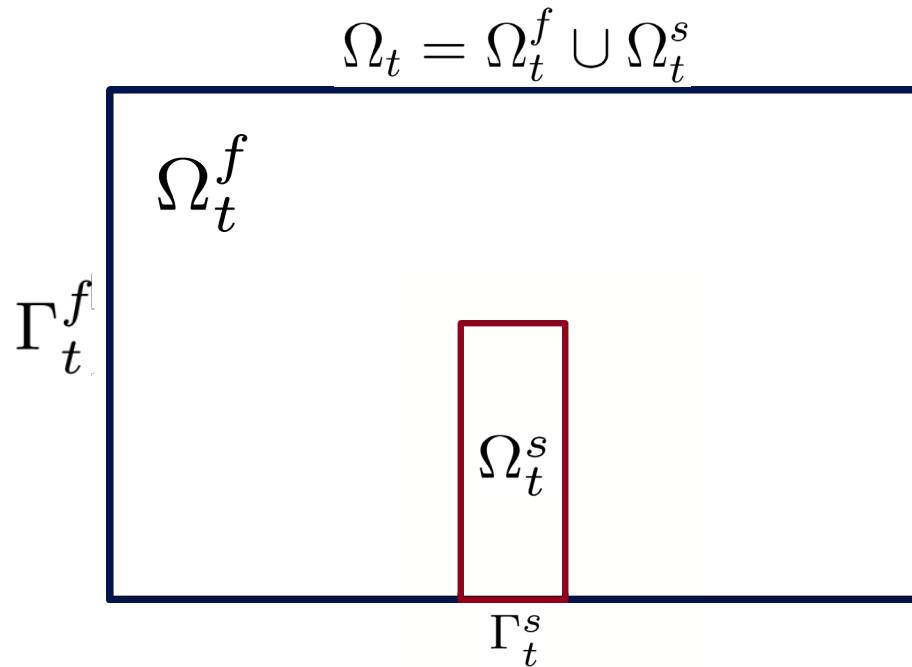
2D		3D	
ALE-FSI	IB-FSI	ALE-FSI	IB-FSI
48 h	1h	> 3 weeks (early opening)	30 h (entire simulation)

Bavo et al. PLOS ONE. 2016



Immersed-FSI

FSI METHODS: ALE- VS IMMERSED-FSI



ALE-FSI Momentum:

$$\int_{\Omega_t^f} \mathbf{w}^f \cdot \rho^f \left(\frac{\partial \mathbf{v}^f}{\partial t} + (\mathbf{v}^f - \hat{\mathbf{v}}^f) \cdot \nabla \mathbf{v}^f - \mathbf{f} \right) d\Omega + \int_{\Omega_t^f} \nabla \mathbf{w}^f : \boldsymbol{\sigma}^f d\Omega - \int_{\Gamma_{t,N}^f} \mathbf{w}^f \cdot \mathbf{h}_f d\Gamma$$

$$+ \int_{\Omega_t^s} \mathbf{w}^s \cdot \rho^s (\ddot{\mathbf{u}}^s - \mathbf{f}) d\Omega + \int_{\Omega_t^s} \nabla \mathbf{w}^s : \boldsymbol{\sigma}^s d\Omega - \int_{\Gamma_{t,N}^s} \mathbf{w}^s \cdot \mathbf{h}_s d\Gamma = 0.$$

$$\left(- \int_{\Gamma_t^{fsi}} \left(\mathbf{w}^f \cdot \boldsymbol{\sigma}^f \mathbf{n}^f + \mathbf{w}^s \cdot \boldsymbol{\sigma}^s \mathbf{n}^s \right) d\Gamma \right)$$

$$\mathbf{v}^f = \mathbf{v}^s \text{ on } \Gamma_t^{fsi} \quad \mathbf{w}^f = \mathbf{w}^s \text{ on } \Gamma_t^{fsi}$$

Fictitious Domain (FD) -FSI Momentum: (Immersed FSI)

Glowinski et al. IJMF. 1999
Hesch et al. CMAME. 2014
Black & Park. CMAME. 2025

$$\int_{\Omega} \mathbf{w}^f \cdot \rho^f (\dot{\mathbf{v}}^f - \mathbf{f}) d\Omega + \int_{\Omega} \nabla \mathbf{w}^f : \boldsymbol{\sigma}^f d\Omega - \int_{\Gamma_{t,N}^f} \mathbf{w}^f \cdot \mathbf{h}_f d\Gamma + \int_{\Omega_t^s} \mathbf{w}^s \cdot (\rho^s - \rho^f) (\dot{\mathbf{v}}^s - \mathbf{f}) d\Omega + \int_{\Omega_t^s} \nabla \mathbf{w}^s : (\boldsymbol{\sigma}^s - \boldsymbol{\sigma}^f) d\Omega = 0$$

$$\mathbf{v}^f = \mathbf{v}^s \text{ in } \Omega_t^s \quad \mathbf{w}^f = \mathbf{w}^s \text{ in } \Omega_t^s$$

IMMERSED FSI METHOD

Glowinski et al. IJMF. 1999
Hesch et al. CMAME. 2014
Black & Park. CMAME. 2025

Immersed nearly-incompressible solids

$$\int_{\Omega} q \nabla \cdot \mathbf{v}^f d\Omega = 0 \quad \mathbf{v}^f = \mathbf{v}^s \text{ in } \Omega_t^s$$



$$\begin{aligned} & \int_{\Omega} \mathbf{w}^f \cdot \rho^f (\dot{\mathbf{v}}^f - \mathbf{f}) d\Omega + \int_{\Omega} \nabla \mathbf{w}^f : \boldsymbol{\sigma}^f d\Omega - \int_{\Gamma_{t,N}^f} \mathbf{w}^f \cdot \mathbf{h}_f d\Gamma + \int_{\Omega_t^s} \mathbf{w}^s \cdot (\rho^s - \rho^f) (\dot{\mathbf{v}}^s - \mathbf{f}) d\Omega \\ & + \int_{\Omega_t^s} \nabla \mathbf{w}^s : \boldsymbol{\sigma}^s d\Omega - \int_{\Omega_t^s} 2\mu^f \boldsymbol{\epsilon}(\mathbf{w}^s) : \boldsymbol{\epsilon}(\mathbf{v}^f) d\Omega = 0. \end{aligned}$$

Viscous fluid stress

Solid constitutive modeling:

Holzapfel. Nonlinear Solid Mechanics. 2000

$$\psi(\mathbf{C}) = \psi_{vol}(J) + \psi_{iso}(\bar{\mathbf{C}})$$

$$\bar{\mathbf{C}} = \bar{\mathbf{F}}^T \bar{\mathbf{F}} \quad \mathbf{F} = J^{1/3} \bar{\mathbf{F}}$$

IMMERSED FSI VARIATIONAL PROBLEM

Glowinski et al. IJMF. 1999
Hesch et al. CMAME. 2014
Black & Park. CMAME. 2025

Find $(\mathbf{v}_h^f, \dot{\mathbf{u}}_h^s, p_h, \boldsymbol{\lambda}_h) \in \mathcal{S}_{vf}^h \times \mathcal{S}_{vs}^h \times \mathcal{S}_p^h \times \mathcal{S}_\lambda^h$ such that $\forall (\mathbf{w}_h^f, \mathbf{w}_h^s, q_h, \delta \boldsymbol{\lambda}_h) \in \mathcal{S}_{vf,0}^h \times \mathcal{S}_{vs,0}^h \times \mathcal{S}_{p,0}^h \times \mathcal{S}_\lambda^h$

Fluid

$$\int_{\Omega} \mathbf{w}_h^f \cdot \rho^f \left(\frac{\partial \mathbf{v}_h^f}{\partial t} + \mathbf{v}_h^f \cdot \nabla \mathbf{v}_h^f - \mathbf{f} \right) d\Omega - \int_{\Omega} p_h \nabla \cdot \mathbf{w}_h^f d\Omega + \int_{\Omega} 2\mu_f \boldsymbol{\epsilon}(\mathbf{w}_h^f) : \boldsymbol{\epsilon}(\mathbf{v}_h^f) d\Omega - \int_{\Gamma_{t,N}^f} \mathbf{w}_h^f \cdot \mathbf{h}_f d\Gamma + \int_{\Omega_t^s} \mathbf{w}_h^f \cdot \boldsymbol{\lambda}_h d\Omega = 0$$

$$\int_{\Omega} q_h \nabla \cdot \mathbf{v}_h^f d\Omega = 0$$

Solid

$$\int_{\Omega_t^s} \mathbf{w}_h^s \cdot (\rho^s - \rho^f) (\ddot{\mathbf{u}}_h^s - \mathbf{f}) d\Omega + \int_{\Omega_t^s} \nabla \mathbf{w}_h^s : \boldsymbol{\sigma}^s d\Omega - \int_{\Omega_t^s} 2\mu^f \boldsymbol{\epsilon}(\mathbf{w}_h^s) : \boldsymbol{\epsilon}(\mathbf{v}_h^f) d\Omega - \int_{\Omega_t^s} \mathbf{w}_h^s \cdot \boldsymbol{\lambda}_h d\Omega = 0$$

FSI Constraint: Weak enforcement of $\mathbf{v}^f = \mathbf{v}^s$ in Ω_t^s using a Lagrange multiplier

$$\int_{\Omega_t^s} \delta \boldsymbol{\lambda}_h \cdot (\mathbf{v}_h^f - \dot{\mathbf{u}}_h^s) d\Omega = 0$$

IMMERSED FSI VARIATIONAL PROBLEM

Glowinski et al. IJMF. 1999
Hesch et al. CMAME. 2014
Black & Park. CMAME. 2025

Find $(\mathbf{v}_h^f, \dot{\mathbf{u}}_h^s, p_h, \boldsymbol{\lambda}_h) \in \mathcal{S}_{vf}^h \times \mathcal{S}_{vs}^h \times \mathcal{S}_p^h \times \mathcal{S}_\lambda^h$ such that $\forall (\mathbf{w}_h^f, \mathbf{w}_h^s, q_h, \delta \boldsymbol{\lambda}_h) \in \mathcal{S}_{vf,0}^h \times \mathcal{S}_{vs,0}^h \times \mathcal{S}_{p,0}^h \times \mathcal{S}_\lambda^h$

Fluid

$$\begin{aligned} & \int_{\Omega} \mathbf{w}_h^f \cdot \rho^f \left(\frac{\partial \mathbf{v}_h^f}{\partial t} + \mathbf{v}_h^f \cdot \nabla \mathbf{v}_h^f - \mathbf{f} \right) d\Omega - \int_{\Omega} p_h \nabla \cdot \mathbf{w}_h^f d\Omega + \int_{\Omega} 2\mu_f \boldsymbol{\epsilon}(\mathbf{w}_h^f) : \boldsymbol{\epsilon}(\mathbf{v}_h^f) d\Omega - \int_{\Gamma_{t,N}^f} \mathbf{w}_h^f \cdot \mathbf{h}_f d\Gamma + \int_{\Omega_t^s} \mathbf{w}_h^f \cdot \boldsymbol{\lambda}_h d\Omega \\ & + \int_{\Omega} \nabla \mathbf{w}_h : (\tau_m \mathbf{r}_m \otimes \mathbf{v}_h) d\Omega - \int_{\Omega} \nabla \mathbf{v}_h : (\mathbf{w}_h \otimes \tau_m \mathbf{r}_m) d\Omega - \int_{\Omega} \frac{1}{\rho^f} \nabla \mathbf{w}_h : (\tau_m \mathbf{r}_m \otimes \tau_m \mathbf{r}_m) d\Omega + \int_{\Omega} \rho^f (\nabla \cdot \mathbf{w}_h) \tau_c r_c d\Omega \\ & + \int_{\Omega} \tau_b (\tau_m \mathbf{r}_m \cdot \nabla \mathbf{w}^f) \cdot (\tau_m \mathbf{r}_m \cdot \nabla \mathbf{v}) d\Omega = 0 \\ & \int_{\Omega} q_h \nabla \cdot \mathbf{v}_h^f d\Omega - \int_{\Omega} \frac{1}{\rho^f} \nabla q_h \cdot \tau_m \mathbf{r}_m d\Omega = 0 \end{aligned}$$

Variational Multiscale Method (VMS):

- Improved accuracy on under-resolved grids
- Robustness for advection dominated flows

Bazilevs et al. CMAME. 2007
Bazilevs et al. Comput. Mech. 2008

Solid

$$\int_{\Omega_t^s} \mathbf{w}_h^s \cdot (\rho^s - \rho^f) (\ddot{\mathbf{u}}_h^s - \mathbf{f}) d\Omega + \int_{\Omega_t^s} \nabla \mathbf{w}_h^s : \boldsymbol{\sigma}^s d\Omega - \int_{\Omega_t^s} 2\mu^f \boldsymbol{\epsilon}(\mathbf{w}_h^s) : \boldsymbol{\epsilon}(\mathbf{v}_h^f) d\Omega - \int_{\Omega_t^s} \mathbf{w}_h^s \cdot \boldsymbol{\lambda}_h d\Omega = 0$$

FSI Constraint

$$\int_{\Omega_t^s} \delta \boldsymbol{\lambda}_h \cdot (\mathbf{v}_h^f - \dot{\mathbf{u}}_h^s) d\Omega = 0$$

GENERALIZED- α TIME INTEGRATION

Chung & Hilbert. *J Appl Mech.* 1993
Jansen et al. *CMAME.* 2000
Liu et al. *IJNME.* 2021

Solve the following nonlinear equations to go from t_n to t_{n+1} :

$$\mathbf{R}^m(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Fluid Momentum}$$

$$\mathbf{R}^c(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Fluid Mass}$$

$$\mathbf{R}^s(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Solid Momentum}$$

$$\mathbf{R}^{fsi}(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{FSI Constraint}$$

Newton-Raphson
(Predictor-Multicorrector)

Full linearized system Black & Park. *CMAME.* 2025

$$\begin{bmatrix} \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{V}}} & \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{P}}} & \mathbf{0} & \frac{\partial \mathbf{R}^m}{\partial \boldsymbol{\lambda}} \\ \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{V}}} & \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{P}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\partial \mathbf{R}^s}{\partial \ddot{\mathbf{U}}} & \frac{\partial \mathbf{R}^s}{\partial \boldsymbol{\lambda}} \\ \frac{\partial \mathbf{R}^{fsi}}{\partial \dot{\mathbf{V}}} & \mathbf{0} & \frac{\partial \mathbf{R}^{fsi}}{\partial \ddot{\mathbf{U}}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta \dot{\mathbf{V}} \\ \Delta \dot{\mathbf{P}} \\ \Delta \ddot{\mathbf{U}} \\ \Delta \boldsymbol{\lambda} \end{bmatrix} = - \begin{bmatrix} \mathbf{R}^m \\ \mathbf{R}^c \\ \mathbf{R}^s \\ \mathbf{R}^{fsi} \end{bmatrix}$$

Eliminate $\boldsymbol{\lambda}$, $N_A^\lambda = \delta(\mathbf{X} - \mathbf{X}_A) \rightarrow \text{IFEM}$

Zhang et al. *CMAME.* 2004

$$\begin{bmatrix} \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{V}}} + \mathbf{C}^\lambda \frac{\partial \mathbf{R}^s}{\partial \ddot{\mathbf{U}}} (\mathbf{C}^\lambda)^T & \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{P}}} \\ \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{V}}} & \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{P}}} \end{bmatrix} \begin{bmatrix} \Delta \dot{\mathbf{V}} \\ \Delta \dot{\mathbf{P}} \end{bmatrix} = - \begin{bmatrix} \mathbf{R}^m + \mathbf{C}^\lambda \mathbf{R}^s \\ \mathbf{R}^c \end{bmatrix}$$

$$\mathbf{C}^\lambda = \frac{\partial \mathbf{R}^{fsi}}{\partial \dot{\mathbf{V}}} \quad \mathbf{C}_{AB}^\lambda = N_A^v(\phi(\mathbf{X}_B, t)) \mathbf{I}$$

GENERALIZED- α TIME INTEGRATION

Chung & Hilbert. *J Appl Mech.* 1993
Jansen et al. *CMAA*. 2000
Liu et al. *IJNME*. 2021

Solve the following nonlinear equations to go from t_n to t_{n+1} :

$$\mathbf{R}^m(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Fluid Momentum}$$

$$\mathbf{R}^c(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Fluid Mass}$$

$$\mathbf{R}^s(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{Solid Momentum}$$

$$\mathbf{R}^{fsi}(\mathbf{V}, \mathbf{P}, \mathbf{U}, \boldsymbol{\lambda}) = \mathbf{0} \quad \text{FSI Constraint}$$

Newton-Raphson
(Predictor-Multicorrector)

Newton-Raphson method: Black & Park. *CMAA*. 2025

1) Predictor

$$\mathbf{Y} = \{\mathbf{V}, \mathbf{P}\}^T$$

$$\mathbf{Y}_{n+1,(0)} = \mathbf{Y}_n, \quad \dot{\mathbf{Y}}_{n+1,(0)} = \frac{\gamma-1}{\gamma} \dot{\mathbf{Y}}_n$$

$$\ddot{\mathbf{U}}_{n+1,(0)} = (\mathbf{C}^\lambda)^T \dot{\mathbf{V}}_{n+1,(0)},$$

$$\dot{\mathbf{U}}_{n+1,(0)} = (\mathbf{C}^\lambda)^T \mathbf{V}_{n+1,(0)},$$

$$\mathbf{U}_{n+1,(0)} = \mathbf{U}_n + \Delta t_n \dot{\mathbf{U}}_n + \frac{\Delta t_n^2}{2} \left((1-2\beta) \ddot{\mathbf{U}}_n + 2\beta \ddot{\mathbf{U}}_{n+1,(0)} \right)$$

2) Multi-corrector

a) Compute intermediate (alpha-level) values

b) Solve linearized system at intermediate values

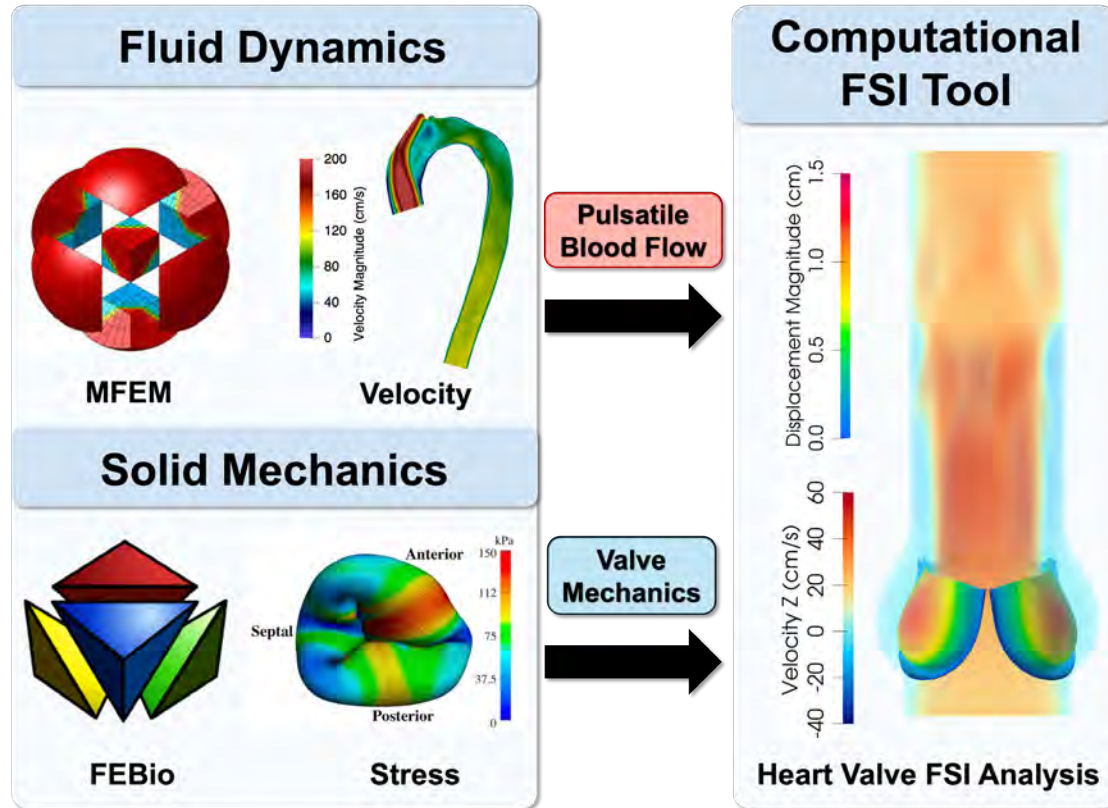
$$\begin{bmatrix} \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{V}}} + \mathbf{C}^\lambda \frac{\partial \mathbf{R}^s}{\partial \ddot{\mathbf{U}}} (\mathbf{C}^\lambda)^T \frac{\partial \mathbf{R}^m}{\partial \dot{\mathbf{P}}} \\ \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{V}}} \frac{\partial \mathbf{R}^c}{\partial \dot{\mathbf{P}}} \end{bmatrix} \begin{bmatrix} \Delta \dot{\mathbf{V}} \\ \Delta \dot{\mathbf{P}} \end{bmatrix} = - \begin{bmatrix} \mathbf{R}^m + \mathbf{C}^\lambda \mathbf{R}^s \\ \mathbf{R}^c \end{bmatrix}$$

c) Update variables

$$\Delta \dot{\mathbf{Y}}_{n+1,(l)} = \{\Delta \dot{\mathbf{V}}_{n+1,(l)}, \Delta \dot{\mathbf{P}}_{n+1,(l)}\}^T$$

$$\Delta \ddot{\mathbf{U}}_{n+1} = (\mathbf{C}^\lambda)^T \Delta \dot{\mathbf{V}}_{n+1}$$

FEBio MFEM iFSI Solver



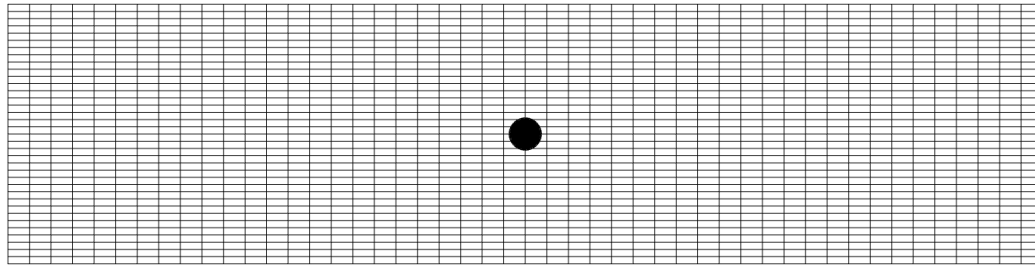
Key Features

- **FEBio**
 - Large material library
 - Advanced modeling capabilities
 - Contact, Three-field formulation
 - E.g. Growth & Remodeling
 - Formation of calcifications
 - Leaflet damage after repair
 - Tissue remodeling
- **MFEM**
 - Built for modern computing architectures
 - Scalable (MPI), state-of-the-art performance
 - GPU-ready infrastructure
 - Wide range of FEM technologies

Black R. T. et al. An open-source computational framework for immersed fluid-structure interaction modeling using FEBio and MFEM. *Engineering with Computers*. 2026.

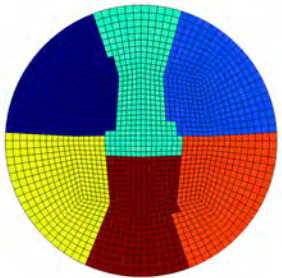
COMPUTING PARALLEL MESH COUPLING TERM

$$\mathbf{C}^\lambda = \frac{\partial \mathbf{R}^{fsi}}{\partial \dot{\mathbf{V}}} \quad \mathbf{C}_{AB}^\lambda = N_A^v(\phi(\mathbf{X}_B, t)) \mathbf{I}$$

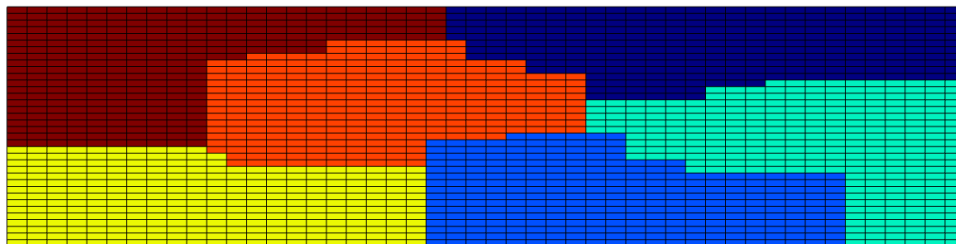


6 processors

Solid



Fluid



Column partition

p0	x	x	x	x			x	x	x
	x	x	x	x			x		x
	x	x	x						
p1				x			x	x	x
							x	x	x
							x	x	x
p2									
	x								

Row partition

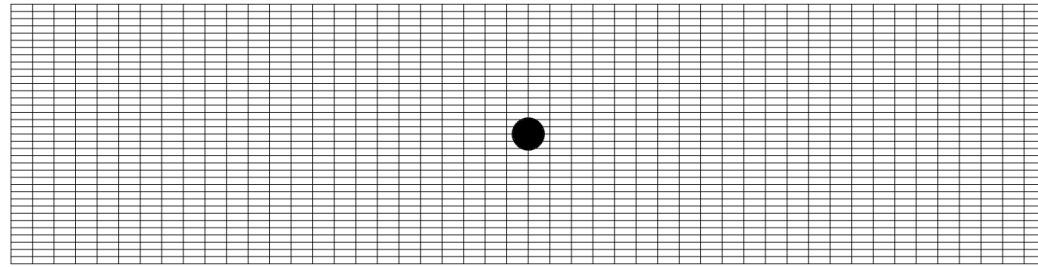
hypre

HypreIJ Interface →

Allows to set parallel matrix values as if setting (i,j)

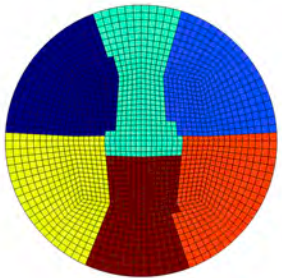
COMPUTING PARALLEL MESH COUPLING TERM

$$C^\lambda = \frac{\partial \mathbf{R}^{fsi}}{\partial \dot{\mathbf{V}}} \quad C_{AB}^\lambda = N_A^v(\phi(\mathbf{X}_B, t)) \mathbf{I}$$

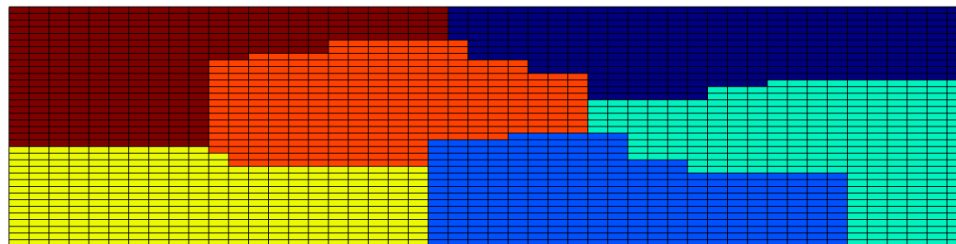


6 processors

Solid



Fluid

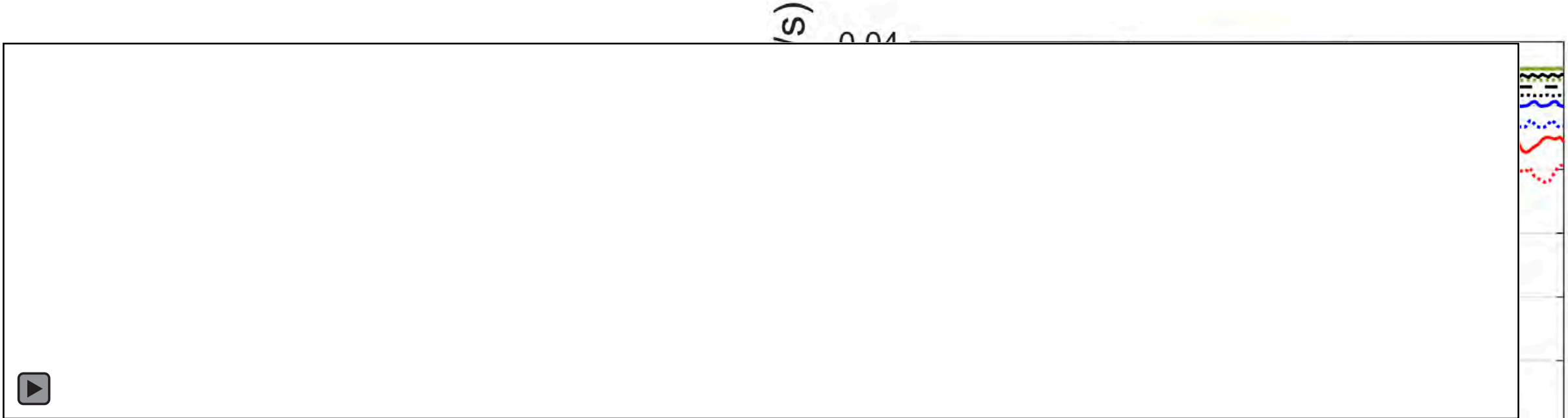


Algorithm 1 C^f Parallel Matrix Assembly

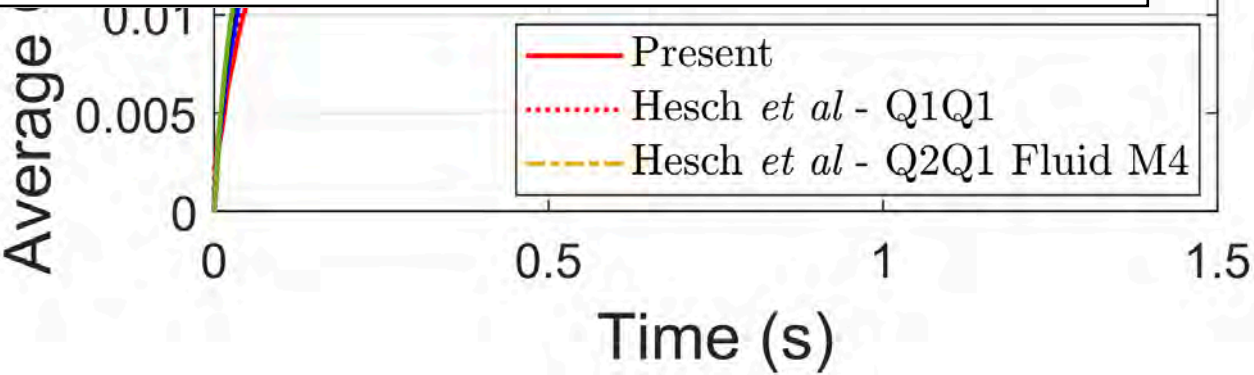
- 1: **Given:** Set of solid nodes $\{x_A\}_{A \in \omega_i}$ on processor i
 - 2: **Start point search from processor i**
 - 3: **Found point on processor j**
 - 4: **for** each point found on processor j **do**
 - 5: Store POINT data = {parametric position ξ_A , element number, and processor number j }
 - 6: **end for**
 - 7: **Transfer POINT data on processor i to target processor j**
 - 8: **On target processor j :**
 - 9: **for** each received point **do**
 - 10: Evaluate fluid shape functions for the containing element at the point
 - 11: **if** Fluid DOF is owned by this processor **then**
 - 12: Store value in the parallel matrix
 - 13: **else**
 - 14: Store DOF data = {value, global row, global column, owning processor}
 - 15: **end if**
 - 16: **end for**
 - 17: **Transfer DOF data to owning processors**
 - 18: **On owning processor:**
 - 19: **for** each received DOF **do**
 - 20: Store value in the parallel matrix
 - 21: **end for**
-

FREE FALLING CYLINDER

Hesch et al. CMAME. 2012



Mesh	# of elements	$h(m)$	h_{fluid}/h_{solid}
Solid	1,600	0.0001429	—
Fluid M1	1,728	0.0019245	13.468
Fluid M2	6,912	0.0009623	6.734
Fluid M3	13,248	0.0004811	3.367
Fluid M4	110,592	0.0002405	1.683



$\Delta t = 1 \times 10^{-3} \text{ s}$

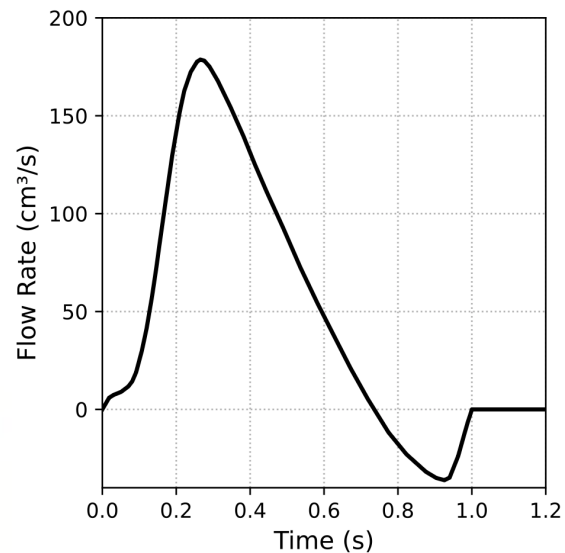
3D VALVE FSI

Fluid:

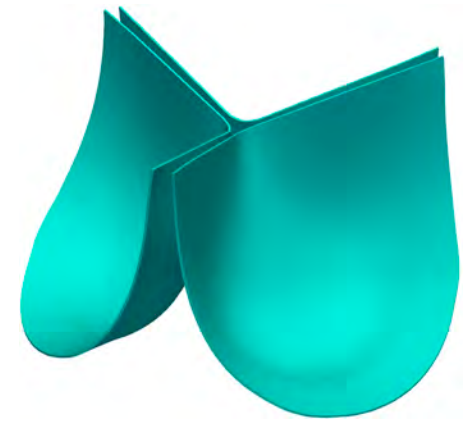
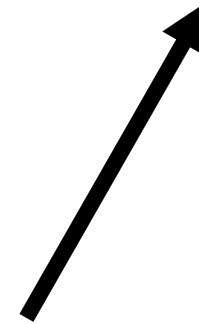
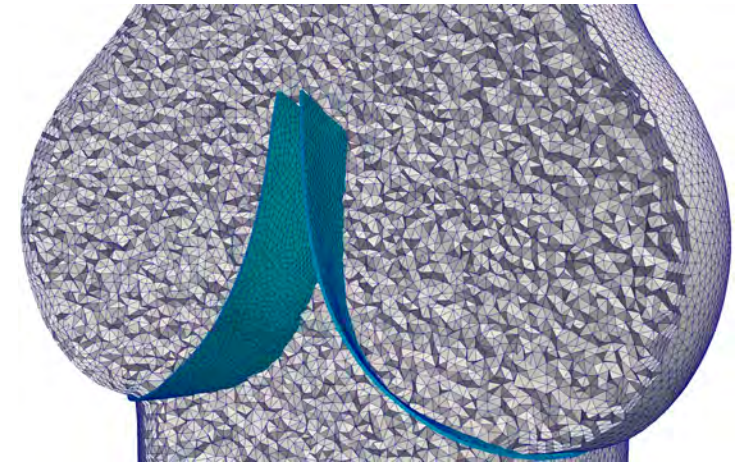
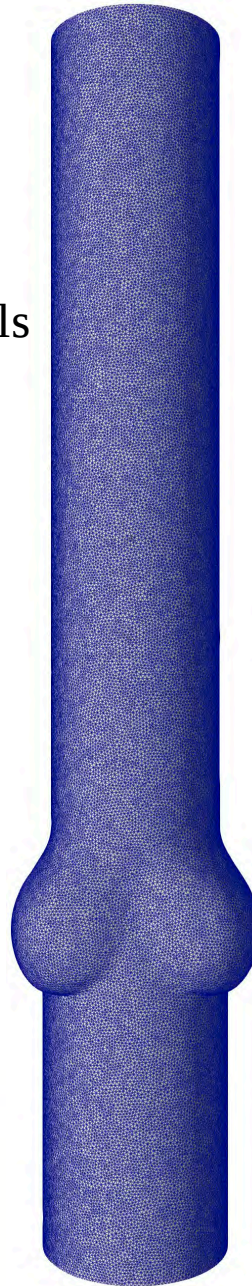
Newtonian Fluid

$$\rho^f = 1.1 \text{ g/cm}^3$$

$$\mu^f = 0.036 \text{ dyn s/cm}^2$$



Rigid No-Slip Walls



Valve:

Compressible Neo Hookean

$$\psi = \frac{\mu^s}{2} (\text{tr}(\mathbf{C}) - 3) - \mu^s \ln J + \frac{\lambda^s}{2} (\ln J)^2$$

$$\rho_0^s = 1 \text{ g/cm}^3$$

$$E = 2 \times 10^6 \text{ dyn/cm}^2$$

$$\nu = 0.4$$

3D VALVE FSI



Fluid:

- 8.5M tetrahedral elements
- 5.6M DOFs

Valve:

- 63k hexahedral and pentahedral elements
- 215k DOFs

Simulation:

- 1 heartbeat
- **32.5 hrs on 256 CPU cores**

Black R. T. et al. An open-source computational framework for immersed fluid-structure interaction modeling using FEBio and MFEM. *Engineering with Computers*. 2026.

CURRENT EFFORTS

CPU

GPU



Simulation:

- 1 heartbeat
- **32.5 hrs on 256 CPU cores**



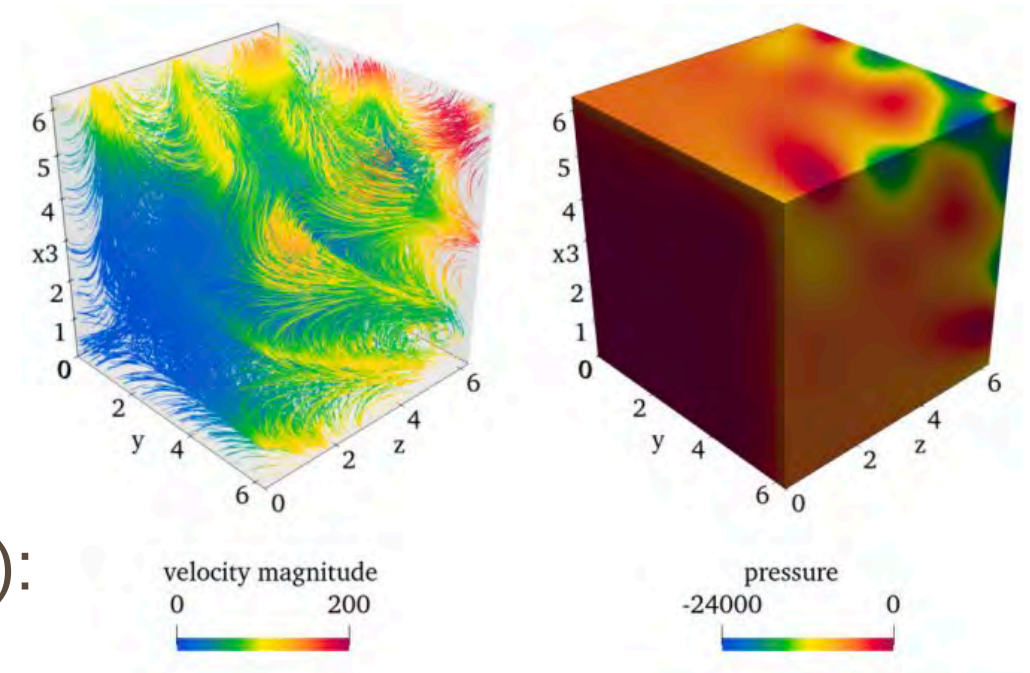
Roadmap:

- Discretization:
 - Full assembly → Partial assembly
 - GPU Kernels
 - Fluid (VMS) residual + tangent
 - Immersed solid residual + tangent
 - FSI coupling term
- Linear Solve:
 - Matrix-free velocity-pressure block system
 - Suitable block preconditioners

BELTRAMI FLOW: BENCHMARK

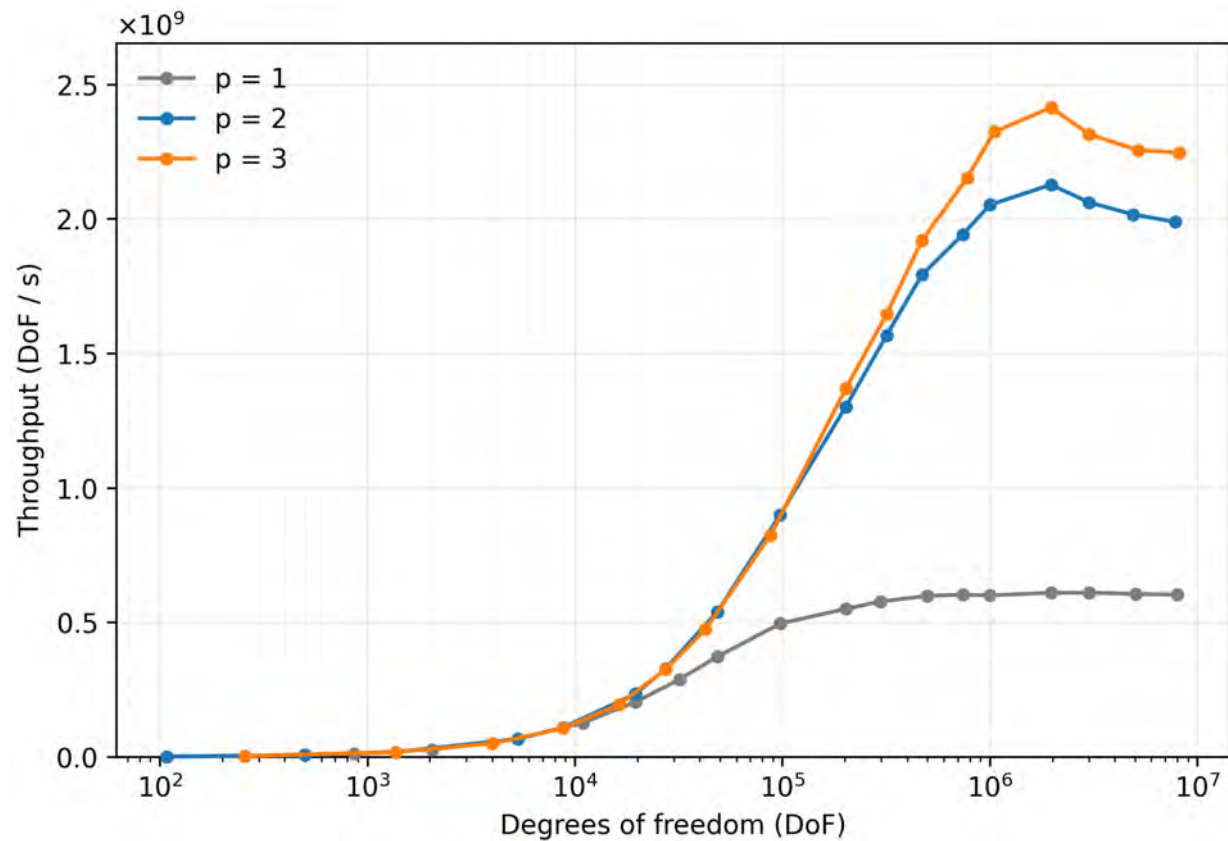
Ross Ethier and Steinman. IJNMF. 1994

- Exact 3D solution for incompressible Navier-Stokes
- Benchmark fluid VMS operators across problem size and order
- Measure operator throughput (DoF/s):
 - Residual Evaluation
 - Tangent Application



Africa et al. Comput Phys Comm. 2024

BELTRAMI FLOW: RESIDUAL EVALUATION



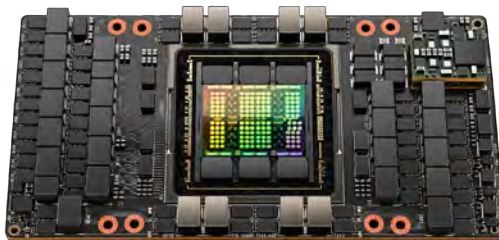
5M DoF	Speedup		
	CPU FA / CPU PA	CPU PA / GPU PA	CPU FA / GPU PA
p=1	8.6×	7.5×	64.6×
p=2	11.9×	15.2×	180.9×
p=3	16.8×	17.6×	295.5×

Algorithmic

Hardware

Overall

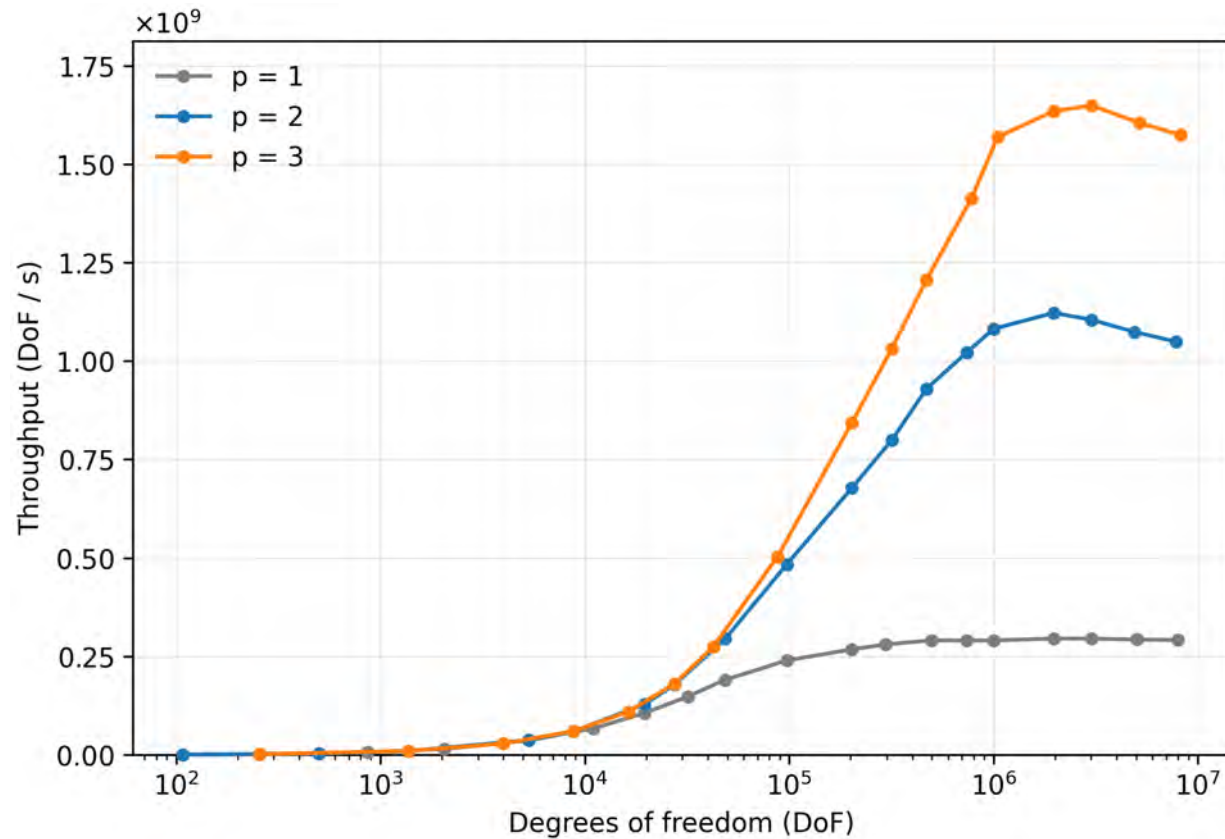
Nvidia H100
PCIe



AMD EPYC 7763
64-Core



BELTRAMI FLOW: TANGENT APPLICATION



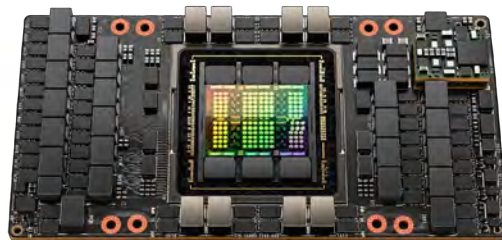
5M DoF	Speedup		
	CPU FA / CPU PA	CPU PA / GPU PA	CPU FA / GPU PA
p=1	0.5×	5.5×	2.8×
p=2	2.4×	9.8×	23.9×
p=3	6.7×	10.0×	67.5×

Algorithmic

Hardware

Overall

Nvidia H100
PCIe

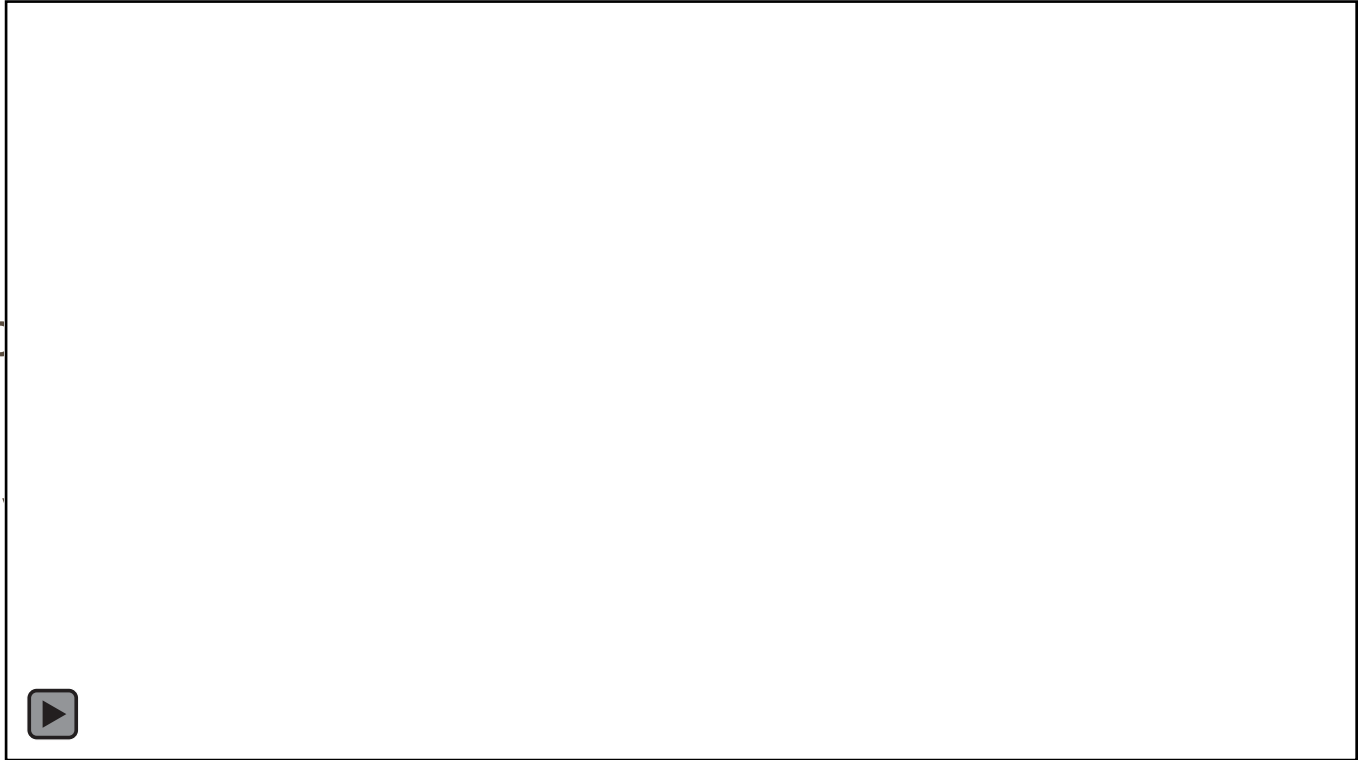


AMD EPYC 7763
64-Core



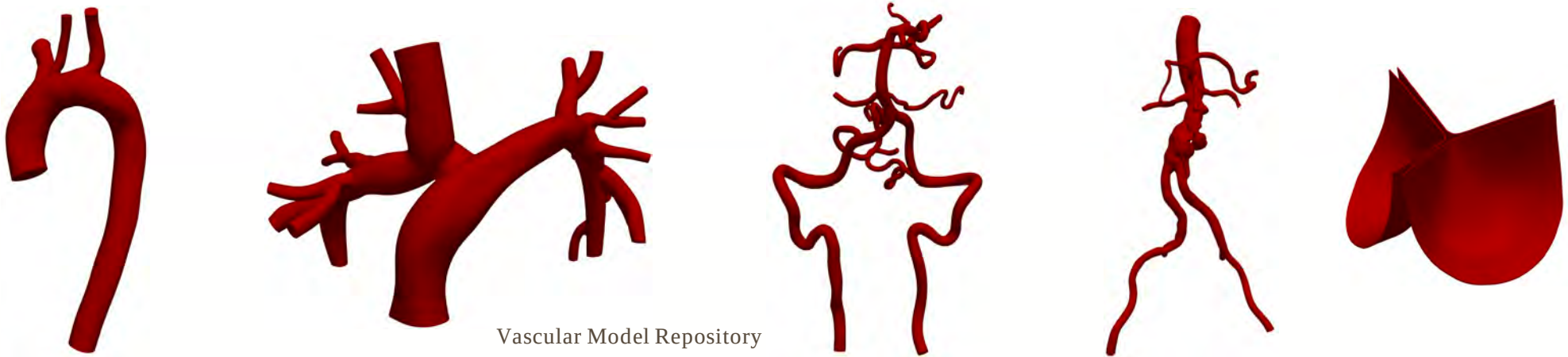
SUMMARY

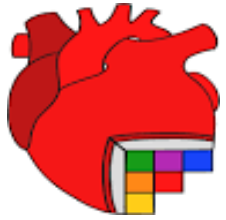
- Challenges with heart valve treatments
- Computational modeling can inform patient-specific valve repair
- Open-source immersed FSI solver
 - Method
 - Implementation
 - Examples
 - GPU acceleration (ongoing)



FUTURE WORK

- Integration with ∂FEM
 - Differentiable FSI code
 - Provide gradients of simulation outputs wrt simulation inputs
 - Support: training neural surrogates, UQ, optimization
- Recent developments with simplices and partial assembly
 - Tetrahedral meshing for complex cardiovascular structures





SlicerHeart

<https://slicerheart.org/>



FEBio

<https://febio.org/>



FEBio MFEM iFSI Solver